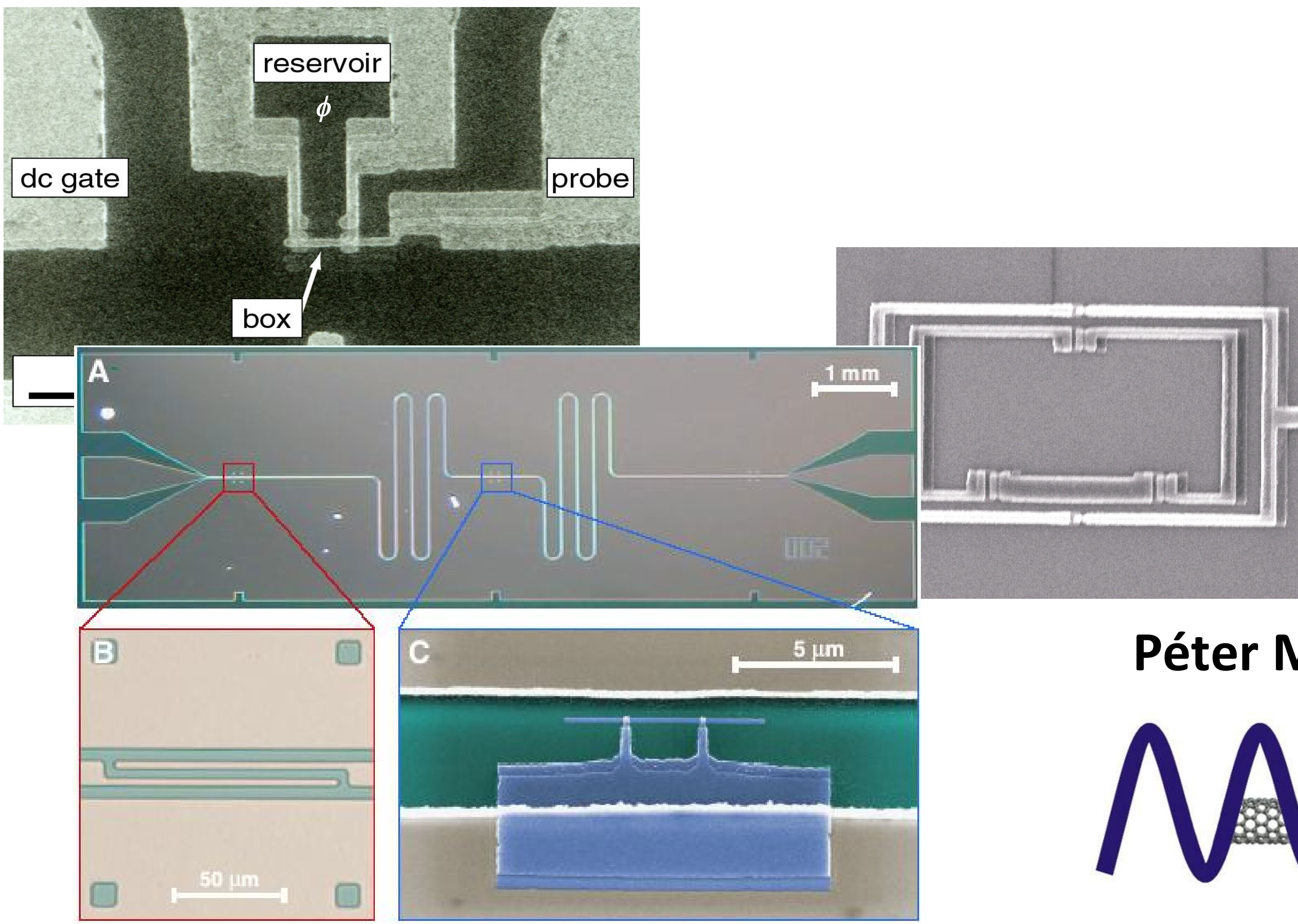
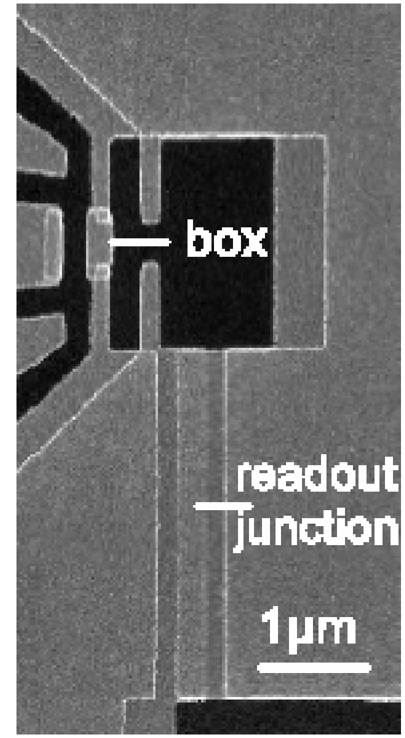
Superconducting quantum bits





Péter Makk

Qubits

Qubit = quantum mechanical two level system

DiVincenzo criteria for quantum computation:

1. Register of 2-level systems (qubits), $n = 2^N$ states: eg. |101..01> (N qubits)
2. Initialization: e.g. setting it to |000..00>
3. Tools for manipulation: 1- and 2-qubit gates, e.g. Hadamard gates: $U_H|0\rangle = (|0\rangle + |1\rangle)/2$,
and CNOT gates to create entangled states, $U_{CNOT}U_H|00\rangle = (|00\rangle + |11\rangle)/2$
4. Read-out : $|\psi\rangle = a|0\rangle + be^{i\Phi}|1\rangle \rightarrow$ a, b
5. Long decoherence times: > 10^4 2-qubit gate operations needed for error correction to maintain coherence ”forever”.
6. Transport qubits and to transfer entanglement between different coherent systems (quantum-quantum interfaces).
7. Create classical-quantum interfaces for control, readout and information storage.

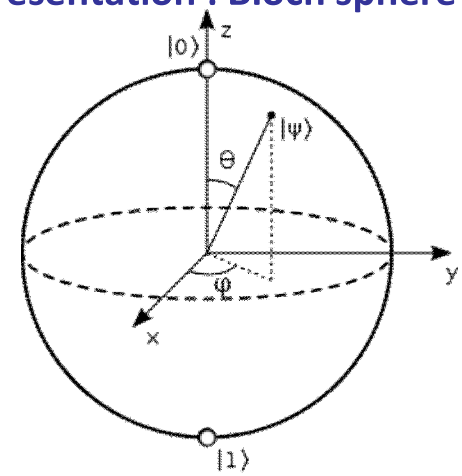
Two level system (quasi-spin)

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

basis, general state by rotation:

$$|\psi\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)e^{i\Phi}|1\rangle$$

Representation : Bloch sphere



Qubits

N-qubit states

$$|\psi\rangle = |\psi_1\rangle|\psi_2\rangle \ldots |\psi_N\rangle = |\psi_1\psi_2 \ldots \psi_N\rangle$$

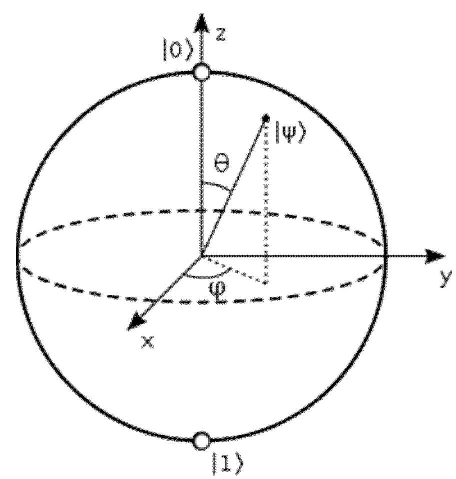
$$|\psi\rangle = c_1|0\ldots00\rangle + c_2|0\ldots01\rangle + \cdots + c_n|1\ldots11\rangle$$

Gates: Unitary operators

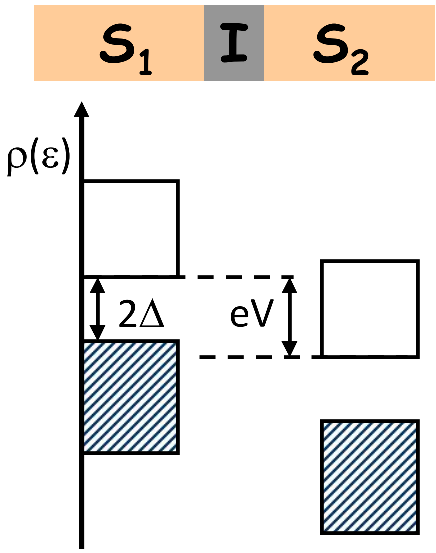
All operations are possible with one-qubit and one two-qubit gates
e.g. CNOT: first control bit, second not

$$\begin{aligned} |0\rangle\,|0\rangle &\to |0\rangle\,|0\rangle \\ |0\rangle\,|1\rangle &\to |0\rangle\,|1\rangle \\ |1\rangle\,|0\rangle &\to |1\rangle\,|1\rangle \\ |1\rangle\,|1\rangle &\to |1\rangle\,|0\rangle. \end{aligned}$$

$$U_{\mathrm{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$



Josephson junctions



$$I = I_c \sin(\delta)$$
$$V = \frac{\Phi_0}{2\pi}\frac{d\delta}{dt}$$

Josephson equations

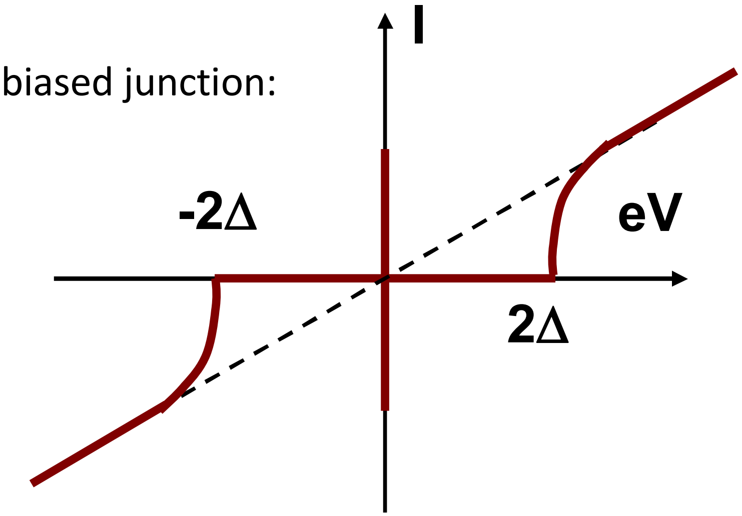
$$\Phi_0 = \frac{h}{2e}$$
$$I_c = \frac{\pi\Delta}{2eR_n}\tanh(\Delta/2T)$$

Applying a constant bias voltage:

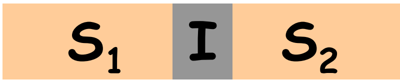
$$I = I_c \sin(\delta_0 + \frac{2eVt}{\hbar})$$

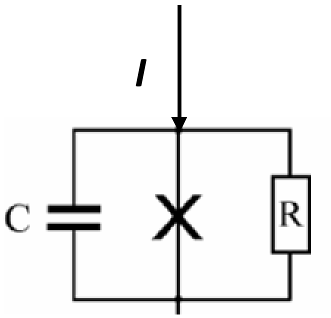
An AC current with ω=2eV/ħ is flowing.
The DC current averages to zero.

Voltage biased junction:



Josephson junctions





Equation of motion

$$I = C\frac{dV}{dt} + \frac{V}{R} + I_c \sin(\delta)$$

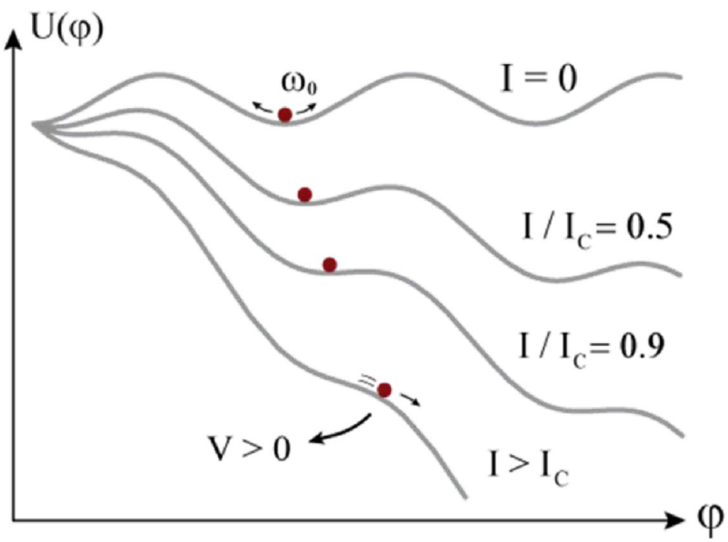
$$V = \frac{\Phi_0}{2\pi}\frac{d\delta}{dt}$$

$$0 = C\frac{\Phi_0}{2\pi}\frac{d^2\delta}{dt^2} + \frac{\Phi_0}{2\pi}\frac{1}{R}\frac{d\delta}{dt} - \frac{d}{d\delta}[I_c \cos(\delta + I\delta)]$$

$\updownarrow$

$$F(t) = m\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} - \frac{dU(x)}{dx}$$

„washboard potential”



$$U(\delta) = -\frac{\Phi_0}{2\pi}[I_c \cos(\delta) + I\delta]$$ *potential*

$$\omega_p = \left(\frac{2\pi I_c}{C\Phi_0}\right)^2 [1 - (I/I_c)^2]^{1/4}$$

$$Q = \omega_p RC$$

plasma oscillations

for $I>I_c$ DC voltage appears

Josephson junctions

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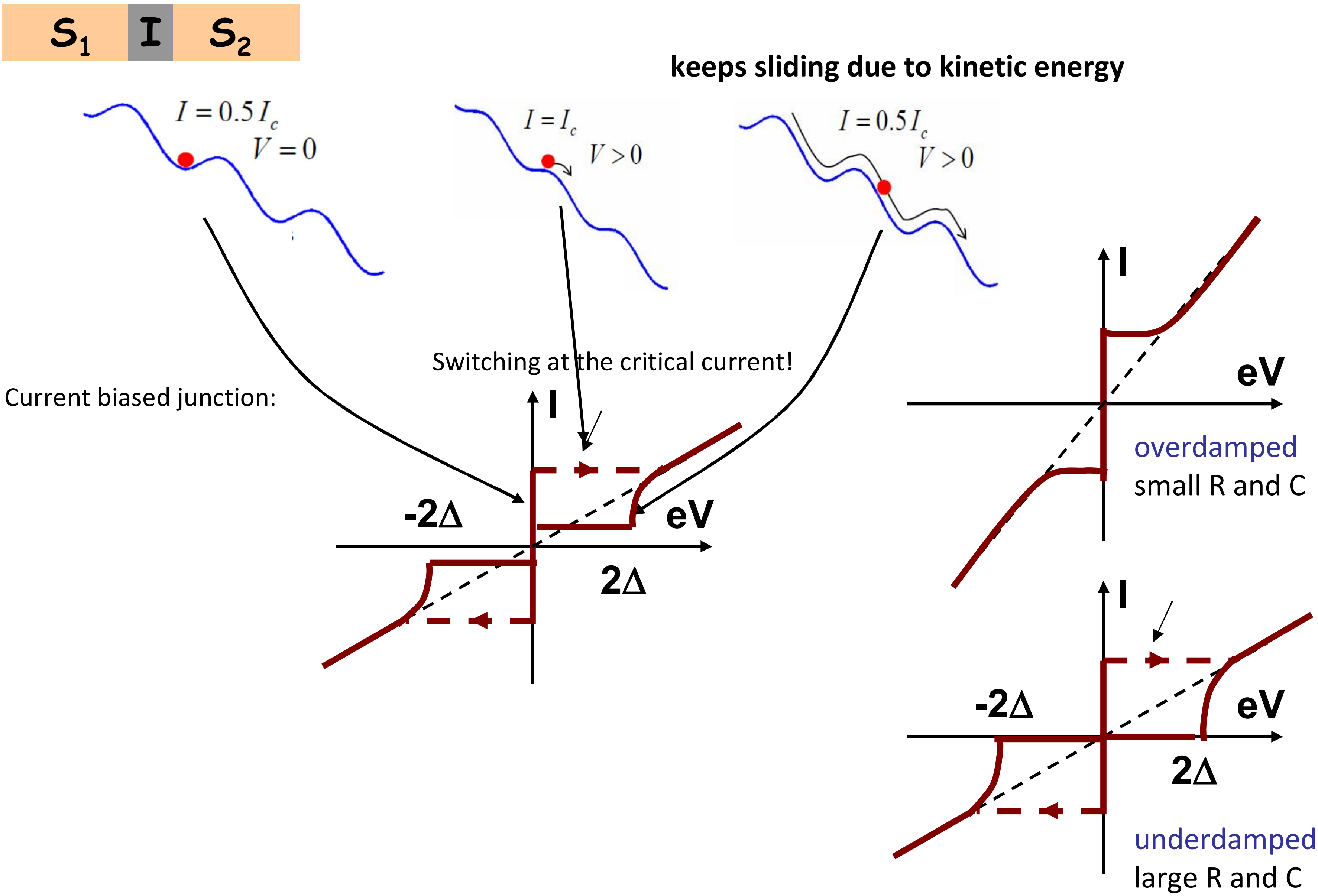
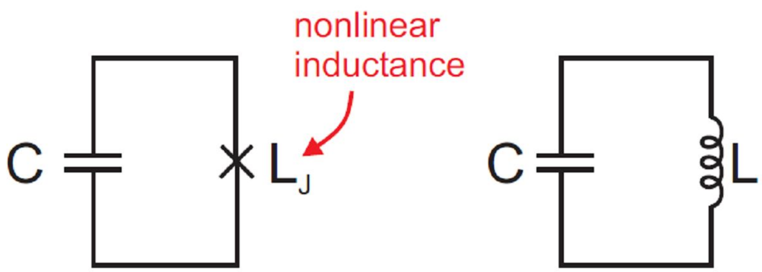


Josephson junctions



$$H = \frac{1}{2}CV^2 + \frac{1}{2}LI^2$$

resonant circuit

$$V = L\frac{dI}{dt} \longrightarrow \Phi = LI \qquad P_\Phi = CV = C\dot{\Phi}$$

$$H = \frac{1}{2C}p_\Phi^2 + \frac{1}{2L}\Phi^2$$

Josephson junctions is a *non-linear inductance*
the energy spectra is anharmonic

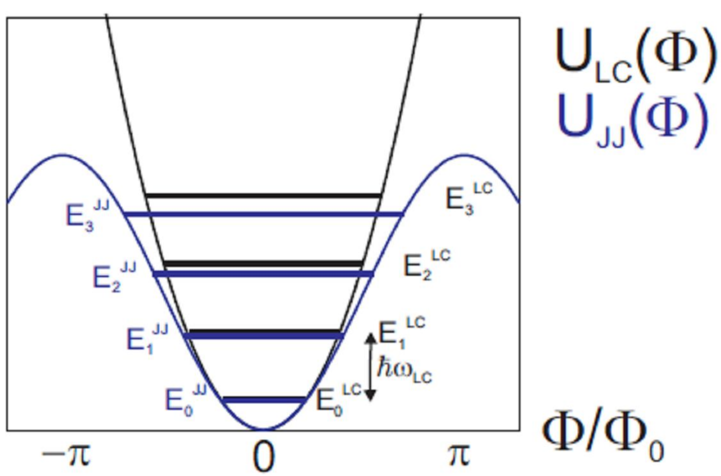
$$I = I_c \sin(2\pi\Phi/\Phi_0)$$

$$L_J = \frac{\Phi_0}{2\pi I_c}$$

for small Φ

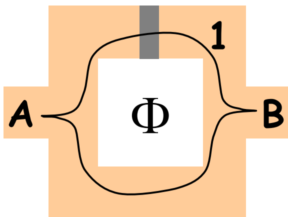
$$I = \frac{\Phi_0}{2\pi L_j} sin(2\pi\Phi/\Phi_0) \qquad I \simeq \frac{\Phi}{L_J}$$

$$H = \frac{1}{2}\frac{1}{C}p_\Phi^2 + \frac{(\Phi_0/(2\pi))^2}{L_J}\left(1 - \cos 2\pi\frac{\Phi}{\Phi_0}\right)$$



$\longrightarrow$ qubit is separateable

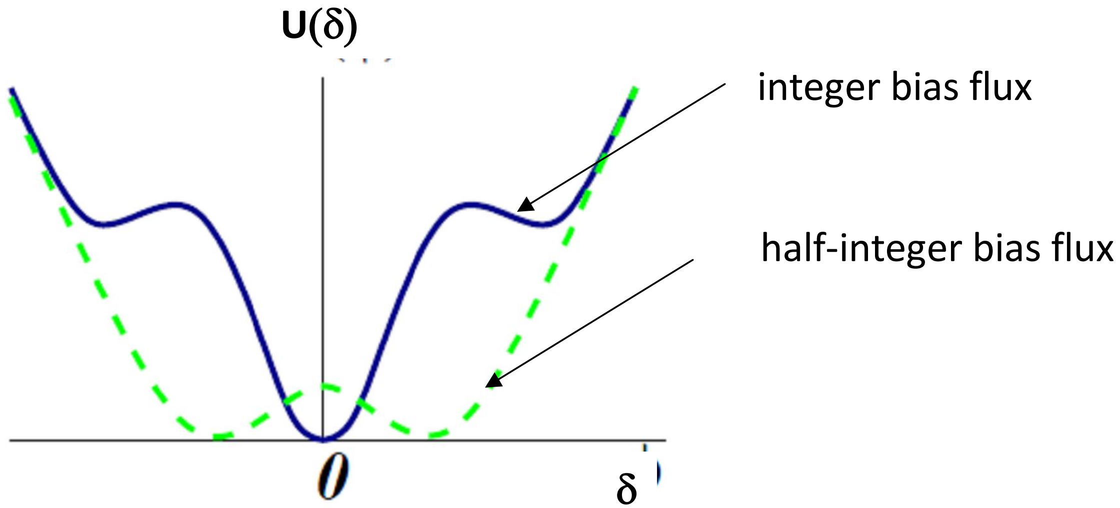
rf-squid



Equation of motion

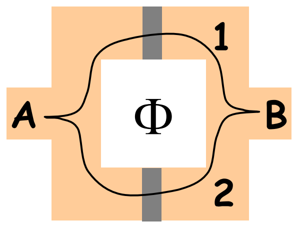
$$0 = C\frac{\Phi_0}{2\pi}\frac{d^2\delta}{dt^2} + \frac{\Phi_0}{2\pi}\frac{1}{R}\frac{d\delta}{dt} - I_c\sin(\delta) - \frac{1}{L}(\delta - \Phi)$$

$$U(\delta) = -\frac{\Phi_0}{2\pi}I_c\cos(\delta) + \frac{1}{2L}(\delta - \Phi)^2$$ *potential*



For half integer quantum, two minima:
two persistent current states, circulating in different direction

dc-squid

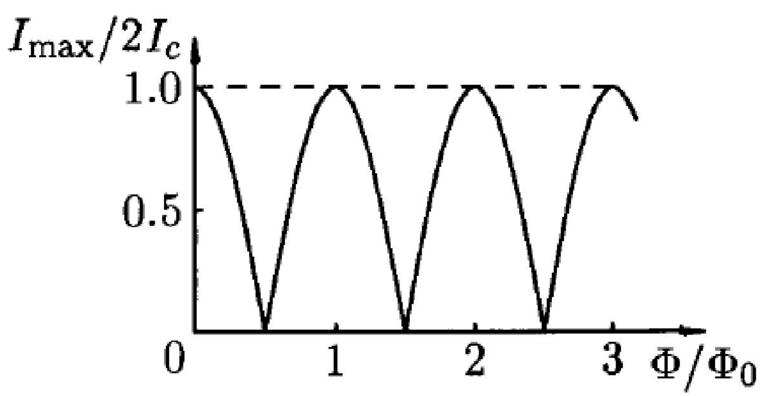


$$I = I_1 + I_2 = I_c\left[\sin\left(\delta_0 + 2\Phi/\hbar\right) + \sin\left(\delta_0 - 2\Phi/\hbar\right)\right] = 2I_c \sin\delta_0 \cos\left(e\Phi/\hbar\right)$$

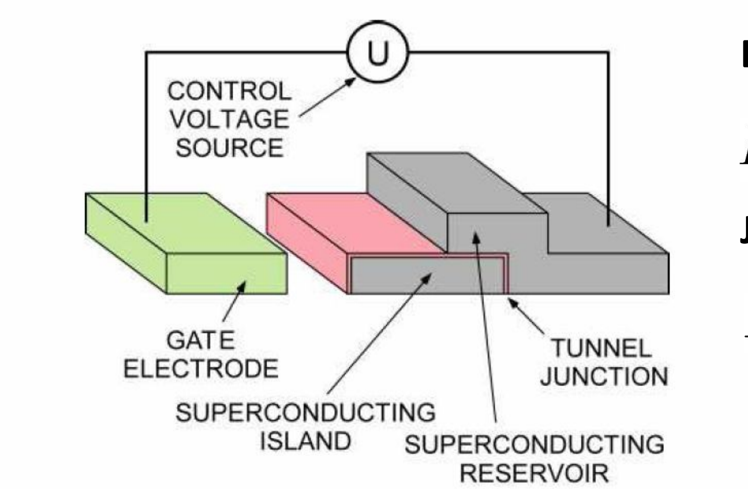
If the critical current of the two qbits are the same
The maximal value of the critical current is tuned by the magnetic flux: $I_{\max} = 2I_c\left|\cos\left(e\Phi/\hbar\right)\right|$

Dependence of the maximum supercurrent (critical current) through the two-junction interferometer on the total magnetic flux through its interior.

The strong dependence of Icon flux makes the DC-SQUID an **extremely sensitive flux detector**.



Cooper pair box

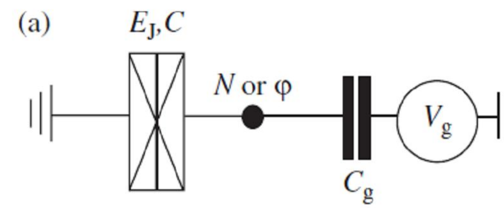


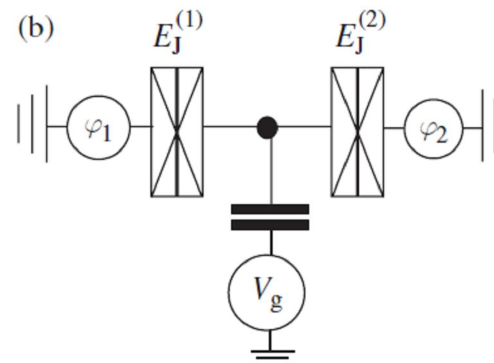
Electrostatic energy (like for a Qdot):

$$H_{\mathrm{el}} = E_C(N-n_g)^2 \qquad E_C = \frac{e^2}{2(C+C_g)}$$

Jospehson energy:

$$H_J = -E_J \cos\delta$$





For two josephson junctions:

$$-E_J^{(1)}\cos(\delta-\delta_1) - E_J^{(2)}\cos(\delta-\delta_2) = -Re\left[\left(\tilde{E}_J^{(1)}+\tilde{E}_J^{(2)}\right)\mathrm{e}^{i\delta}\right]$$

where

$$\tilde{E}_J^{(1,2)} = E_J^{(1,2)}\exp(-i\delta_{1,2})$$

$$E_J^{\mathrm{eff}}(\delta_1-\delta_2) = \left|\tilde{E}_J^{(1)}+\tilde{E}_J^{(2)}\right| = \sqrt{\left(E_J^{(1)}\right)^2+\left(E_J^{(2)}\right)^2+2\cos(\delta_1-\delta_2)E_j^{(1)}E_j^{(2)}}$$

The phase difference can be tuned by applying a magnetic field.
Thus the Josephson energy can be tuned

$$E_J^{\mathrm{eff}}(\Phi) = \sqrt{\left(E_J^{(1)}\right)^2+\left(E_J^{(2)}\right)^2+2\cos(2\pi\Phi/\Phi_0)E_j^{(1)}E_j^{(2)}}$$

Second quantization

Simplification: $\delta << \pi$

$$E_J \cos(\delta) \rightarrow \text{const} + E_J \delta^2/2$$

$I = -I_c \delta$ using the josephson energy

$\dot{I} = I_c 2e/\hbar V \longrightarrow L = V/\dot{I} = \hbar/I_c 2e$ Josephson inductance

Classical equations:

$$\frac{dQ}{dt} = -I(\delta) = -I_c \delta$$

$$\frac{d\delta}{dt} = \frac{2eV}{\hbar} = \frac{2eQ}{C\hbar}$$

$$\longrightarrow \quad \frac{dN}{dt} = \frac{2E_J}{\hbar}\delta \qquad \frac{d\delta}{dt} = \frac{2E_C}{\hbar} N$$

$$I_c = 2eE_J/\hbar$$

$$E_c = e^2/2C$$

Commutation relation

$$\left[\hat{N}, \hat{\delta}\right] = \alpha$$

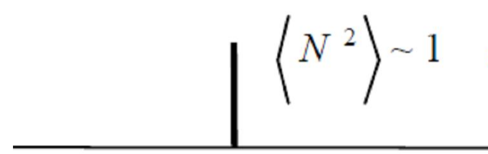
$$\hat{H} = E_J \frac{\hat{\delta}^2}{2} + E_C \frac{\hat{N}^2}{2}$$

$$\frac{dN}{dt} = \frac{i}{\hbar}\left[\hat{H}, \hat{N}\right] = -\frac{i\alpha E_J}{\hbar}\hat{\varphi}$$

$$\frac{d\delta}{dt} = \frac{i}{\hbar}\left[\hat{H}, \hat{\delta}\right] = \frac{i2\alpha E_C}{\hbar}\hat{\delta}$$

$$\longrightarrow \quad \boxed{\alpha = -2i}$$

phase



number

Phase and number are conjugated variables!

Phase representation (analogue: coordinate representation)

N operator and N eigenstates: $-2i\partial/\partial\delta$ and $N \to e^{iN\delta/2}$

$$e^{i\delta}|N\rangle = |N+2\rangle$$

$$\cos(\delta) = \sum_N |N\rangle\langle N+2| + |N+2\rangle\langle N|$$

in the other representation

$$\hat{N}|N\rangle = N|N\rangle$$

$$H_{\rm el} = E_C \sum (N-N_g)^2 |N\rangle\langle|N$$

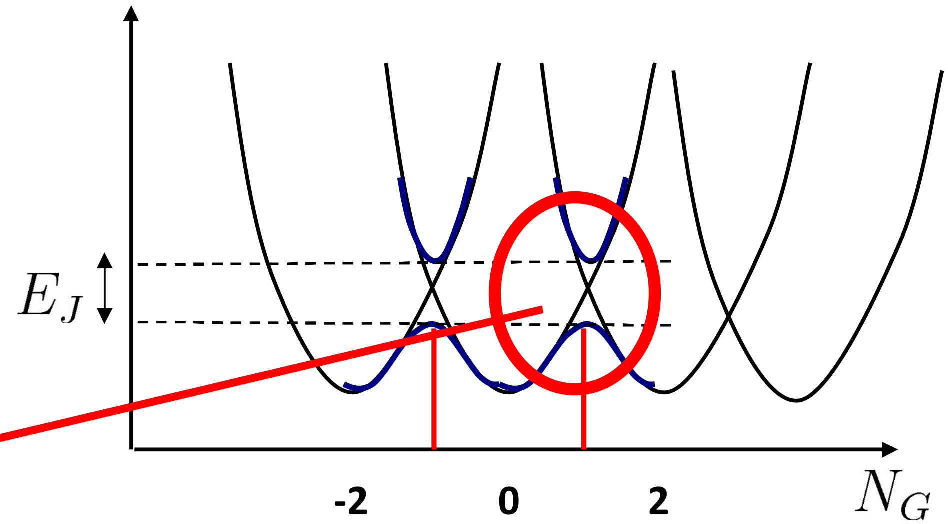
$$H_J = -\frac{E_J}{2}\sum_N \left(|N\rangle\langle N+2| + |N+2\rangle\langle N|\right)$$

transfer of Cooper pairs

Charge qubit

For $E_J << E_C$

The charging energy states are degenerate at the crossing point:
E_J makes a transition and lifts the degeneracy:



The charge qubit near the degeneracy point

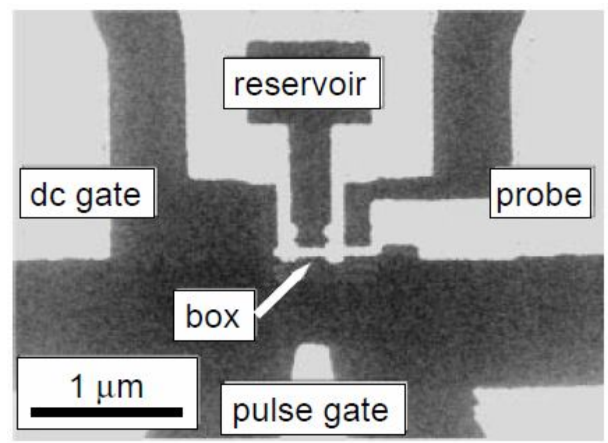
$$\hat{H} = E_C + \begin{pmatrix} \varepsilon & E_J/2 \\ E_J/2 & -\varepsilon \end{pmatrix}$$

where : $\varepsilon = 2E_C(1 - N_G)$

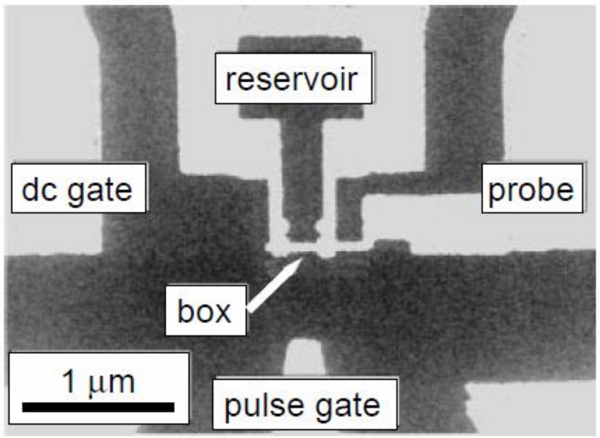
At the degeneracy point the eigenstates are trivial: $|\pm\rangle = (|0\rangle \pm |2\rangle)/\sqrt{2}$

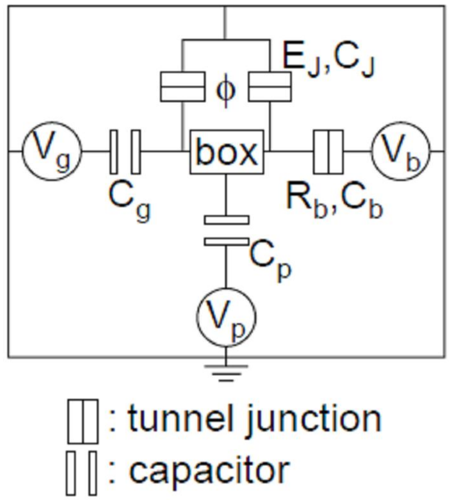
Far from the degeneracy points the original states are good eigenstates.

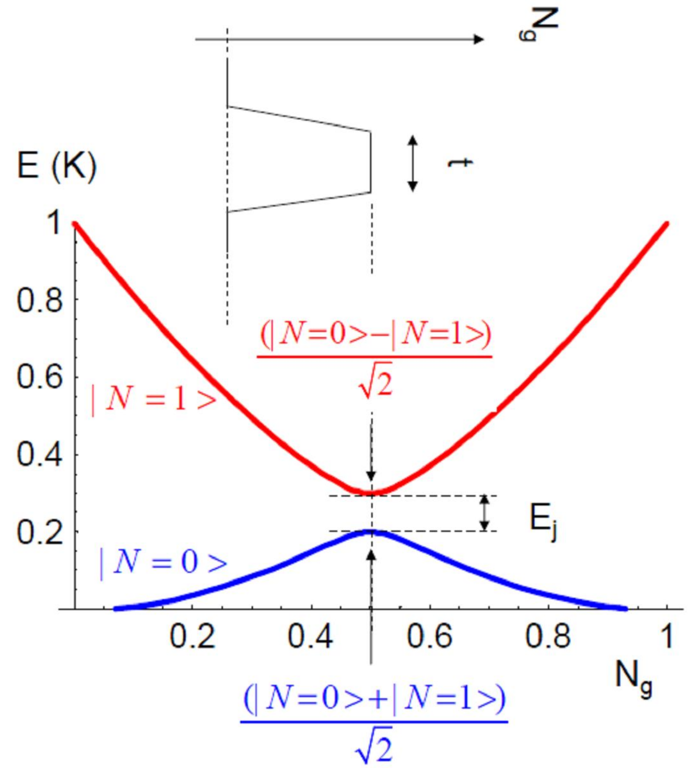
Higher order coupling through EJ are negligible



Charge qbit



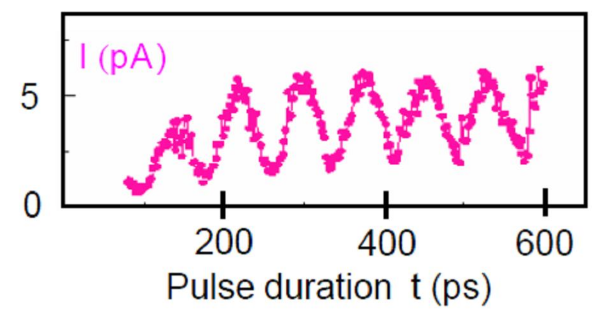




- First the qubit is prepared in state |0>
- Fast DC pulse to the gate → not adiabatic, it remains in |0>
- It starts Rabi-oscillating, and evolves during the pulse length (*t*)

- If after the pulse, the qubit is in |1> it will relax back and the current is detected (high repetition frequency) – **single shot readout**
- Detection: working point for state 1 is above delta in energy →decay to quasi particles

- By adjusting *t*, the length of the pulse Rabi oscillation is seen
- DC gate is need to calibrate the pulse gate
- Relaxation <10ns, probably due to charge fluctuations



Y. Nakamura et al., Nature, ***398****, 786 (1999)*

Rabi oscillations

Oscillations for not energy eigenstates

energy eigenstates

$$\begin{pmatrix}1\\0\end{pmatrix} = \alpha\begin{pmatrix}\alpha\\\beta\end{pmatrix} + \beta\begin{pmatrix}\beta\\-\alpha\end{pmatrix} \xrightarrow{\hat{U}(t)}$$

$$\alpha\begin{pmatrix}\alpha\\\beta\end{pmatrix} e^{-iE_+t/\hbar} + \beta\begin{pmatrix}\beta\\-\alpha\end{pmatrix} e^{-iE_-t/\hbar} = e^{-iE_+t/\hbar}\begin{pmatrix}\alpha^2+\beta^2 e^{-i\mathcal{R}t/\hbar}\\ \alpha\beta(1-e^{-i\mathcal{R}t})\end{pmatrix}$$

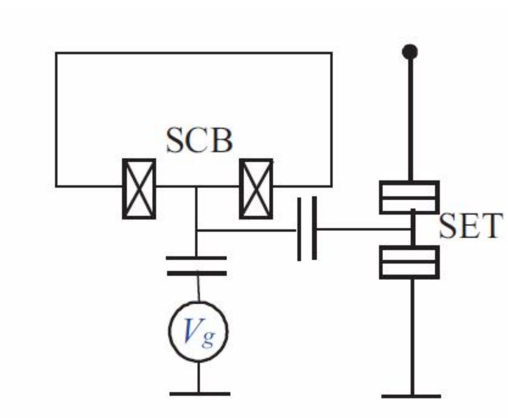
R is Rabi frequency:
$R=E_+-E_-$

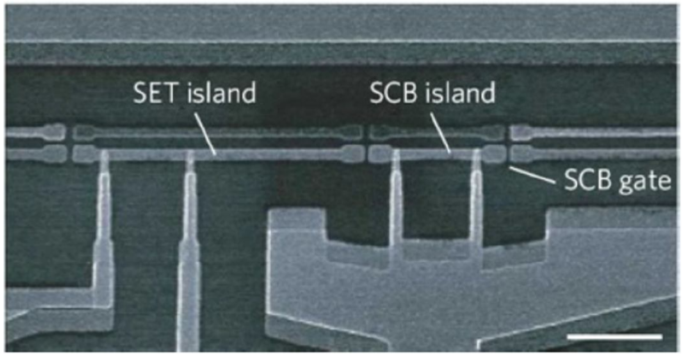
$$P_\uparrow(t) = \left|\alpha^2+\beta^2 e^{-i\mathcal{R}t/\hbar}\right|^2 = \left(\alpha^2-\beta^2\right)^2 + 4\alpha^2\beta^2\cos^2(\mathcal{R}t/2)$$

Probability of being in state (1,0) will oscillate in time

Special case: Spin ½ in magnetic field
If a=1/2 and b=1/2, the oscillation is maximal (spin precesses in the x-y plane)

Other readout for charge qbit: with SET





Relaxation - T_1 and T_2

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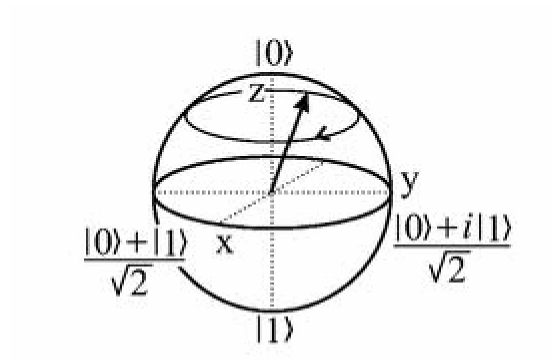
Larmor precession

$$\frac{d\mu}{dt} = \mu \times (\gamma H_0)$$ where the larmor frequency is: $$\omega_L = \gamma H_0$$

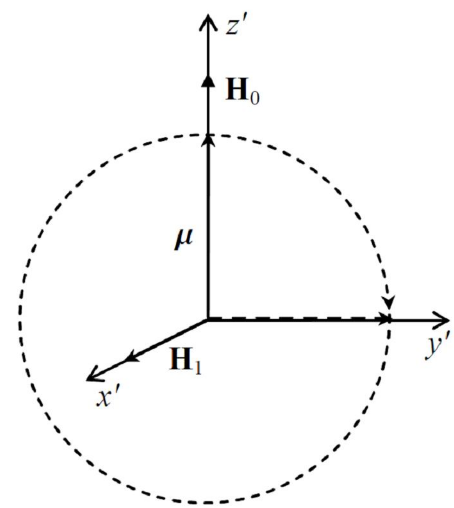


Adding a small modulating field

$$H(t) = (H_1 \cos(\omega t), H_1 \sin(\omega t), H_0)$$

In rotating frame with angular moment (0,0,ω) , H_1 is static: H_1'=(H_1, 0 , 0)

$$\frac{\delta\mu}{\delta t} = \mu \times \gamma[(H_0 + \omega/\gamma)\hat{z}' + H_1\hat{x}']$$



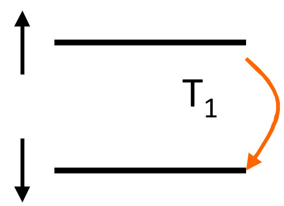
If $\omega=\omega_L$, than the magnetic moment will precess around H_1'
With short pulses, the moment can be rotated with arbitrary angle
E.g. $\pi/2$ pulse rotates to y' axis, a precession in the x-y plane in the lab frame

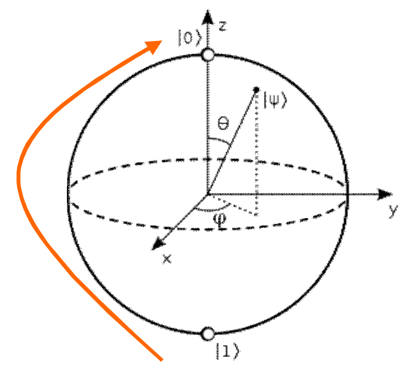
Relaxation – Bloch equations

$$\frac{dM_{x,y}}{dt} = \gamma[\mu \times H_0]_{x,y} - \frac{M_{x,y}}{T_2} \qquad \omega_L = \gamma H_0$$

$$\frac{dM_z}{dt} = \gamma[\mu \times H_0]_z - \frac{M_0 - M_z}{T_1}$$

T_1 (spin-lattice in NMR)



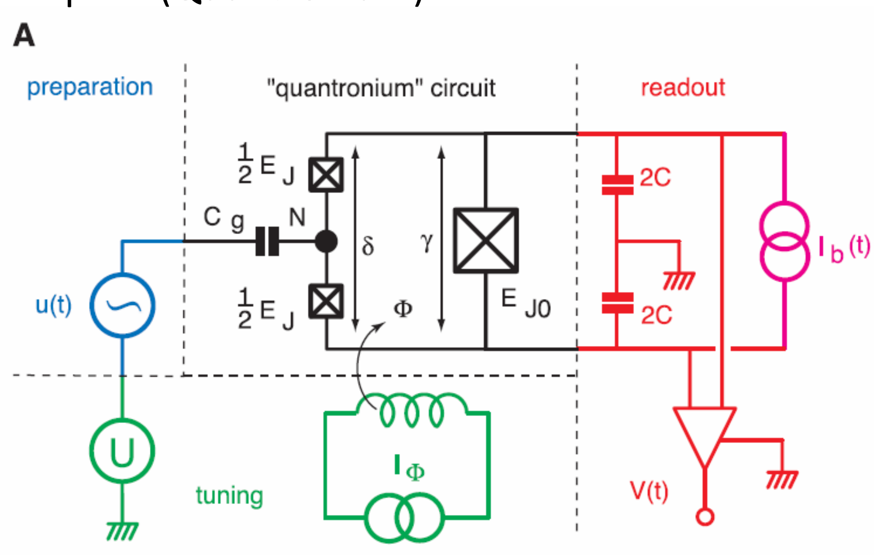


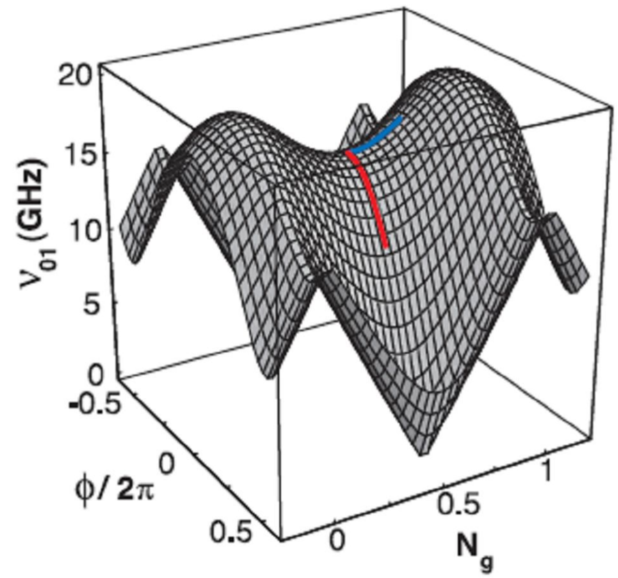
T_2 – decoherence time (spin-spin in NMR)

The time until a superposition preserves its phase
In NMR lot of spins, which precess with different larmor frequency, and the net spin disappears

Quantronium

-charge qubits operated at the degeneracy point are not sensitive to *charge* fluctuations (2nd order) but to *phase* fluctuations
- for charge-flux qubit there is also a sweet spot in the flux-charge plain (**Quantronium**)



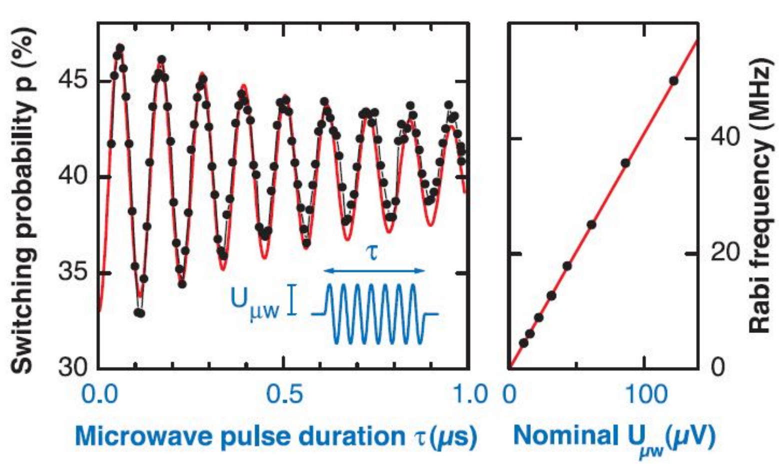


Quantronium is operated at the sweet point, where $I_b=0$.
Except the sweet point a net supercurrent flows, the direction depending on the state
Another, more transparent Josephson junction, with large E_J is added
For readout I_b is applied such, that $I_b+I_0<I_c$, but $I_b+I_1>I_c$, and finite voltage is measured

D. Vion et al., Science 296 (2002)

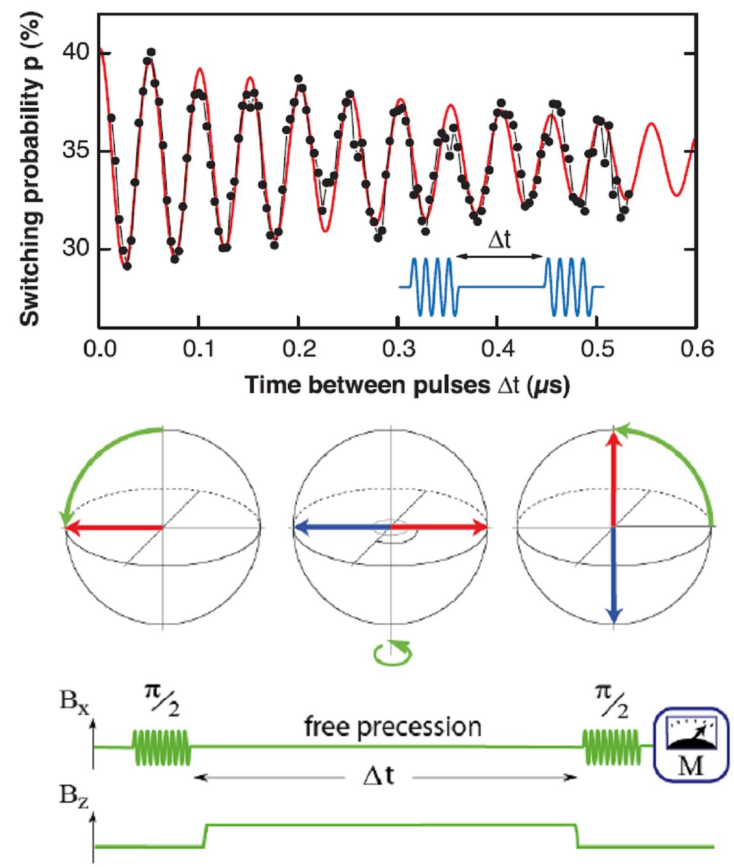
Quantronium

Rabi oscillations



Rotation for τ time, drive between the two states
→ after the rotation measurement
Decay of oscillations go with T_{Rabi}
Frequency depends on the microwave power

Ramsey oscillations



after free precession, π/2 rotates up or down, depending on the acquired phase
T2 follows from the decay of the oscillation

D. Vion et al., Science 296 (2002)

Quantronium

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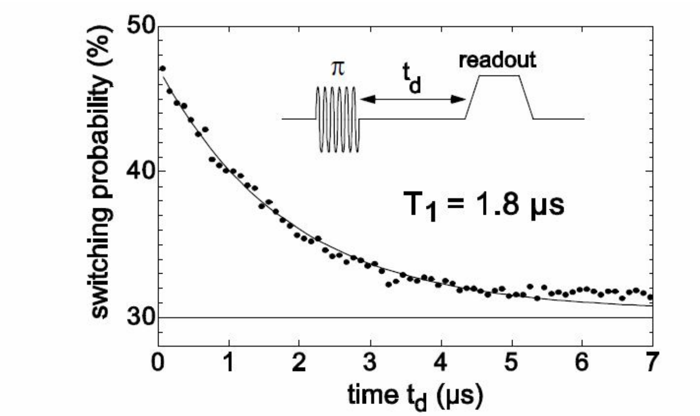
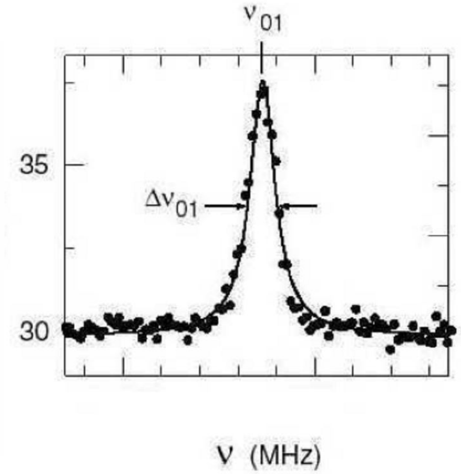
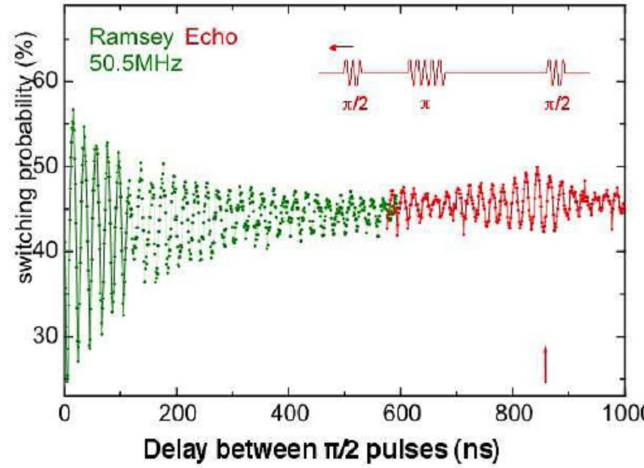
Measurement of T_1

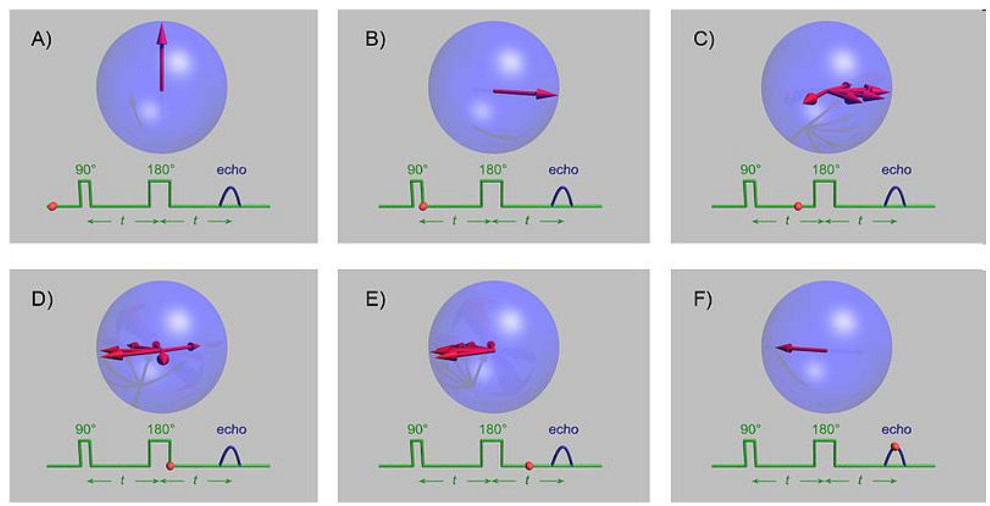




Spin-echo



- T_1 is the longest scale and it has also contribution to T_2
-With echo technique stationary inhomogenities can be filtered out
- from the resonance signal ~ T_2 can be deduced



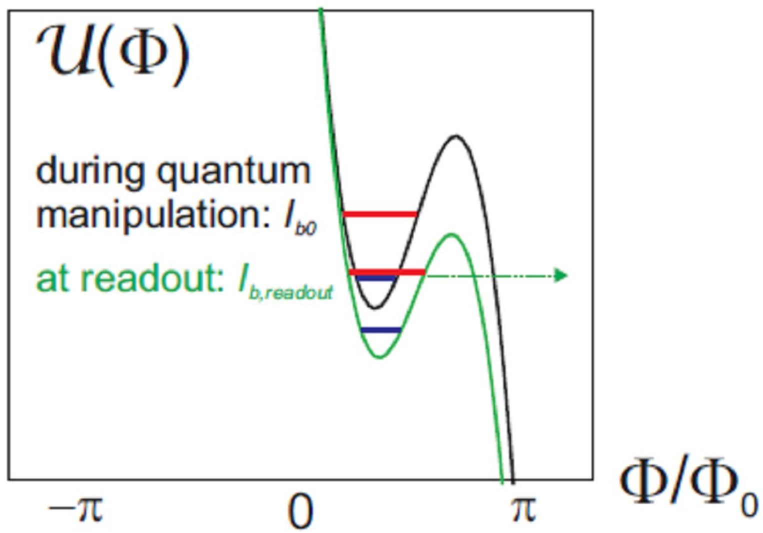
D. Vion et al., Science 296 (2002)
Grégoire Ithier PhD Thesis

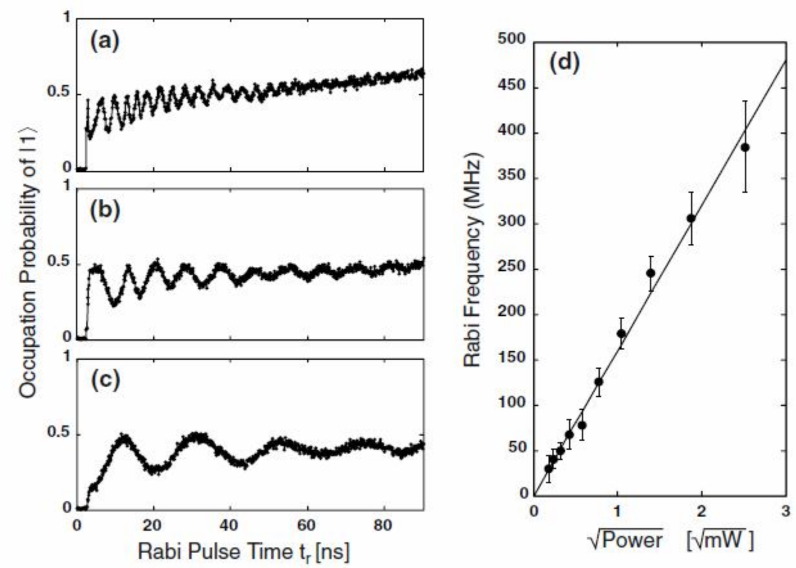
Phase qubit

BUTE, Low temperature
solid state physics laboratory

Tune the potential of a sing JJ (washboard potential) , such, that it is asymmetric and only houses 2-3 levels

$E_c << E_J$, $I < I_c$





Operation:

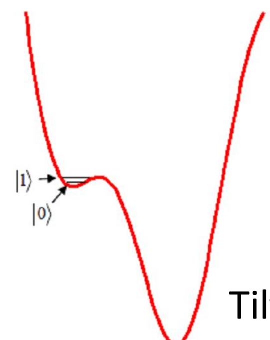
- Prepare eg. to state 1

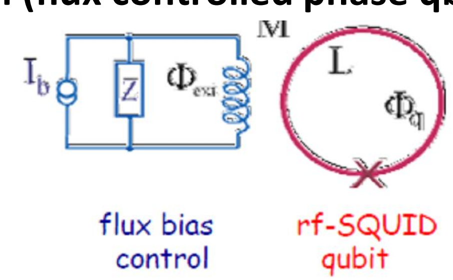
Readout:

- change I_c, such that 1 can tunnel out → finite voltage appears on the junction
- or swap 1-2, and 2 can tunnel out

Rabi oscillations etc…

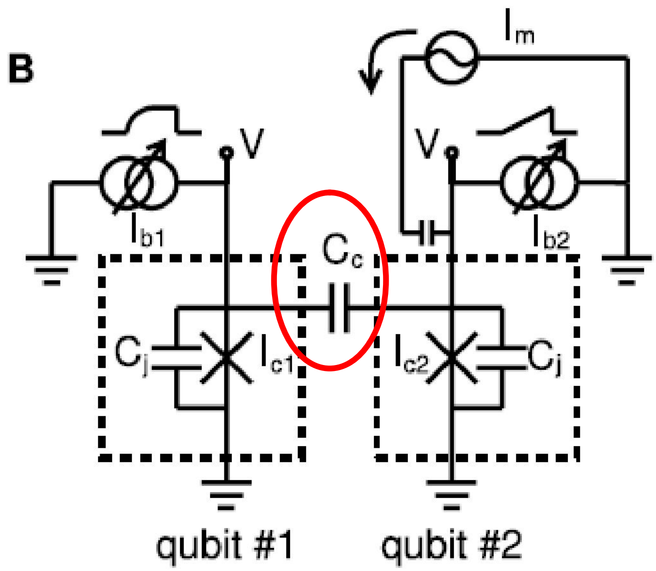
a bit different design (flux controlled phase qbit)





Tilt the potential such, that it only houses 1 or 2 levels

Coupled phase qubits



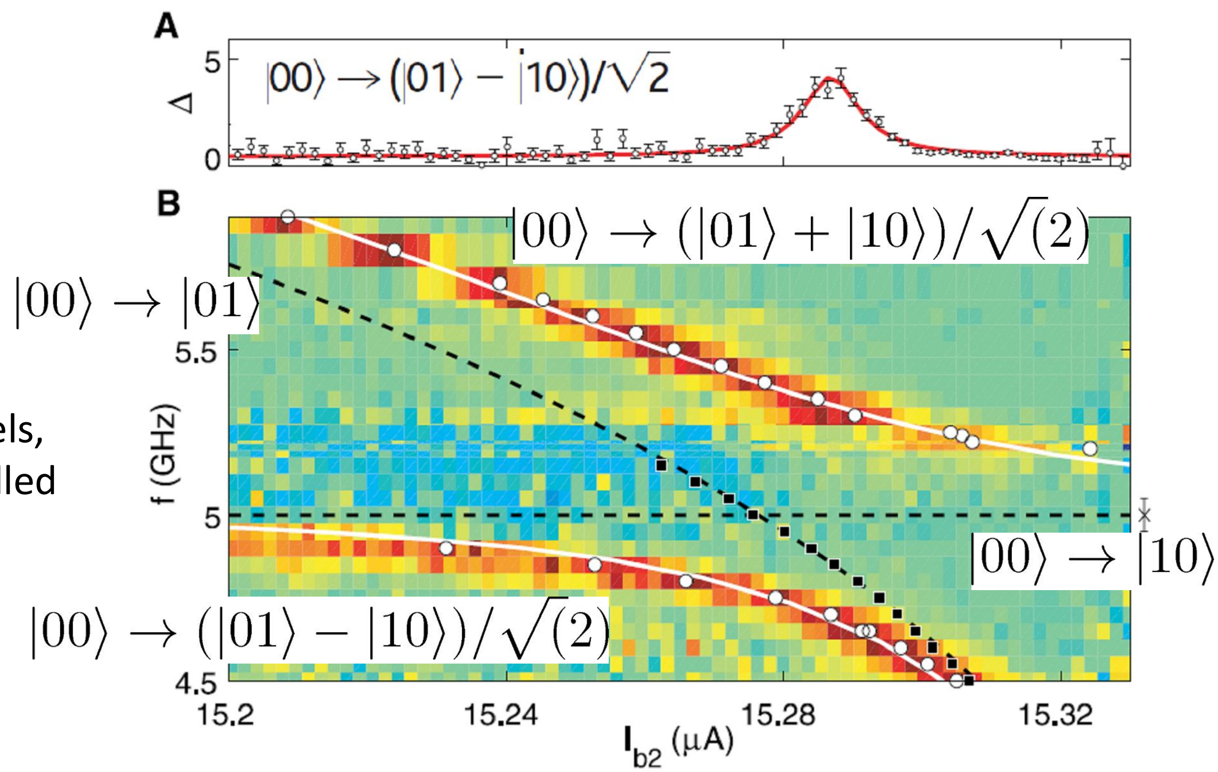
Capacitive coupling

- The levels (splitting) of the single qubits are tunable
-If the level spacing is nearly the same than the states of the two qubits hybridize
-First three levels |00>, |01> - |10> and |01> + |10>

Escape probability is measured

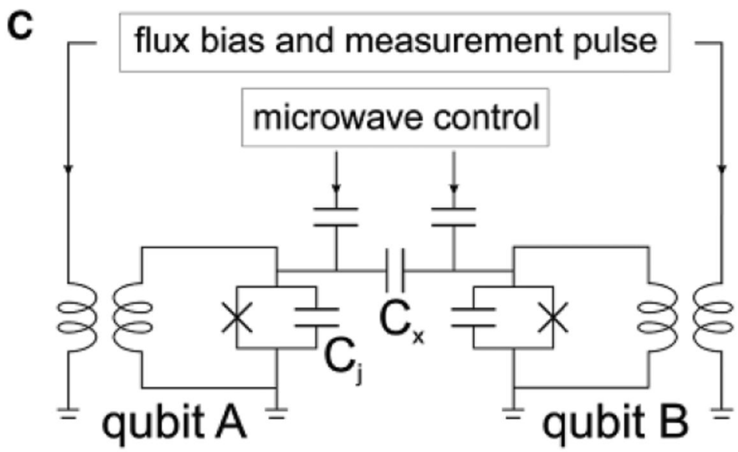
I_{b1} is fix, f is fix and I_{b2} is tuned
For fix f, I_{b2} will tune the coupled levels, and for some I_{b2} the condition is fulfilled

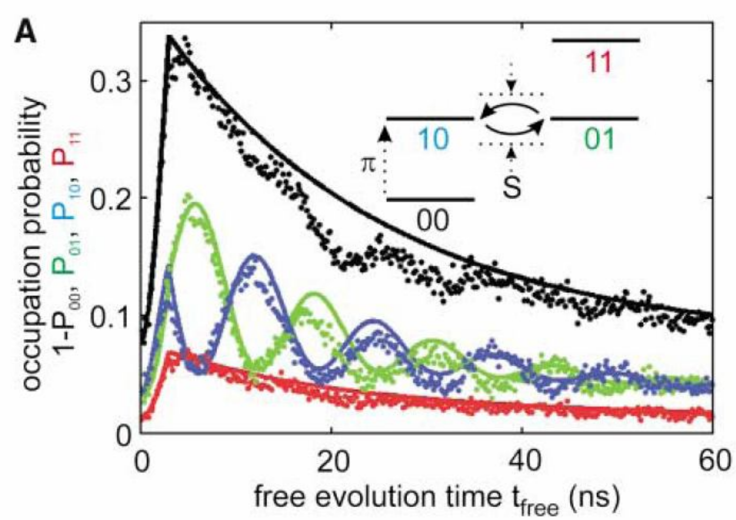
On the **f--$\mathbf{I_{b2}}$** map avoided crossing is seen→coupling



R. McDermott et al., Science 307 (2005)

Phase qubit





Readout of both qubits at the same time
Qbits are brought to resonance
At resonance |01> and |10> are not eigenstates
Prepare |00>, than excite qbit1 → |10>
This is not an eigenstate and start to precess
Measurement of the two qubits is anticorrelated
Coherence time is the same as for single qubit

$$|\psi(t)\rangle = \cos(St/2\hbar)|10\rangle + i\sin(St/2\hbar)|01\rangle$$

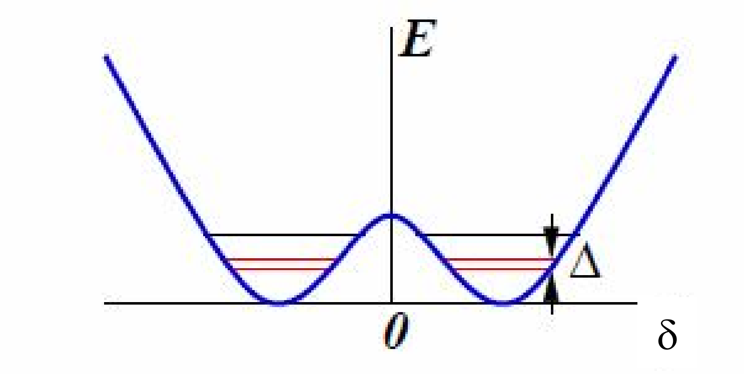
$$H_{int} = S/2(|10\rangle\langle 01|\rangle|01\rangle\langle 10|)$$

At t=hbar/2S, |10> goes to i| 01> → iSwap

$$U(\hbar/2S) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R. McDermott et al., Science 307 (2005)

Flux qbit



potential for half integer flux bias

$$U(\delta) = E_J(1-\cos(\delta)) + E_L(\delta-\Phi)^2/2$$
$$\sim E_L(-\varepsilon\tilde{\delta}^2/2 - f\tilde{\delta} + (1+\varepsilon)/24\tilde{\delta}^4)$$

where $\tilde{\delta} = \delta - \pi \quad f = \Phi - \pi$

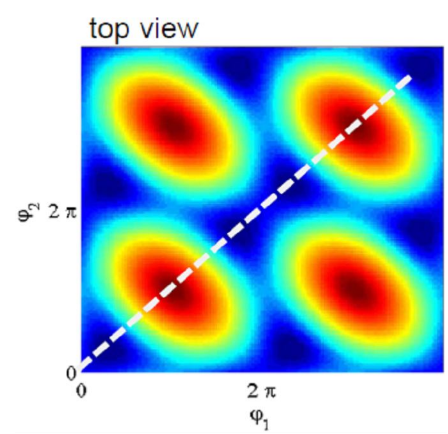
$$\hat{H} = -1/2(\epsilon\sigma_z + \Delta\sigma_x) \quad \varepsilon = E_J/E_L - 1 << 1$$
$$\epsilon = E_r - E_l$$

Two wells → two levels – for symmetric potential degenerate flux states
The two states have opposite persistent current
If tunneling in switched on (Δ), states hybridize and the macroscopic tunneling determines the separation

Hard to fabricate, big loop is needed for inductance matching (big decoherence)→ 3 JJ-s qbit

3-junction flux qubit

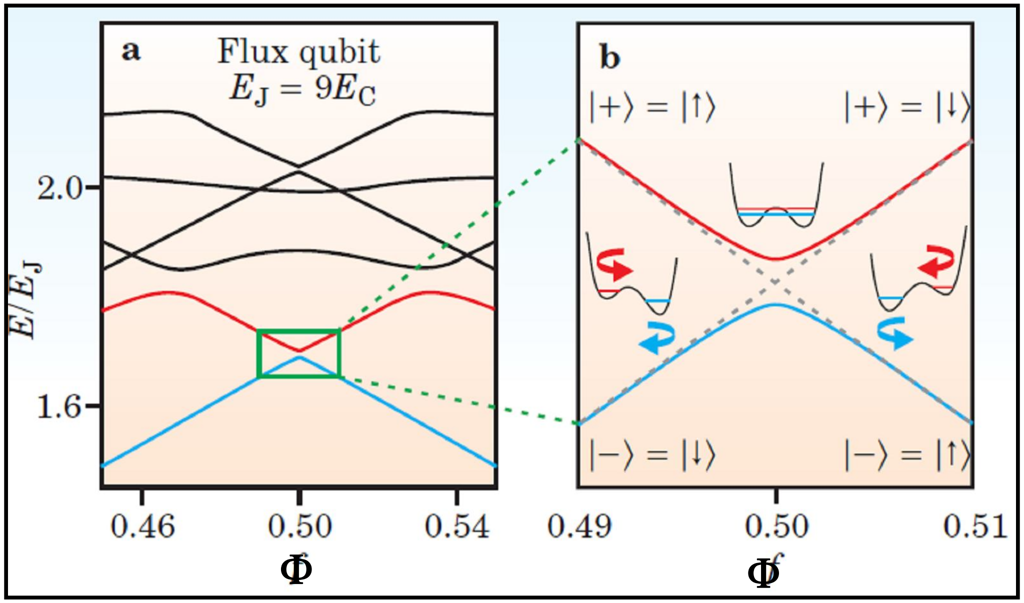
$$\varphi_1 + \varphi_2 + \varphi_3 + 2\pi\Phi/\Phi_0 = 2\pi n$$
$$U = -E_j\left[\cos(\varphi_1) + \cos(\varphi_2) + \alpha\cos(\varphi_1 - \varphi_2 - 2\pi\Phi_{ext}/\Phi_0)\right]$$

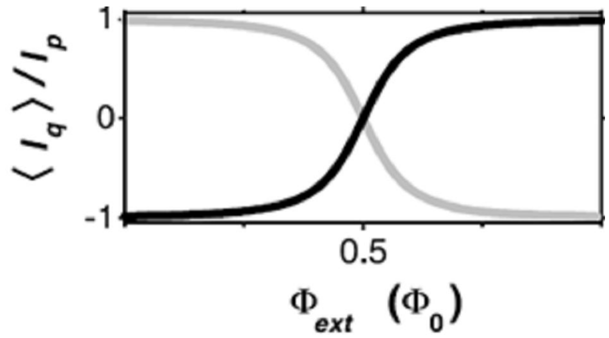


the potential is parabolic on the white intersection

I. Chiorescu et al., Science 299 (2003)

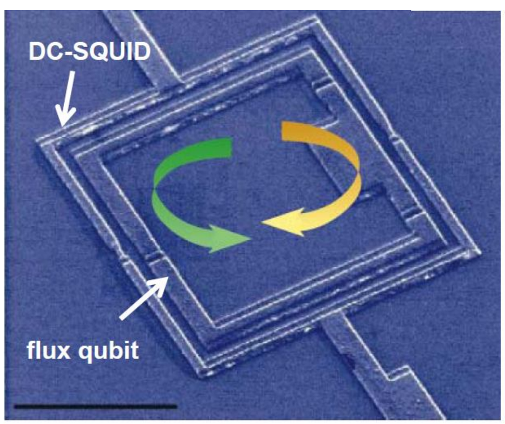
Flux qubit

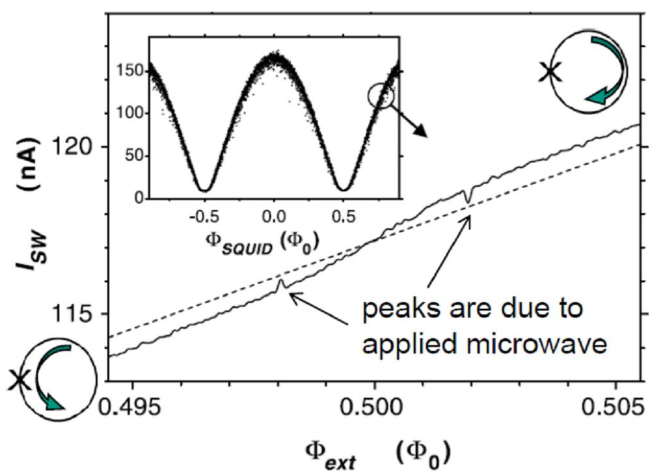




The current as a function of the flux
Away from half flux quanta, pure flux states

Detection – by DC squid measuring the opposite supercurrents

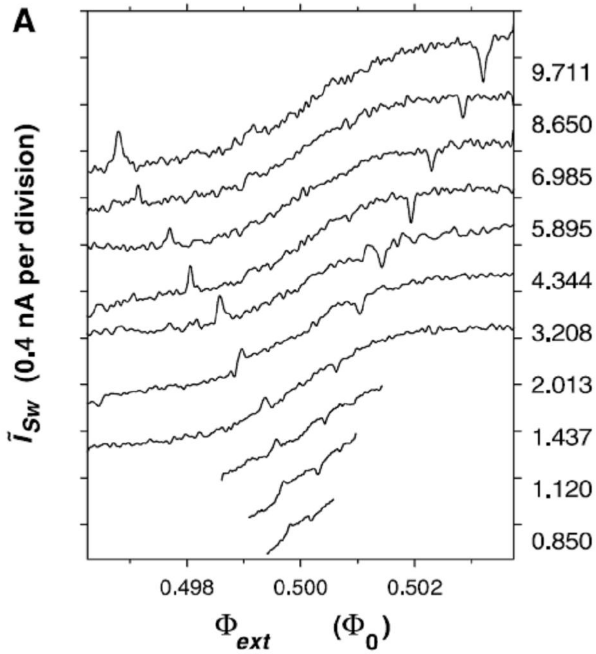


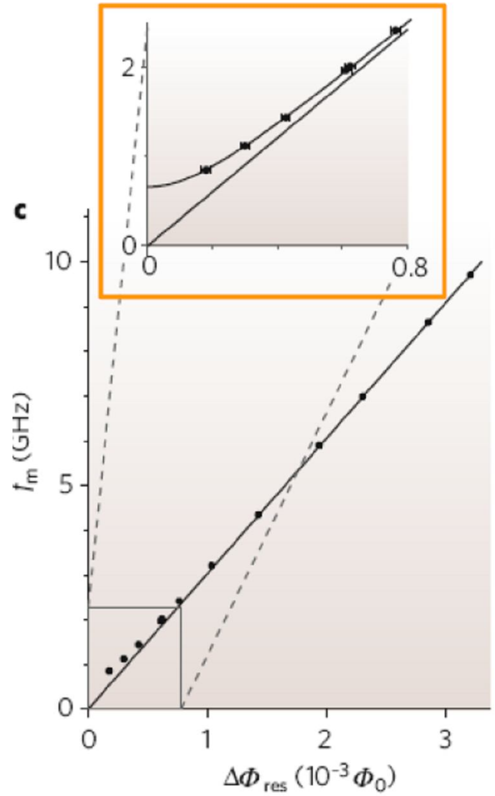


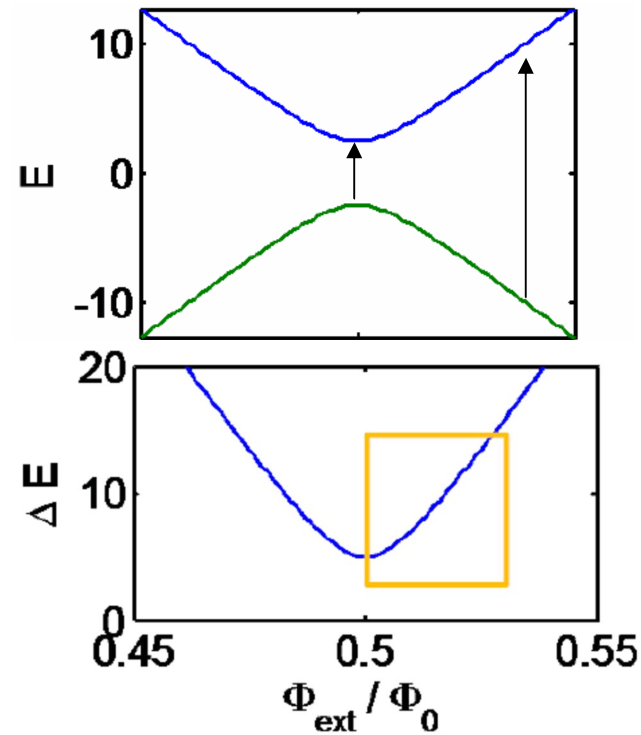
A small change in the squid signal
shows the persistent current states

Caspar H. van der Wal et al., Science 290 (2000)

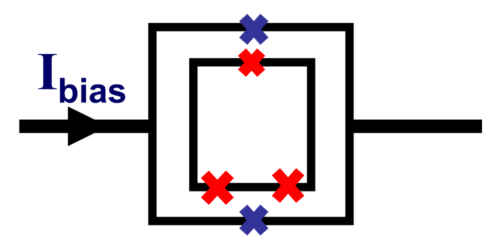
Flux qbit







During the sweeping of the magnetic field, microwave applied
- the resonance seen for different frequencies at different flux points
- peaks indicate switching between flux states
- the spectra is nicely reproduced



Caspar H. van der Wal et al., Science 290 (2000)

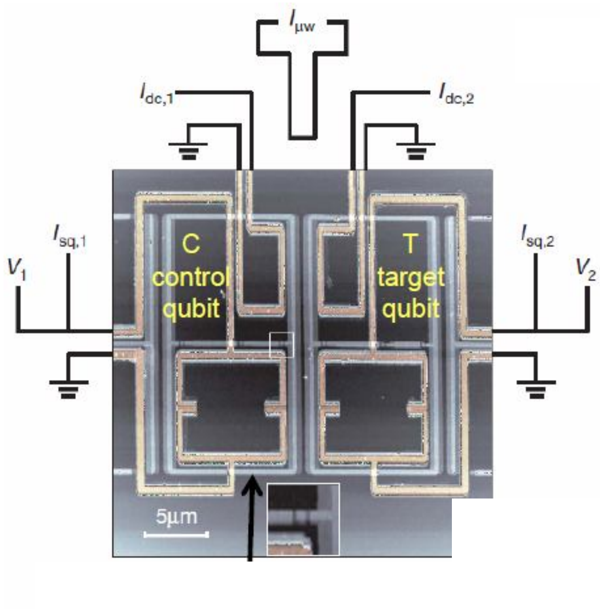
C-NOT with coupled flux qubits

BUTE, Low temperature
solid state physics laboratory

If control qubit is 1, it flips the target qubit
here, the first qubit is the control qubit, |10>→|11> and |10>→|11>

$$U_{CNOT} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

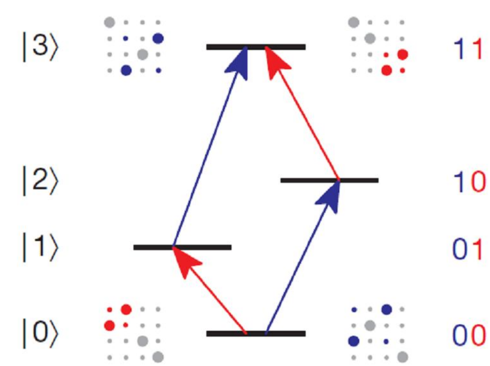
$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$



Qubits are operated far from the degeneracy point
Main idea: If one of the qubits manipulated, e.g.. qubit 1 is excited, the change of the persistent current will change the resonance frequency of qubit 2
Qubit1= Control, Qubit2=Target
states 0_C0_T, 0_C1_T, 1_C0_T, 1_C1_T
Using microwave pulses for rotations (its phase determines the axes)

$$\hat{R}_{1_C0_T-1_C1_T}(\omega,\tau) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\frac{\omega\tau}{2} & i\sin\frac{\omega\tau}{2} \\ 0 & 0 & i\sin\frac{\omega\tau}{2} & \cos\frac{\omega\tau}{2} \end{pmatrix}$$

With this rotation qubit 1 controlled CNOT is realized (almost, but phase can be easily shifted)



J. H. Plantenberg et al., NATURE 447 (2007)

C-NOT with coupled flux qubits

BUTE, Low temperature
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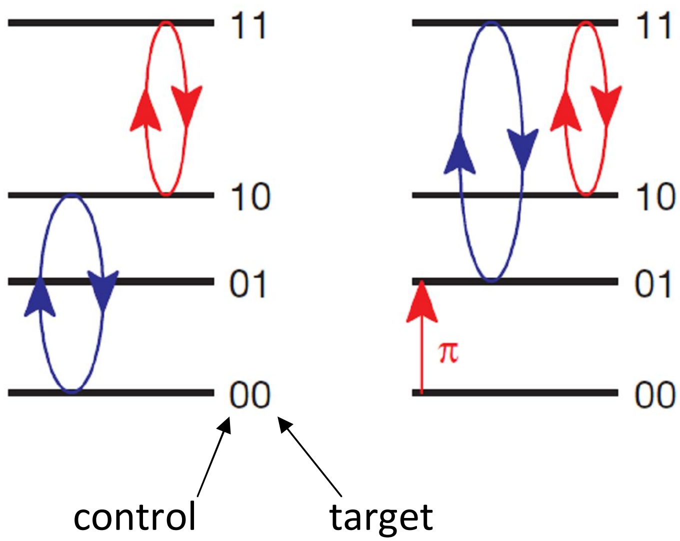
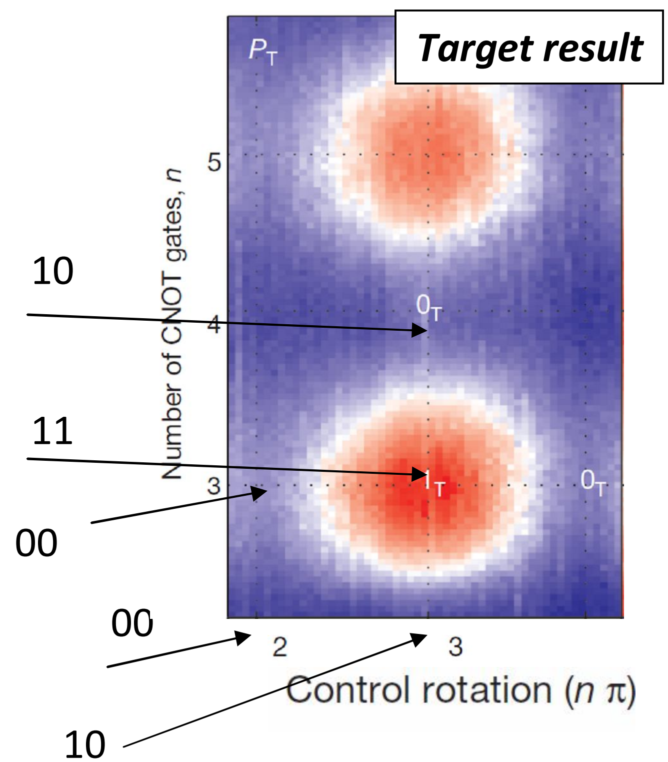
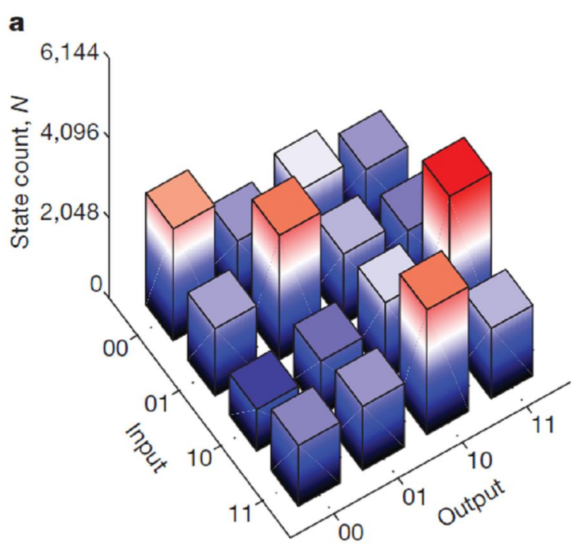


- preparation of state
- CNOT
- measurement of qubit states

e.g. first rotation prepares from |00> → a|00>+ b|10>
than the CNOT is only resonant for |10>
|10>→|11>
|00>→|00>
With other preparation sequence other 2 relations
From readout P_{00} and P_{10} … can be determined



$$U_{CNOT}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$



J. H. Plantenberg et al., NATURE 447 (2007)

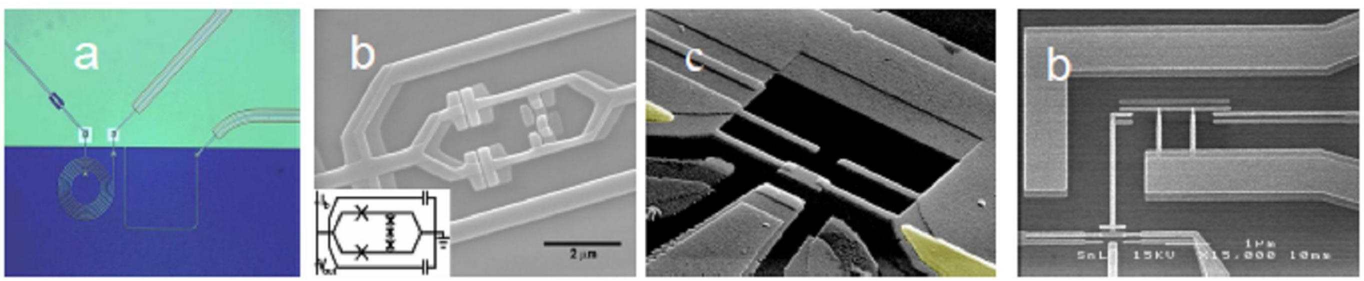
Summary

	Charge	Charge–flux	Flux	Phase
E_J/E_c	0.1	1	10	10^6
ν_{01}	10 GHz	20 GHz	10 GHz	10 GHz
T_1	1–10 μs	1–10 μs	1–10 μs	1–10 μs
T_2	0.1–1 μs	0.1–1 μs	1–10 μs	0.1–1 μs

Superconducting Circuits and Quantum Information
J. Q. You and Franco Nori
Phys. Today 58(11), 42 (2005)

Superconducting Quantum Circuits, Qubits and Computing
G. Wendin and V.S. Shumeiko
arXiv:cond-mat/0508729v1

phase | flux | charge-flux | charge



$E_j \gg E_c$ | $E_j \geq 10E_c$ | $E_j \approx E_c$ | $E_j \ll E_c$