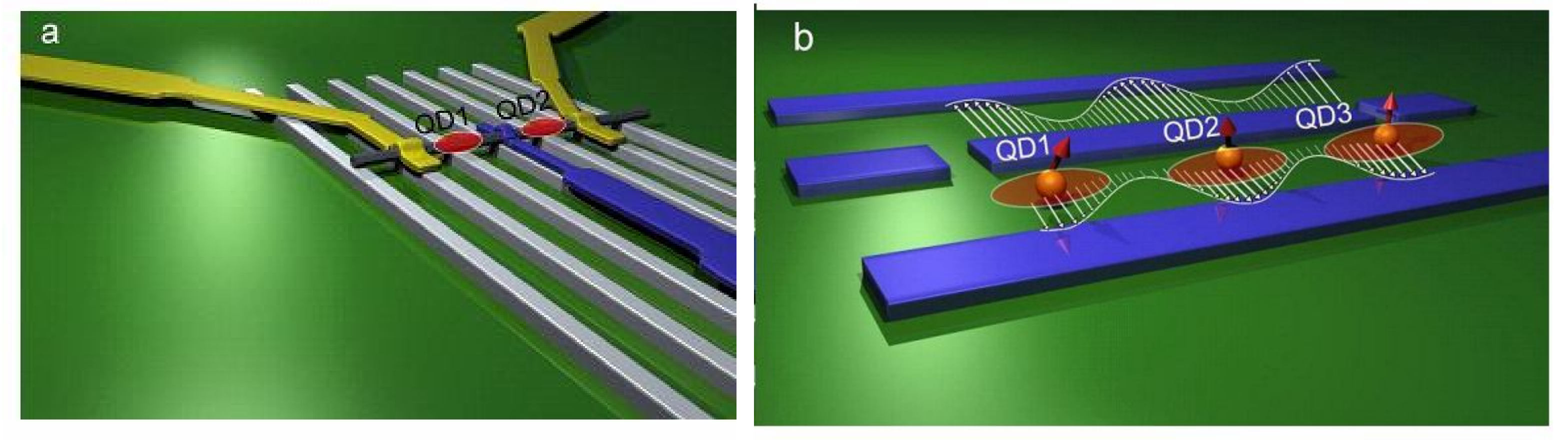
The interaction of microwave cavities with quantum dots



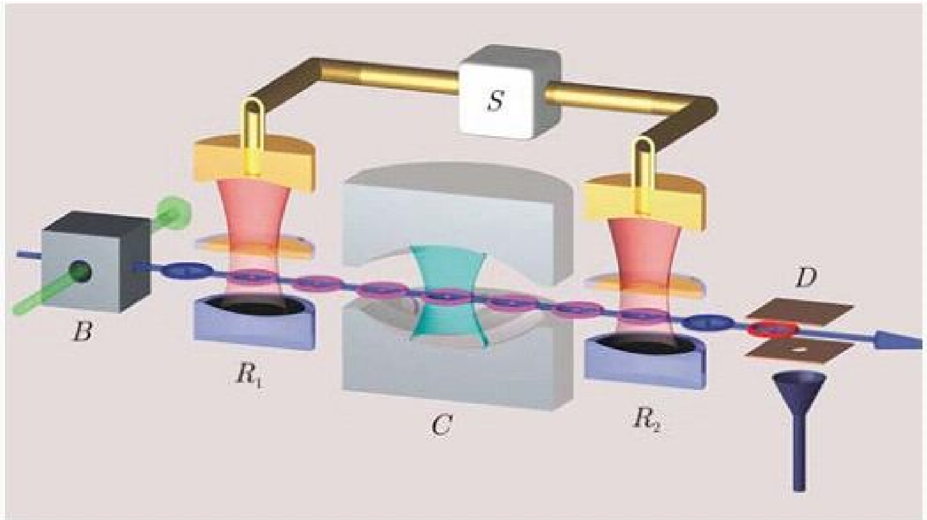
Péter Makk

2012.10.11.

Nobel Prize 2012

Serge Haroche, David J. Wineland

Interaction of atoms with the photons of the cavity



Jaynes-Cummings

2 level atom

$$\hat{H}_{atom} = \frac{1}{2}\hbar\omega_a \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{2}\hbar\omega_a\sigma_z \qquad |g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad |e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Photon field

$$\hat{H}_{photon} = \hbar\omega_r a^\dagger a$$

Interaction

$$\hat{H}_{int} = -\hat{\vec{d}}\hat{\vec{E}}$$

Dipole moment

$$\hat{d} = \frac{d}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{d}{2}(\sigma_+ + \sigma_-)$$

Electric field

$$\hat{E} = \sqrt{\frac{\hbar\omega_r}{\epsilon_0 V}}\sin(Kz)(\hat{a} + \hat{a}^\dagger)$$

$$\hat{H}_{int} =\sim \hbar g(a^\dagger\sigma_- + a\sigma_+ + \cancel{a\sigma_- + a^\dagger\sigma_+})$$

Rotating wave approx.

$$g = \frac{d}{2\hbar}\sqrt{\frac{\hbar\omega_r}{\epsilon_0 V}}\sin(Kz)$$

Coupling

$$\boxed{\hat{H} = \frac{1}{2}\hbar\omega_a \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \hbar\omega_r a^\dagger a + \hbar g(a^\dagger\sigma_+ + a\sigma_-)}$$

Jaynes-Cummings Hamiltonian

In a new basis:

$$|e, n\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |g, n+1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{H} = (n + 1/2)\omega_r \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -\delta/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & \delta/2 \end{pmatrix}$$

Detuning: $\delta = \omega_r - \omega_a$

$\delta = 0$ **Resonant case**

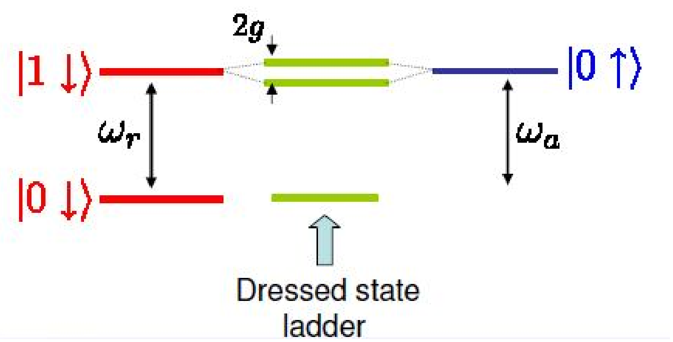
$$\hat{H} = (n+1/2)\omega_r \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -\delta/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & \delta/2 \end{pmatrix}$$

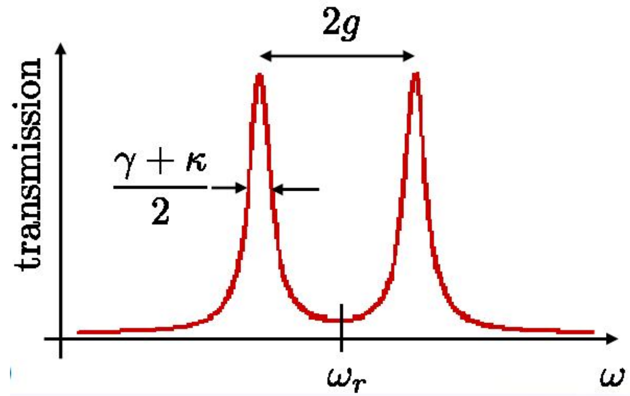
$$E_n = (n+1/2)\omega_r \pm R/2$$

$$R = 2g\sqrt{(n+1)} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$|e, n\rangle$ Not an eigenstate, Rabi oscillations between the states
Mixed photon numbers

$$P_e(t) = \frac{1}{2}(1 + \cos(Rt))$$





Jaynes-Cummings

$\delta \neq 0$ **Off-resonance**

$$E_n = (n+1/2)\omega_r \pm \sqrt{\delta^2/4 + g^2(n+1)}$$

$$E_n \sim (n+1/2)\omega_r \pm (\delta/2 + \frac{g^2}{\delta}(n+1))$$

$$= \begin{cases} n\omega_r + \omega_a/2 - \frac{g^2}{\delta}(n+1) \\ (n+1)\omega_r - \omega_a/2 + \frac{g^2}{\delta}(n+1) \end{cases}$$

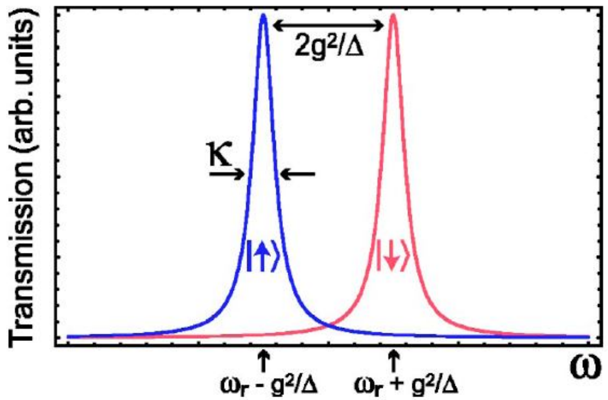
N photons in the cavity, excited atom

N+1 photons, ground state atom

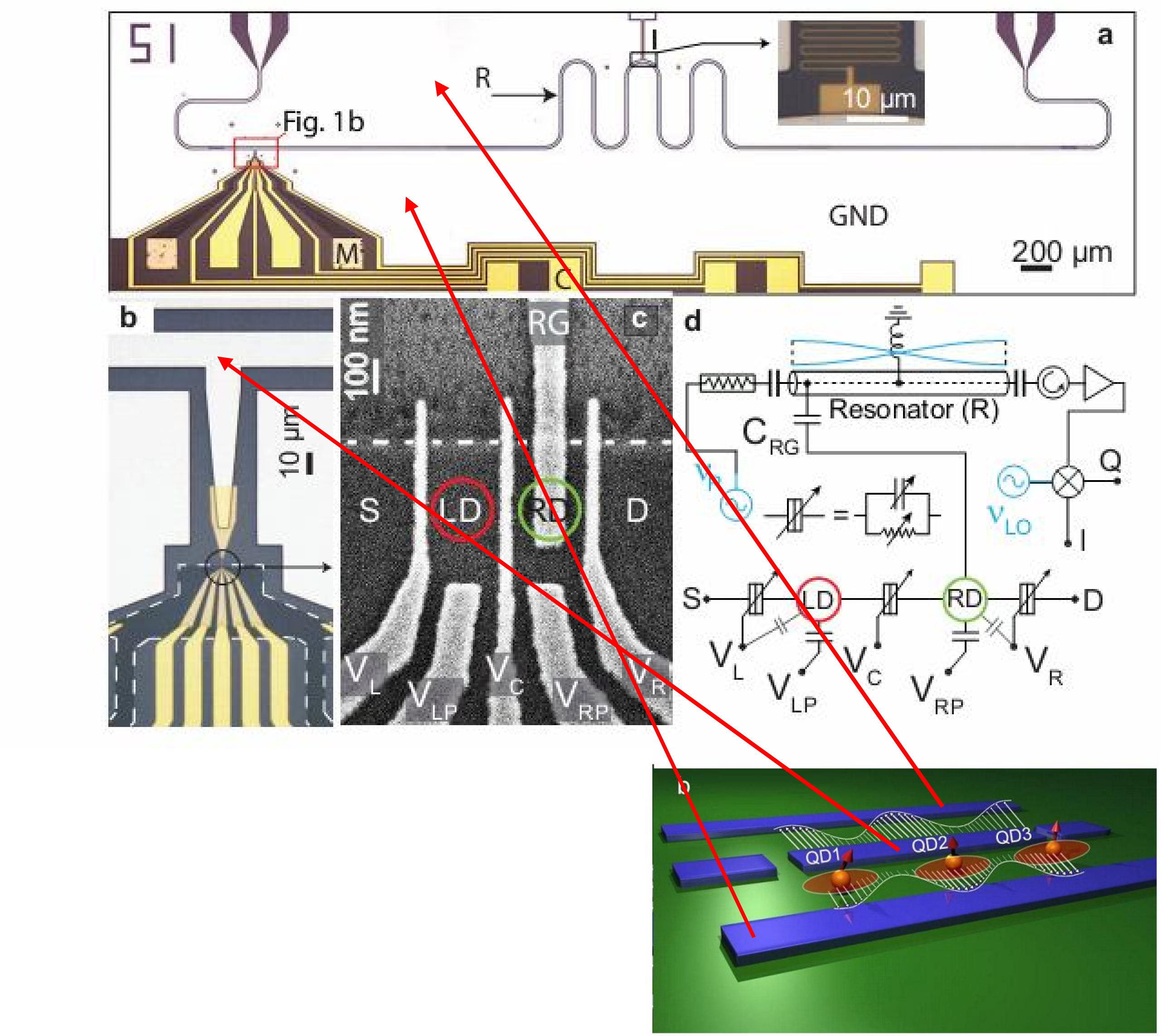
The Hamiltonian after the perturbation calculation

$$\hat{H} = -\frac{1}{2}(\hbar\omega_a + \frac{g^2}{\hbar\delta})\sigma_z + (\hbar\omega_r - \frac{g^2}{\hbar\delta}\sigma_z)a^\dagger a$$

Lamb shift Stark shift



Coupling double Qdot to cavity



Resonator frequency ~6.7 GHz (25μeV)
Q~2600
E_c~ 1 meV, T~135mK
No thermal photons

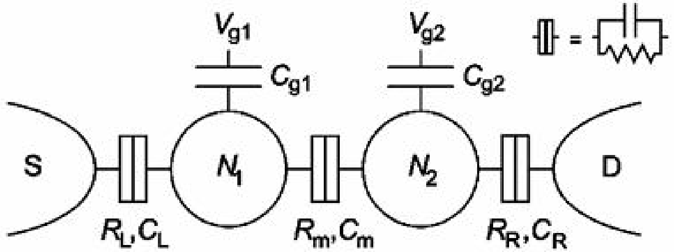
R_g – extends from the resonator to maximize the coupling
This gives a selective capacitive coupling of the resonator to the right quantum dot.

Measure amplitude and phase of transmitted signal by heterodyne detection (mix with a local oscillator and measure low frequency signal).

T. Frey et al., Phys. Rev. Lett. ***108****, 046807 (2012)*

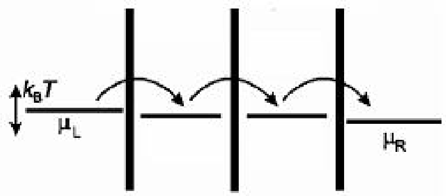
Double dots

Két dot közötti csatolás C_m és R_m-sal jellemezve:
$G_m=4\pi e^2/h(t_c/\Delta)^2$, ha t_c kicsi, e-k jól lokalizáltak.



Transzport kizárólag 3-as pontokban (V_{sd}~ 0)
Egyébként Coulomb-blokád
E vezetés:
$(N_1,N_2) \rightarrow (N_1+1,N_2) \rightarrow (N_1,N_2+1) \rightarrow (N_1,N_2)$

Lyuk vezetés:
$(N_1,N_2) \rightarrow (N_1-1,N_2) \rightarrow (N_1,N_2-1) \rightarrow (N_1,N_2)$



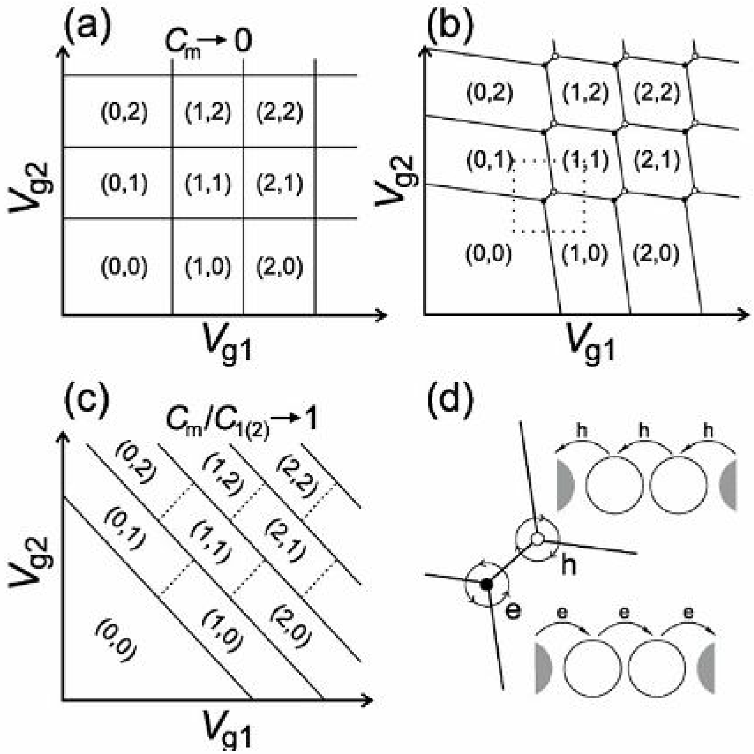
Van der Wiel, Rev. Mod. Phys. 75 1 (2003)

2QDot stabilitás diagrammja

Ahol (N_1,N_2), N_i: elektronok száma az i. doton

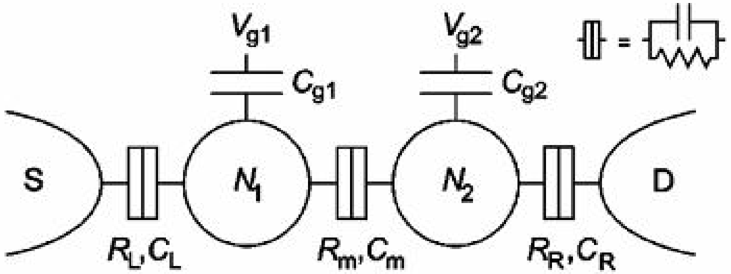
- Teljesen szétcsatolt határeset (a)
- Egy nagy dot határeset (c)
- Hexagonális mintázat két hármas-ponttal (b)

Egyik 3-as pont lyuk, másik e vezetést ad



Double dots

Két dot közötti csatolás C_m és R_m-sal jellemezve:



Transzport kizárólag 3-as pontokban (V_{sd}~ 0)
Egyébként Coulomb-blokád
E vezetés:
$(N_1,N_2) \rightarrow (N_1+1,N_2) \rightarrow (N_1,N_2+1) \rightarrow (N_1,N_2)$

Lyuk vezetés:
$(N_1,N_2) \rightarrow (N_1-1,N_2) \rightarrow (N_1,N_2-1) \rightarrow (N_1,N_2)$

$\mu_i(N_1,N_2)$: az i-ik dot kémiai potenciálja

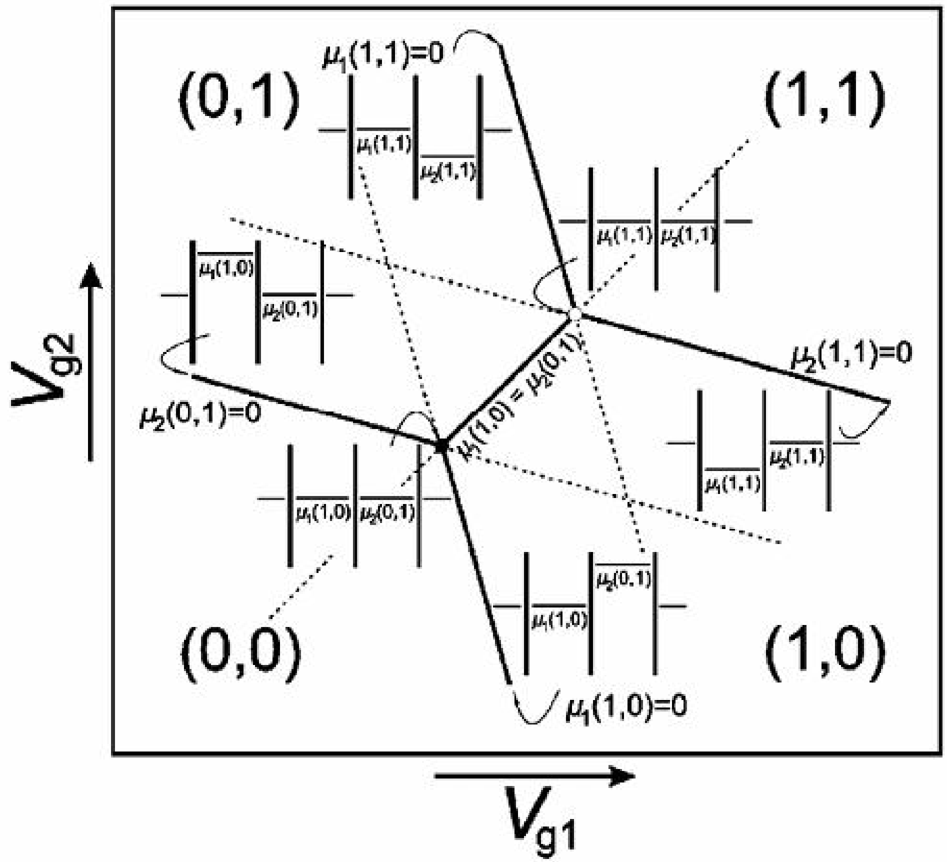
pl: $\mu_i(N_1,N_2) = U(N_1,N_2)-U(N_1-1,N_2)$

Van der Wiel, Rev. Mod. Phys. 75 1 (2003)

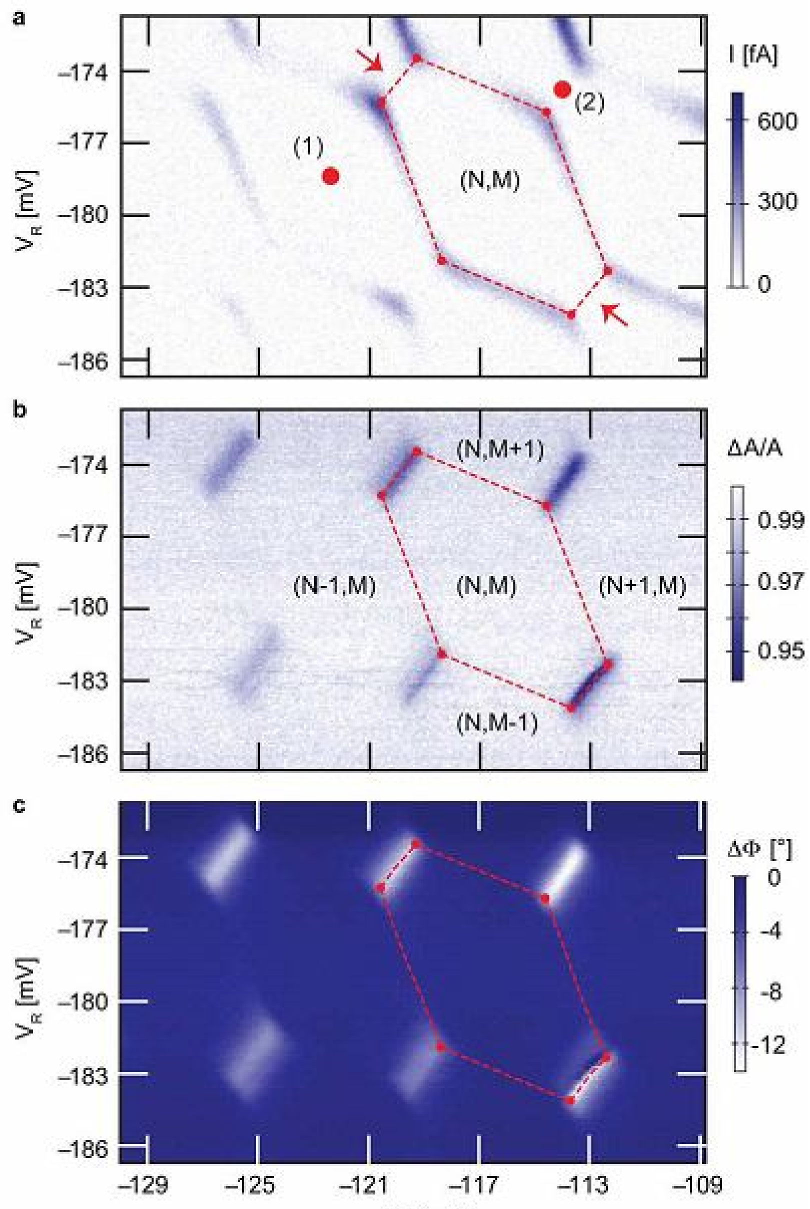
2QDot stabilitás diagrammja

Ahol (N_1,N_2), N_i: elektronok száma az i. doton

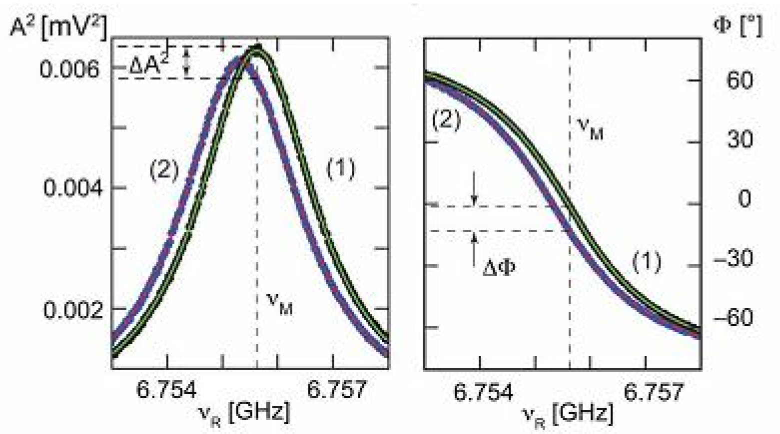
Eneriaszintek a hármas-pontok körül:



Coupling double Qdot to cavity



T. Frey et al., Phys. Rev. Lett. ***108****, 046807 (2012)*



First measure it in DC (V_{bias}=50μV)

Similar pattern observed with microwave

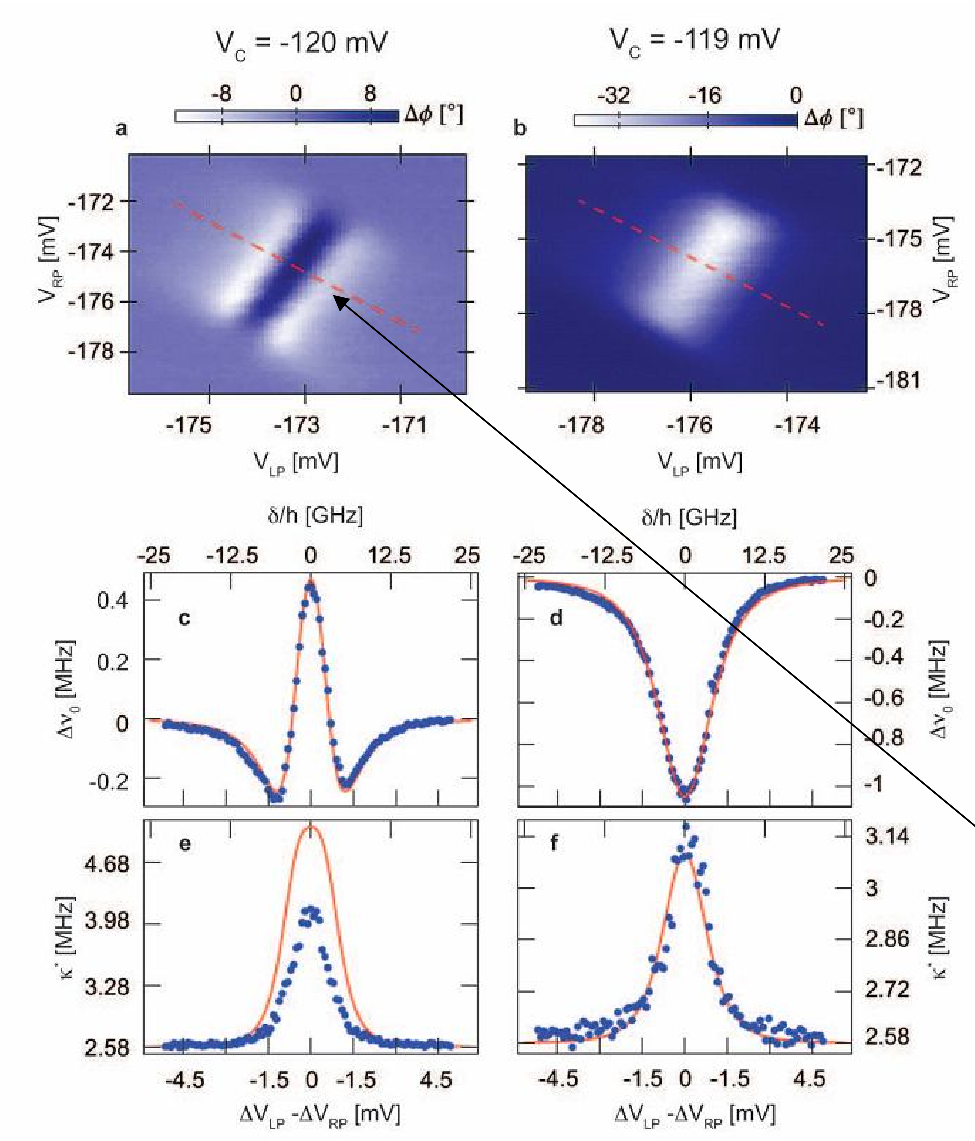
Microwave frequency and phase is sensitive to dot configuration

The double dot is blockaded, however, for (2) interdot transitions are possible

Seen diamond pattern where the current in DC is too small to measure

Investigate one particular transition →

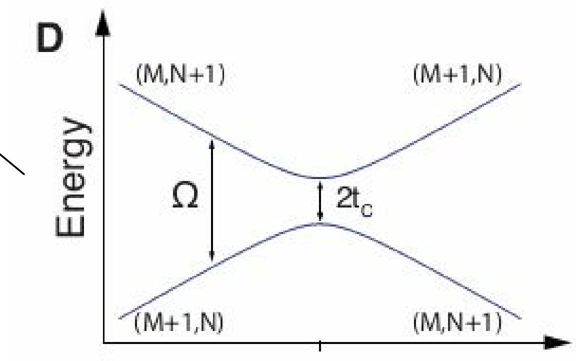
Coupling double Qdot to cavity



Tune interdot coupling by V_c

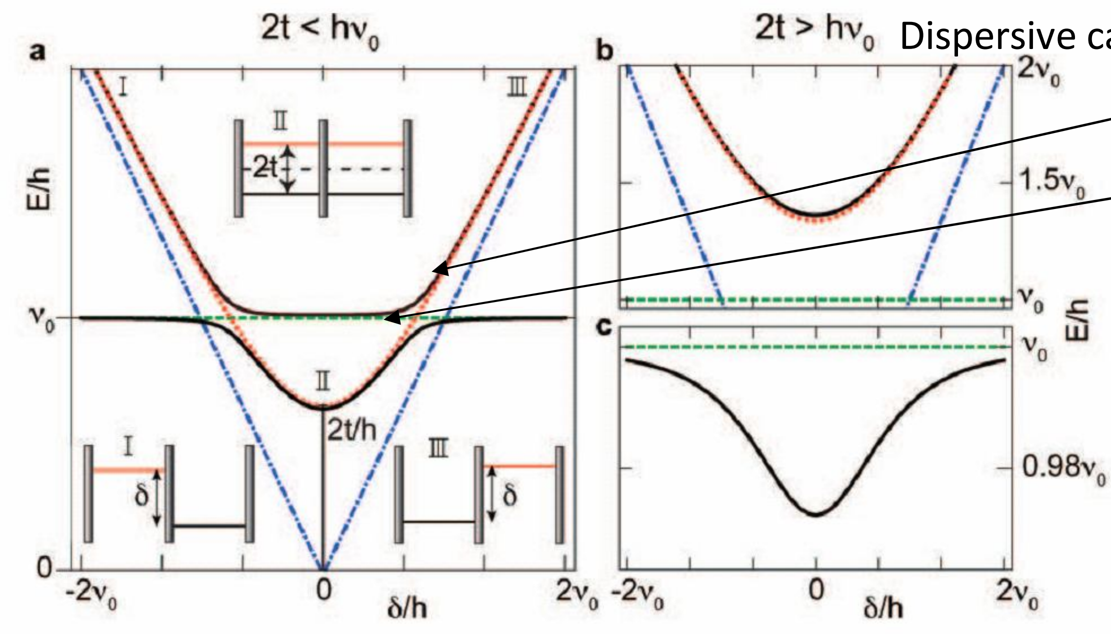
Measure full transmission spectra and determine the frequency and phase shift for different detunings (gate detuning)
These quantities are plotted as a function of detuning (red line on 2D figure)

Two different wave-forms seen for different couplings



'detuning'- red line

Coupling double Qdot to cavity



Dispersive case

Red: bare qbit level

Green: bare resonator

Black: coupled resonator-qubit system

$$H = h\nu_0(\hat{n} + \frac{1}{2}) + \frac{h\nu_{\mathrm{q}}}{2}\hat{\sigma}_{\mathrm{z}} + \hbar g \sin\theta(\hat{a}^{\dagger}\hat{\sigma}^{-} + \hat{a}\hat{\sigma}^{+})$$

$$\nu_{\mathrm{q}} = \sqrt{\delta^2 + 4t^2}, \qquad \sin\theta = 2t/\sqrt{\delta^2 + 4t^2}.$$

Qubit: Two level system with degenerate energies at δ=0, zero detuning
If there is a nonzero *t* coupling, the levels hybridize and split (2*t*) splitting
Avoided level crossing - resonator qubit hybridization
More sophisticated calculations include decoherence etc. and fit the data

Energy scales

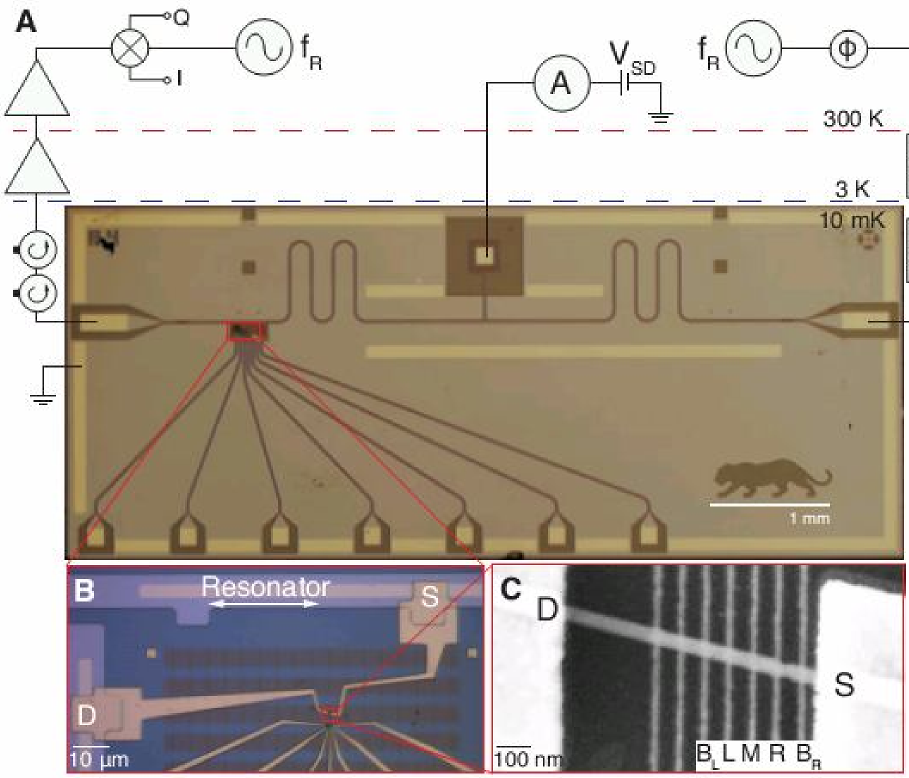
$\gamma_1/2\pi \sim 100\mathrm{MHz}$ Qbit relaxation

$\nu_0 \sim 6.77\mathrm{GHz}$ $\kappa/2\pi \sim 2\mathrm{MHz}$ resonator $g/2\pi \sim 50\mathrm{MHz}$ Weak coupling

$2t/h \sim 9\mathrm{GHz}$ $\gamma_\phi/2\pi \sim 0.9\mathrm{GHz}$ dispersive regime

$2t/h \sim 6\mathrm{GHz}$ $\gamma_\phi/2\pi \sim 3.3\mathrm{GHz}$ Resonant regime, however dephasing is very fast, no effect of the crossing seen, very weak g

Results on InAs

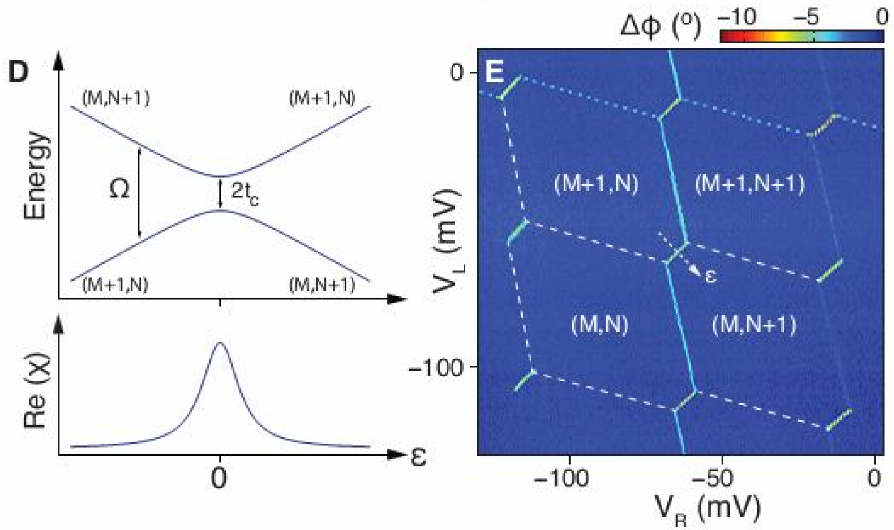


6GHz, Q~2000
Homodyne technique

Double QDot, bottom gates
To maximize field source is connected to node, drain to antinode of the field

~1000a_B dipole moment for an electron

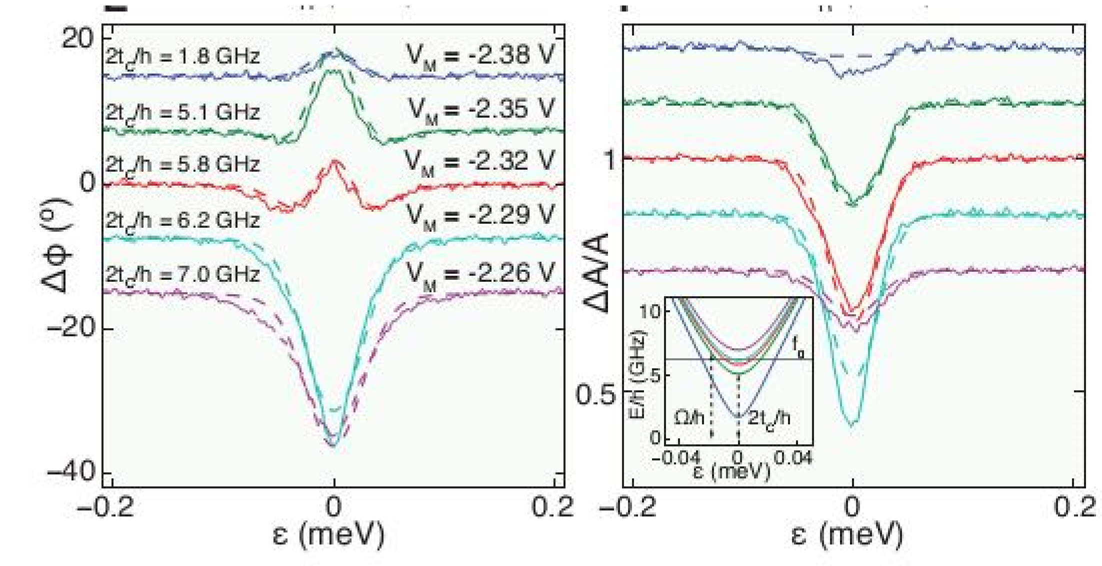
Investigate interdot charge coupling
Splitting…
E_c~12meV, hf_r~25μeV



Detect how the states damp, and shift the cavity freq
Hexagonal diamond patterned measured

K.D. Petersson et al., arXiv:1205.6767v1

Results on InAs

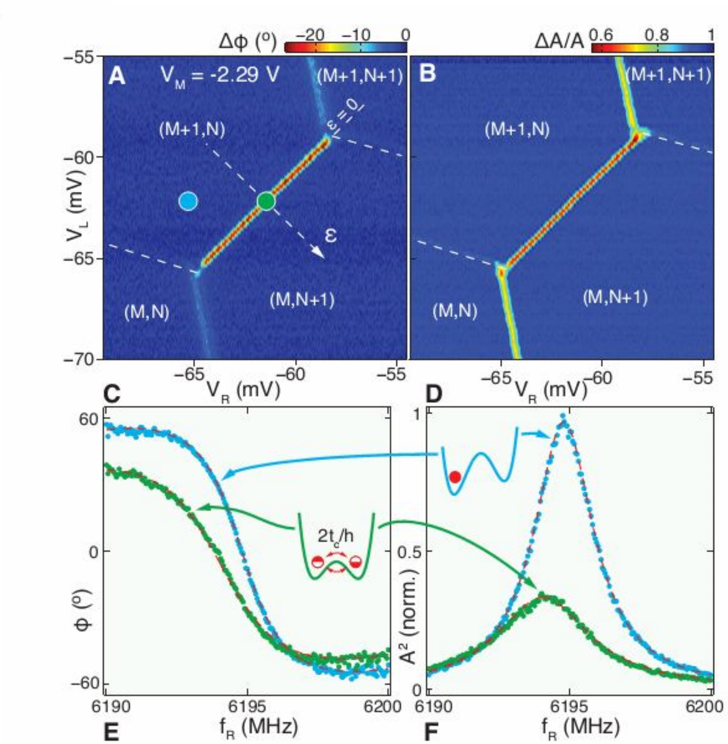


Determine the vacuum Rabi freq, as they change the tunnel coupling

$$g/2\pi \sim 30\mathrm{MHz}$$

$$2t/h \sim 1.8 - 7\mathrm{GHz}$$

Lifetime 15ns



Similar as for the Zurich group, but spin physics is also interesting

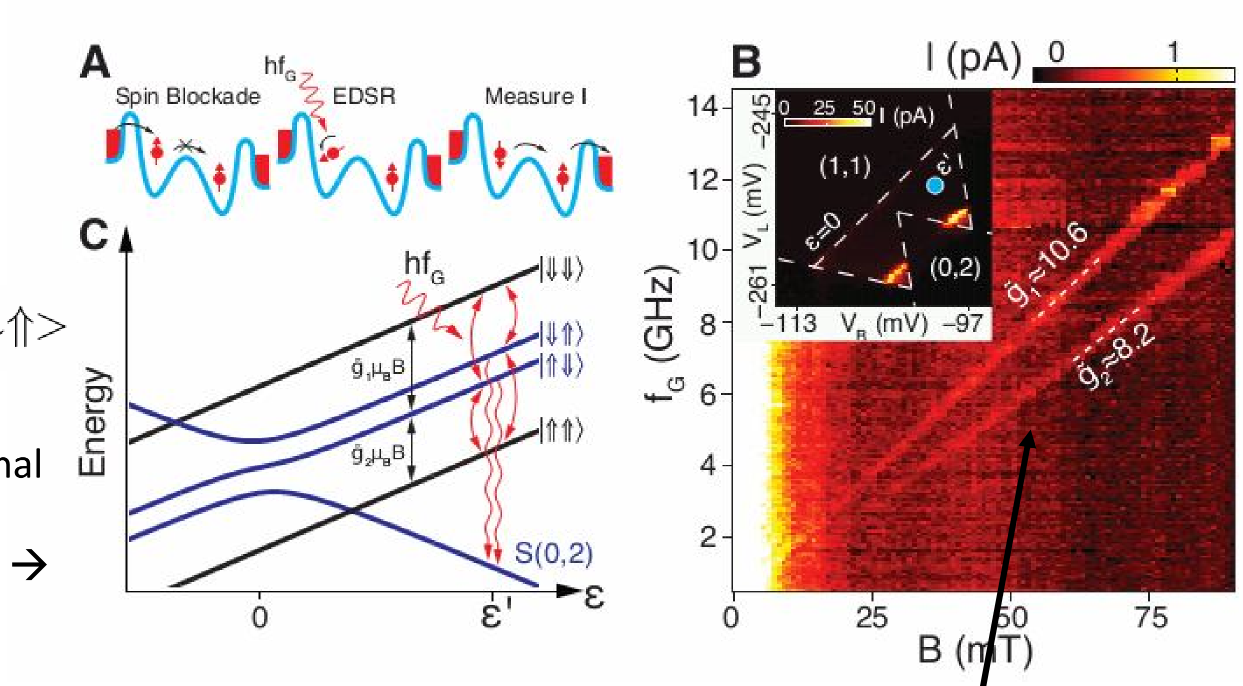
Results on InAs

Investigate spin physics

(0,2) and (1,1)
For (0,2) ground state S(0,2)
For (1,1)

$|\Uparrow\Uparrow>, \ |\Downarrow\Downarrow>, \ |\Uparrow\Downarrow>, \text{ and } |\Downarrow\Uparrow>$

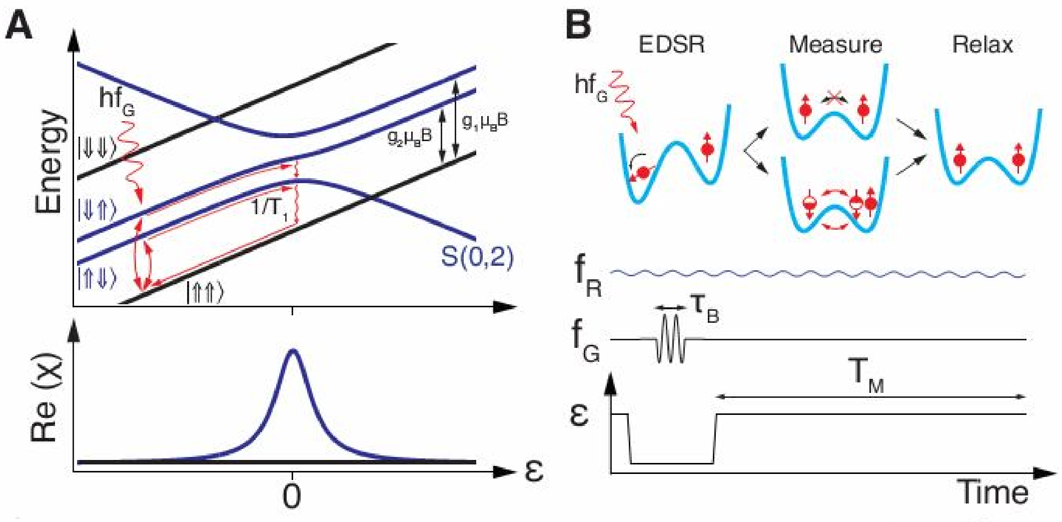
Splitting of the fields with external field
G factors different for two dots, → no twofold degeneracy
At zero detuning tunneling hybridizes S(0,2) with S(1,1) singlet



Pauli blockade (↑ ↑ cannot go into S(0,2))
One can rotate one of the spins selectively (different g factors) by EDSR and lift the blockade: $hf_g = E_Z$
At low fields hyperfine mixes states

EDSR: The ac electric field is generated through the excitation of the gates, which periodically displaces the electron wave function around its equilibrium position. Due to the SOI the electron spin will feel an oscillating effective magnetic field, which is used to rotate the electron spin.

Results on InAs



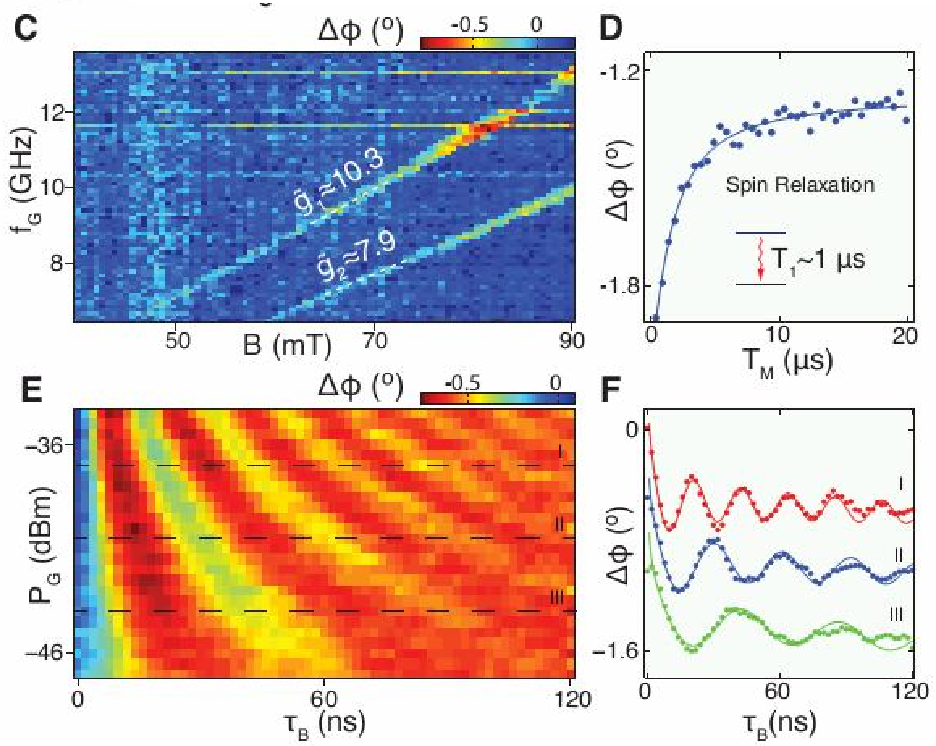
-Prepare in ↑ ↑
- Negative detuning
- Rotate with EDSR
- Zero detuning and measure with cavity: sensitive to phase response

$$E = \frac{1}{2} C V^2 \qquad C_Q \sim \frac{\partial^2 E}{\partial V^2}$$

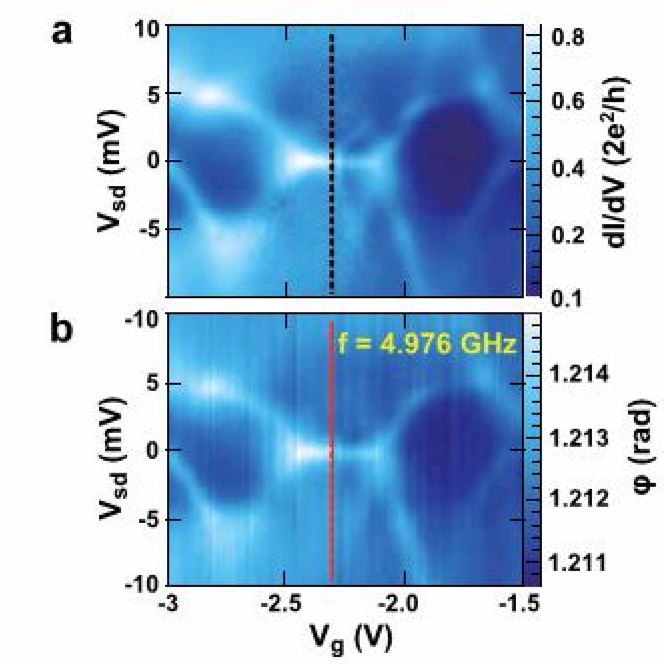
Curvature counts, dipole: detuning dependent

- See the Zeeman-lines in the cavity response
- Measure relaxation by waiting at zero detuning T_1 ~1ms
- Measure Rabi oscillations: vary EDSR pulse length and power

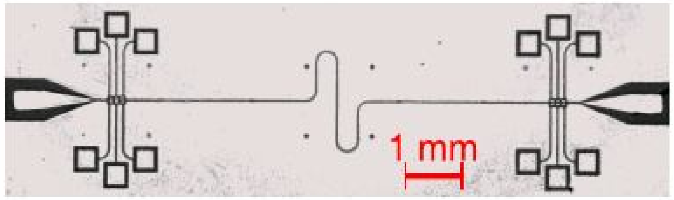
-Estimations: coupling of distant qubits ~1 Mhz
For reaching strong coupling regime need to dechorence rate, have ideas… (dynamical decoupling, GeSi core shell wires, …)

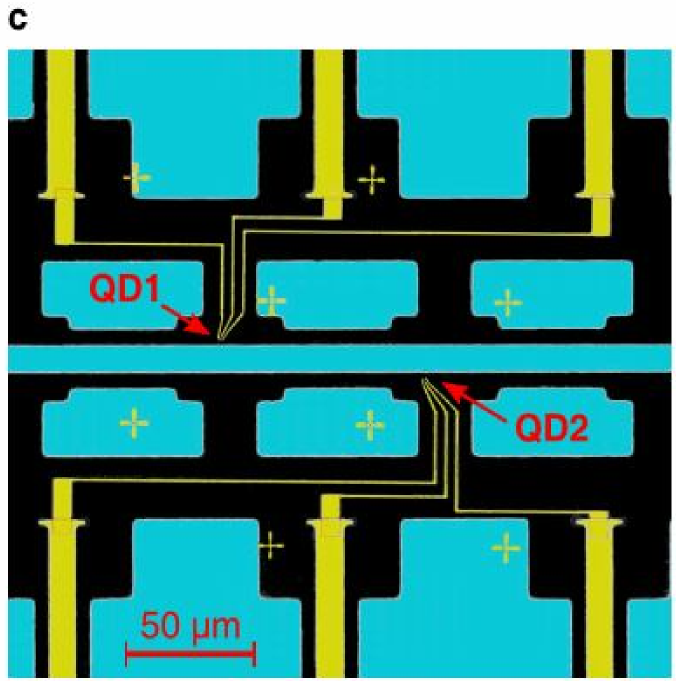


Similar setup in nanotube

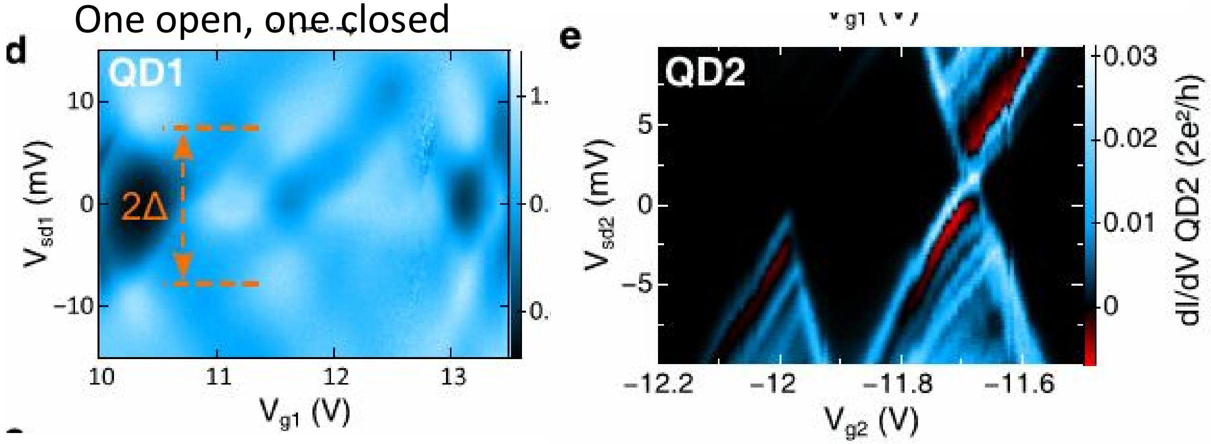


Similar setup and investigate the interaction of Kondo and cavity



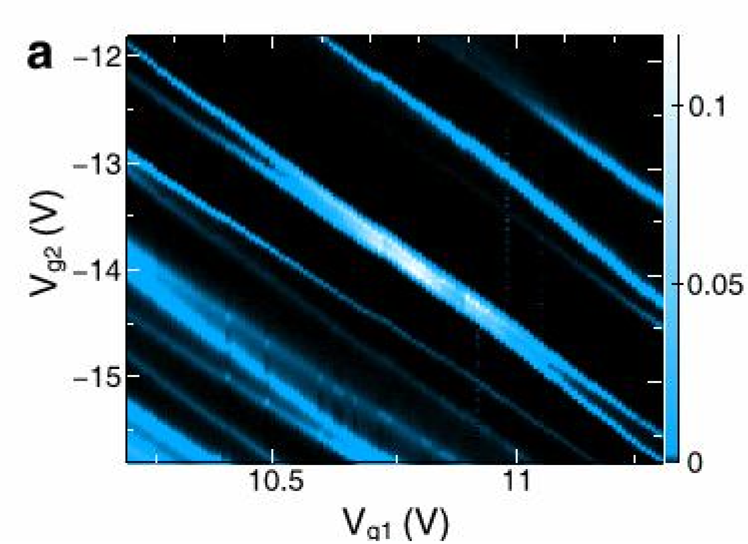


2 dots on two nanotubes
One open, one closed



M. R. Delbecq et al., PRL107,256804 (2011), M.R. Delbecq et al., submitted to Nature Comm.

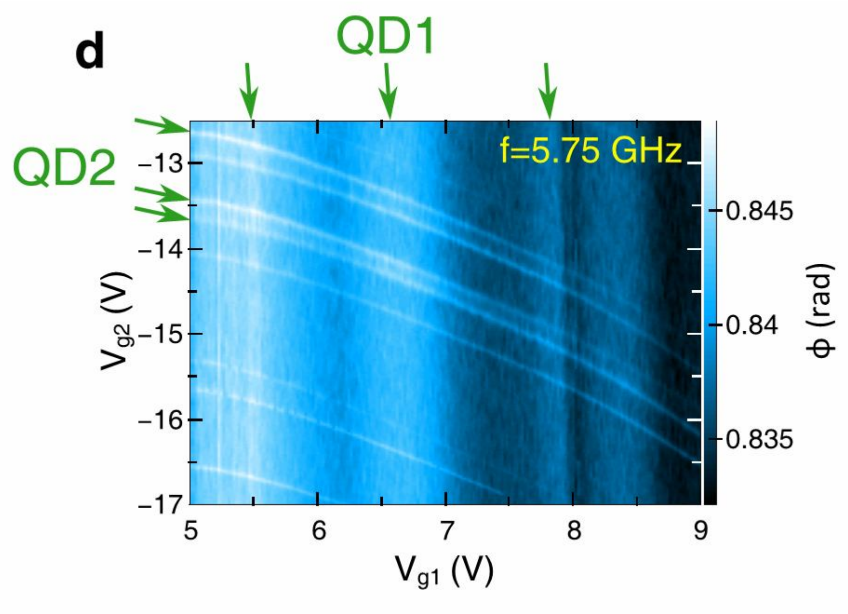
Similar setup in nanotube



Closed dot – sensing

Levels tuned with other gate
No capacitive crosstalk possible
Coupling via virtual exchange of photons

$$\Delta_{1(2)}^{polaron} = -4\pi g_1 g_2 N_{2(1)}/f_0$$

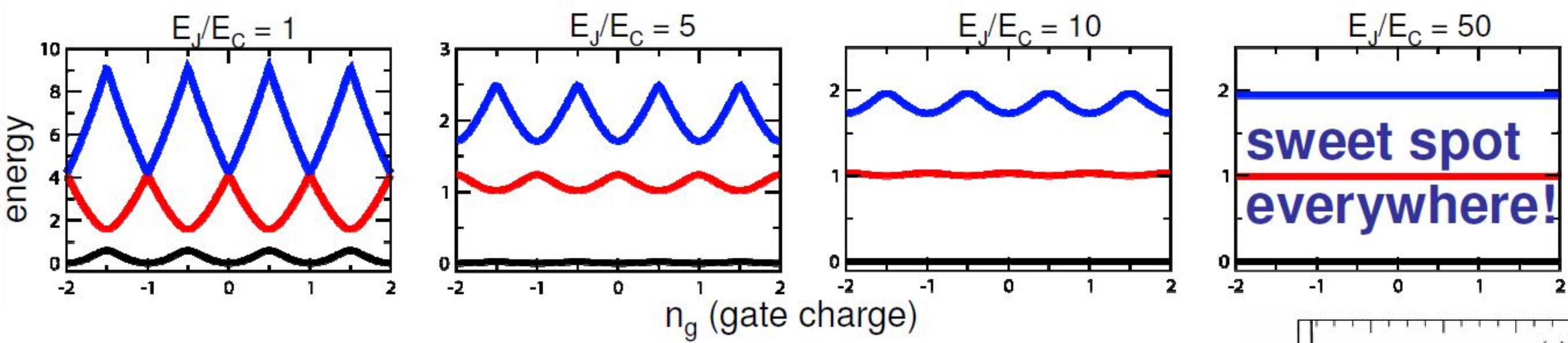


Shift depend on the number of electrons on the tuning dot
If the open dot is the sensing, no effective tuning possible
Crossings not lifted→ no electronic transport, and photon field is also far detuned
Also measured the level shifts by cavity

Transmon

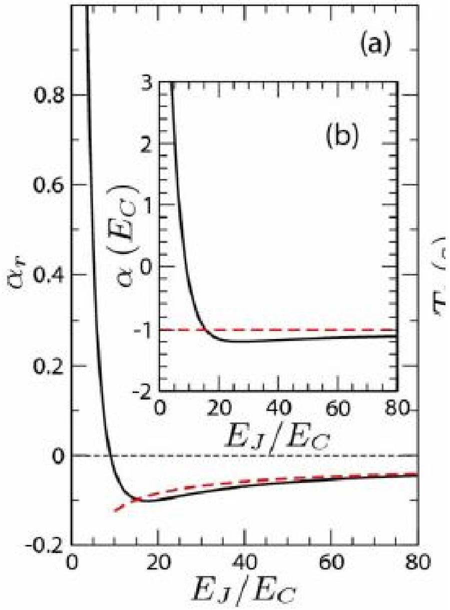
Problem with charge qubit: sensitivity to charge noise, fluctuations of the gate electrode
The degeneracy point is a sweet point, sensitivity to charge fluctuations is second order

By increasing the $\mathbf{E_j/E_c}$ ratio:



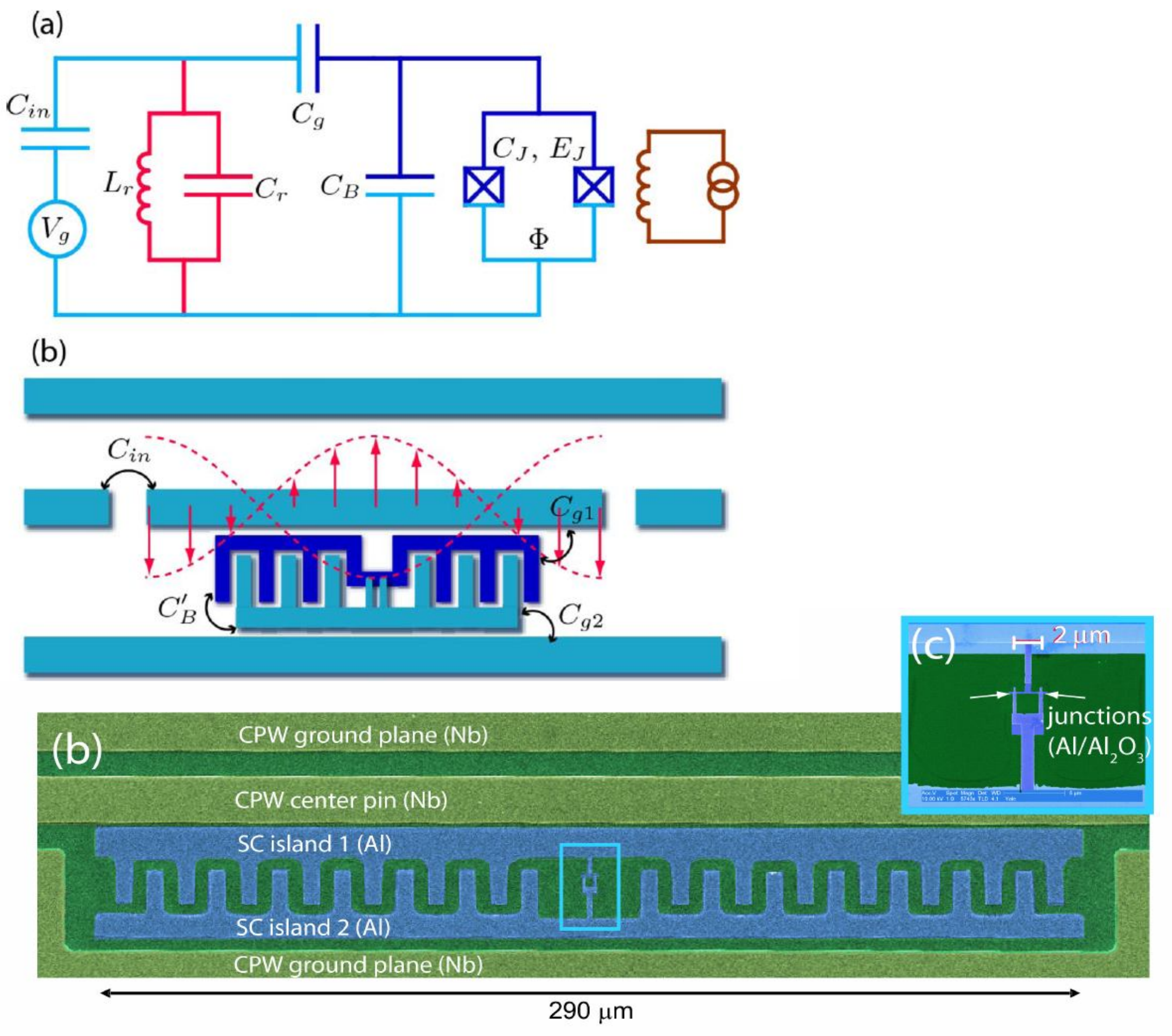
- *good news*: the dispersion of the bands disappear exponentially

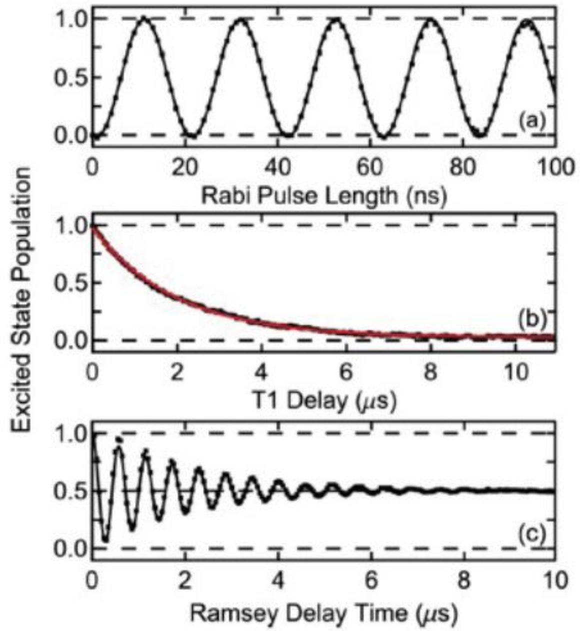
- *bad news:* the anharmonicity decreases (even changes sign), but just linearly, still enough to separate qubit



J. Koch et al., PRA **76**, 042319 (2007)

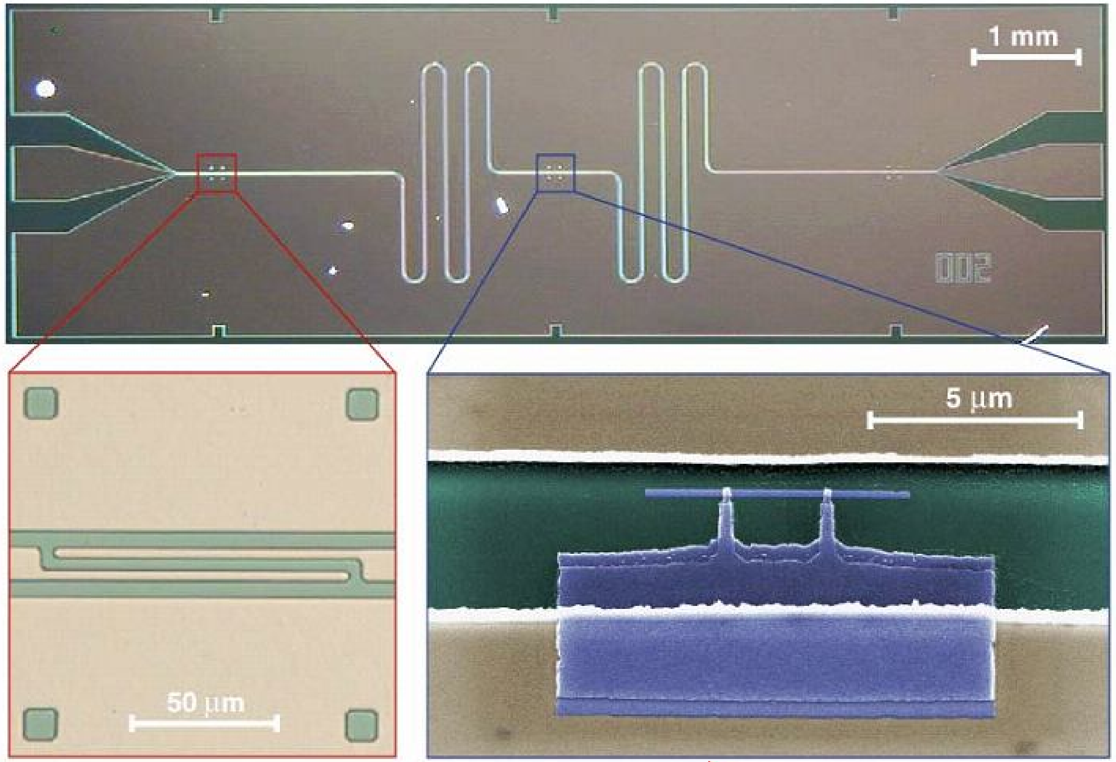
E_j/E_S increased by a shunting capacitance (E_C decreased): design parameter





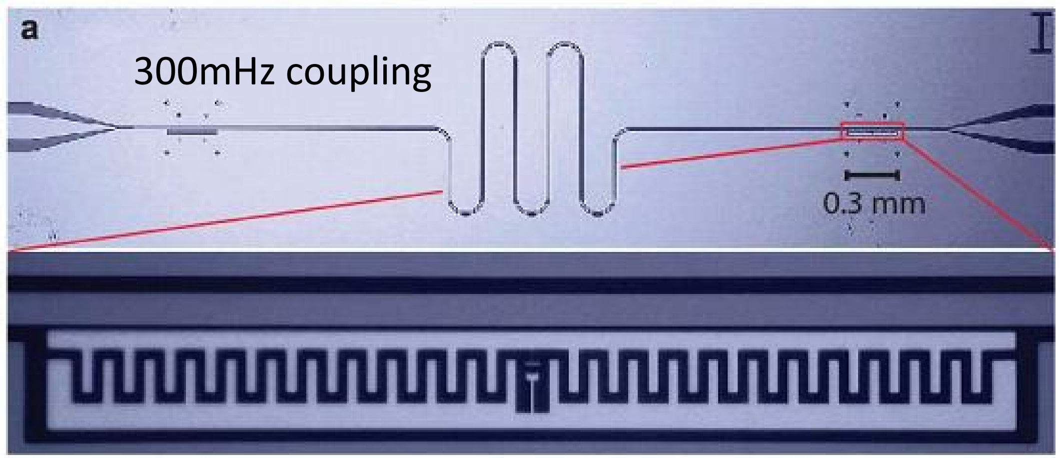
$T_1 \sim \mu s$
$T_\phi \sim 10 \mu s$

Transmon



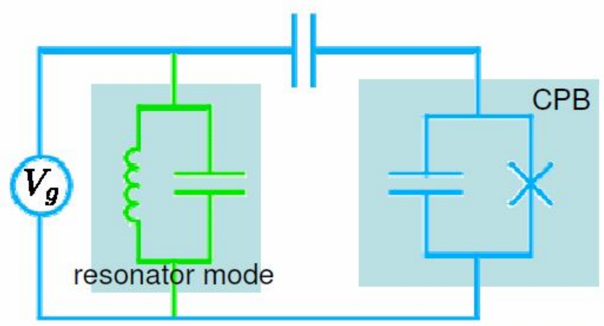
$$g/2\pi \sim 1..100\text{MHz}$$

$$\begin{aligned} \nu_{\text{vac}} &= 11.6\,\text{MHz} \\ (g/2\pi &= 5.8\,\text{MHz}) \\ \kappa/2\pi &= 0.8\,\text{MHz} \\ \gamma/2\pi &= 0.7\,\text{MHz} \end{aligned}$$



Transmon

Coupling to resonator



$$V_g \rightarrow V_g + V_{RMS}(\hat{a} + \hat{a}^\dagger)$$

Coupling:

$$g = dE/\hbar$$

d and E higher than in 3d cavity
easy to arrive at strong coupling regime

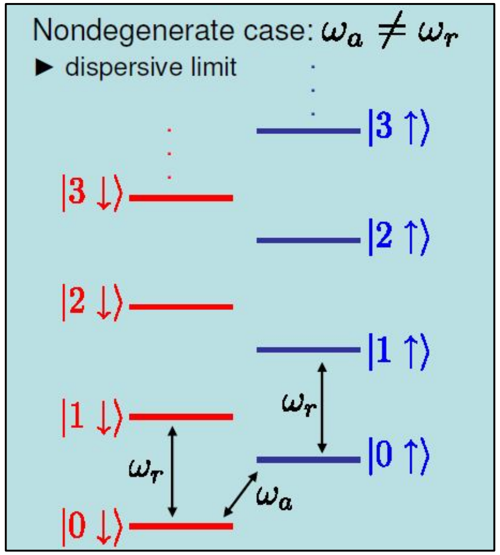
$$H_c = 4/e * E_c C_G V_{RMS}(\hat{n}(\hat{a} + \hat{a}^\dagger))$$

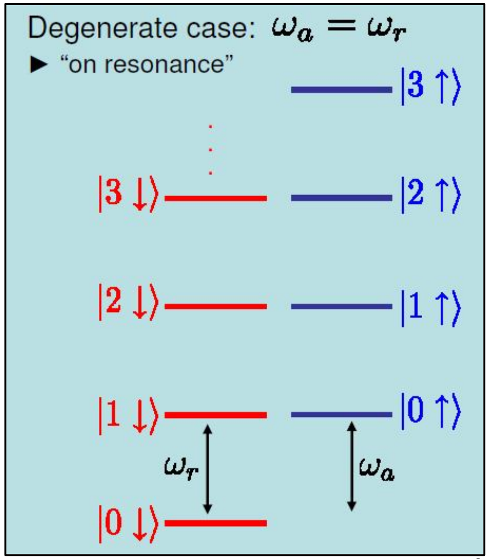
RWA

Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2) + \frac{\hbar\omega_a}{2}\sigma_z + \hbar g(\hat{a}\sigma_+ + \hat{a}^\dagger\sigma_-)$$

without coupling
(g=0)



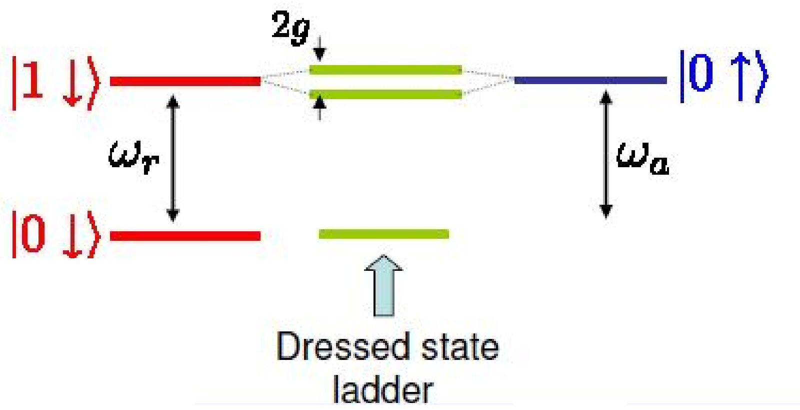


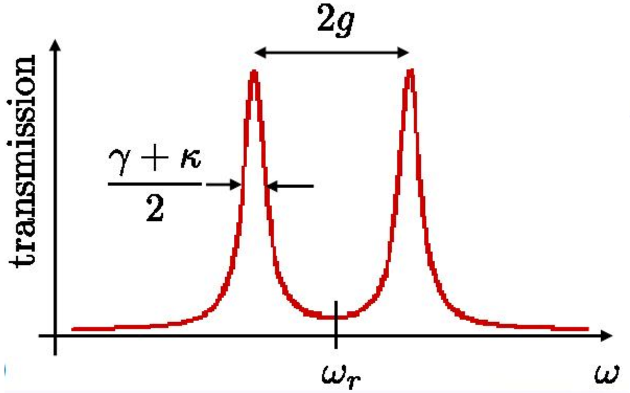
*A. Blais et al., PRA **69**, 062320 (2004)*

Transmon

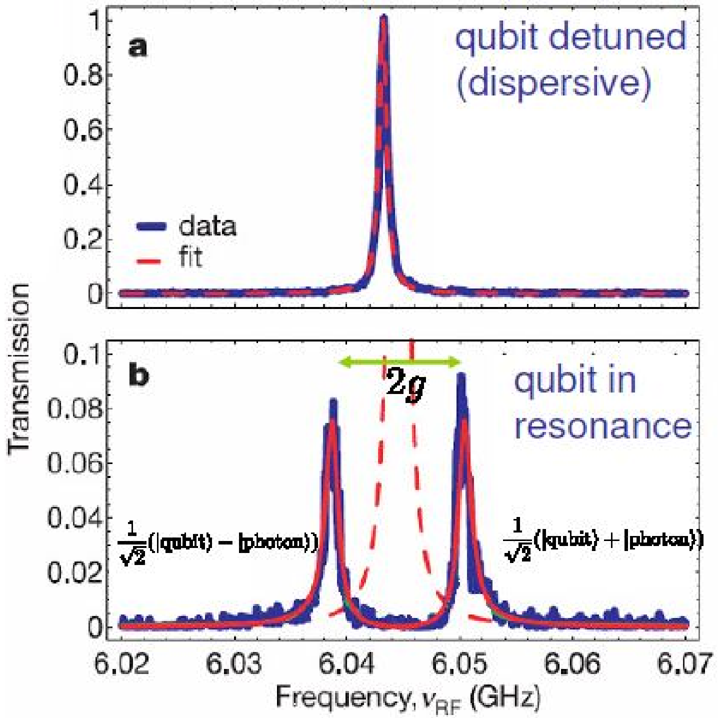
Resonant case, g≠0

$$E_{n\pm} = n\hbar\omega \pm \hbar g\sqrt{n+1}$$





Vacuum Rabi splitting can be seen by measuring the transmission of the cavity



A. Walraff et al., Nature 431, 9 (2004)

Transmon

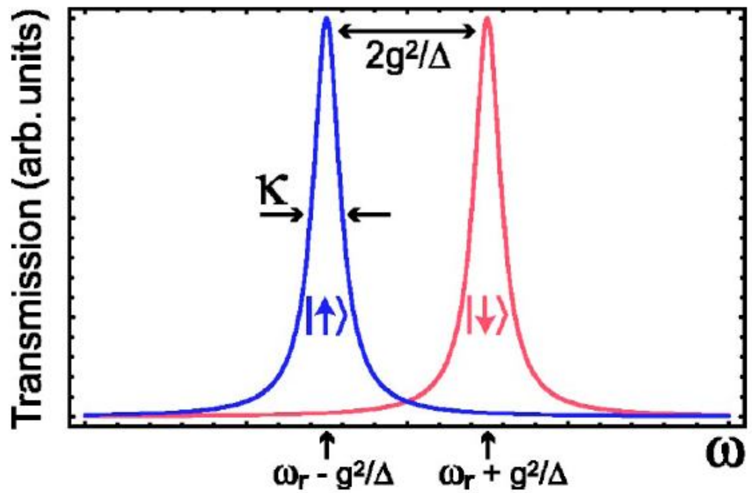
Non resonant case, g≠0
dispersive regime: ω_r-ω_a>> g

- State dependent polarizability of 'atom' pulls the cavity frequency

$$H_{eff} = \hbar(\omega_r + \frac{g^2}{\Delta}\sigma_z) + \frac{1}{2}(\omega_a + \frac{g^2}{\Delta}\sigma_z)$$

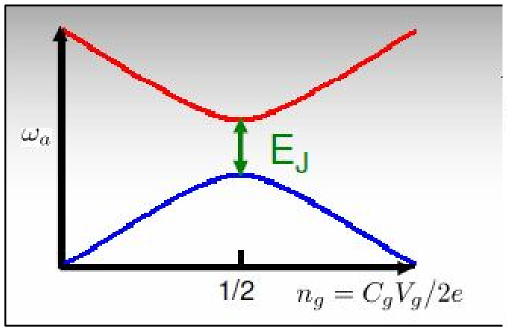
qubit ac Stark shift
and cavity shift

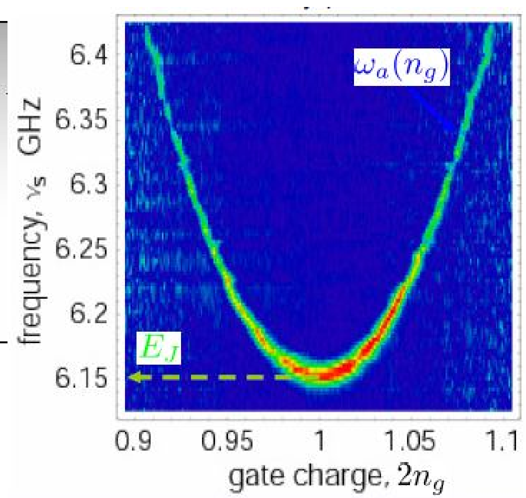
Lamb shift



Example: **spectroscopy of CPB charge qubit**

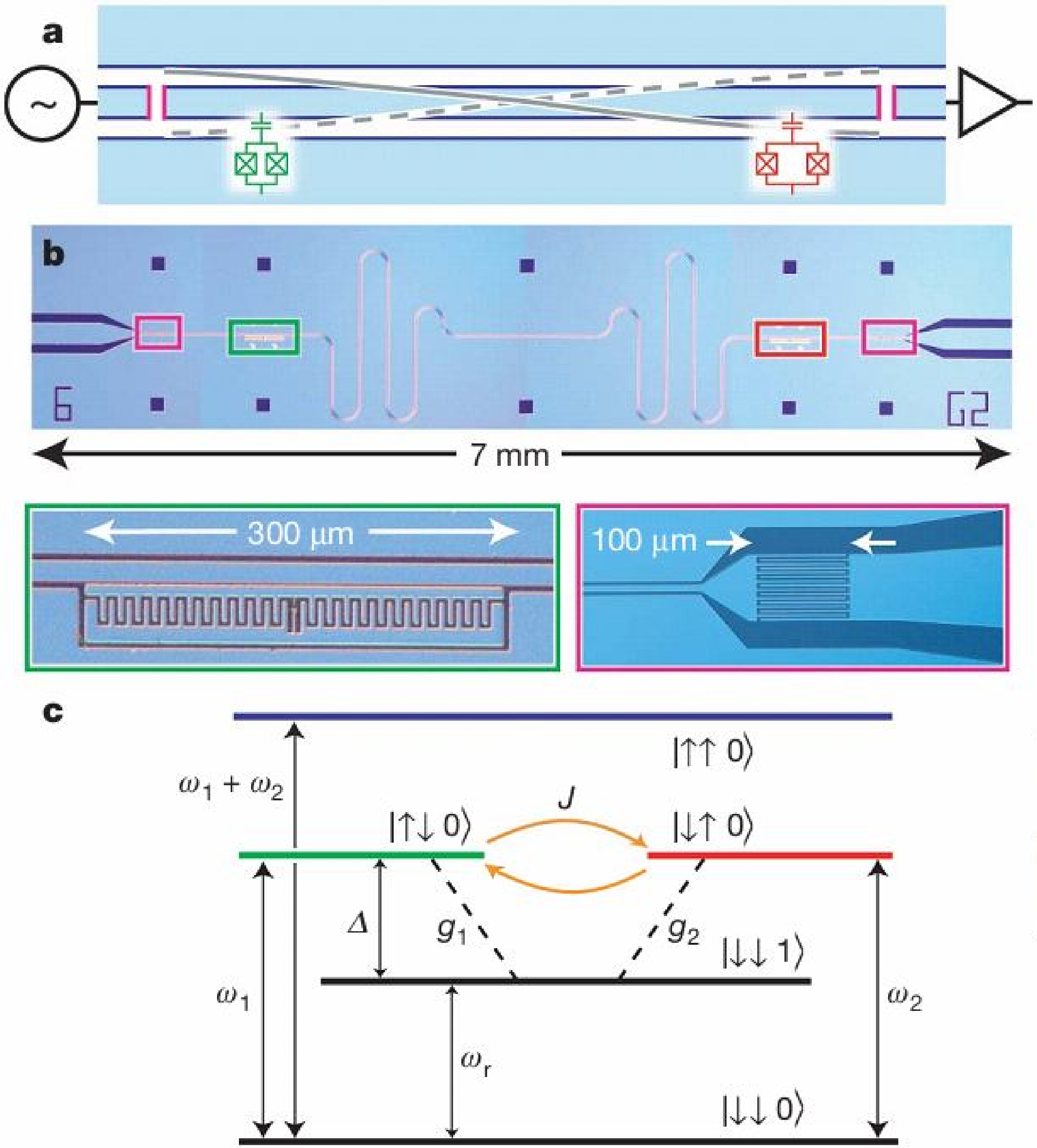
The transition energies mapped with sophisticated setup…





*D. Schuster et al., PRL **94** (2005), 123602.*

Quantum bus



Exchange of excited states via virtual photons in the dispersive regime

Phase qubits…