Current correlations in the interacting Cooper-pair beam-splitter

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(Dated: September 13, 2011)

Using a conserving many-body treatment, we propose an approach allowing the computation of currents and their correlations in interacting multi-terminal mesoscopic systems involving quantum dots coupled to normal and/or superconducting leads. We illustrate our method with the Cooperpair beam-splitter setup recently proposed [1, 2], which we model as a double quantum dot with weak interactions, connected to a superconducting lead and two normal ones. Our results suggest that even a weak Coulomb repulsion tends to favor positive current cross-correlations.



Zoltán Scherübl Nanophysics Seminar BUTE 22.09.2011.



The Hamiltonian

$$\begin{split} H &= \sum_{j,\alpha} \left(H_{\alpha} + H_{j} + H_{j\alpha} \right) \\ H_{\alpha} &= \epsilon_{\alpha} \hat{d}_{\alpha}^{\dagger} d_{\alpha} + \sum_{\beta \neq \alpha} \left(\frac{t_{\alpha\beta}}{2} d_{\alpha}^{\dagger} \sigma_{z} d_{\beta} + h.c. \right) + U_{\alpha} n_{\alpha\uparrow} n_{\alpha\downarrow} \quad \text{(dots)} \\ H_{j} &= \sum_{k} \hat{\Psi}_{jk} (\xi_{k} \sigma_{z} + \Delta_{j} \sigma_{x}) \hat{\Psi}_{jk} \quad \text{(leads)} \\ H_{j\alpha} &= \sum_{k} \left(\hat{\Psi}_{jk}^{\dagger} T_{j\alpha}(t) \hat{d}_{\alpha} + h.c. \right) \quad \text{(hopping)} \\ \text{where} \quad \hat{d}_{\alpha}^{\dagger} &= \left(d_{\alpha\uparrow}^{\dagger} d_{\alpha\downarrow} \right) \\ \hat{\Psi}_{jk}^{\dagger} &= \left(\Psi_{jk\uparrow}^{\dagger} \Psi_{j-k\downarrow} \right) \\ T_{j\alpha}(t) &= t_{j\alpha} \sigma_{z} e^{i\sigma_{z} V_{j} t} \end{split}$$

Numerical method

Based on Kadanoff-Baym-Keldysh perturbation theory The Luttinger-Ward functional: (function of single-particle Green function and sum of all closed-loop two-particle irreducible diagrams

(a)
$$\Phi\left[\check{G}\right] = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{6} + \dots$$

The interacting self-energy is function of VL, but not of the frequency From these the single- and two-particle Green functions can be derived

Calculating the current



FIG. 2: Currents I_{L1} and I_{R2} (arbitrary units) as a function of V_L , for different values of the interaction $U = 0, 0.05, 0.1, 0.15\Delta$, and the parameters (in units of Δ) $\beta = 100$, $\epsilon_1 = 0.5, \epsilon_2 = -0.5, V_R = -0.7$ and $t_{L1} = t_{S1} = t_{S2} = t_{R2} = 0.2$. The upper left panel corresponds to $t_d = 0$, while the lower panels are computed for $t_d = 0.2\Delta$. The upper right panel shows a representation of the setup.

Zero frequency current-current correlation without direct interdot tunneling



FIG. 3: Current cross-correlations (arbitrary units) as a function of V_L , for $U = 0, 0.05, 0.1, 0.15\Delta$, and the same parameters as in Fig. 2, in the absence of direct tunneling.

 VL < ε1: positive, approx. const correlation, strongly enhanced by interaction (due to the twoparticle effects)

• V_L > ϵ 1: CAR is unfavored,

electron co-tunneing expected (negativ correlation), but the interaction changes the sign of the correlation

Zero frequency current-current correlation with direct interdot tunneling



FIG. 4: Current cross-correlations (arbitrary units) as a function of V_L , for $U = 0, 0.05, 0.1, 0.15\Delta$, and the same parameters as in Fig. 2, in the presence of direct tunneling, $t_d = 0.2\Delta$.

• Low voltage: still positive, enhanced by interaction, but more feature at $V_L \approx -\varepsilon 1$ (because of DOS)

 VL > ε1 interactions have weaker effect (reduce the correlations)

Current cross-correlations



Effect of td: Shift the correlations to negativ values Effect of U: Shift the correlations to positive values

→ Competition between direct interdot tunneing and local Coulomb interaction

Possible extension

- Non local Coulomb interaction (between dots)
- Include second set of diagrams into the LWf
 - Freq dependent interacting self-energy