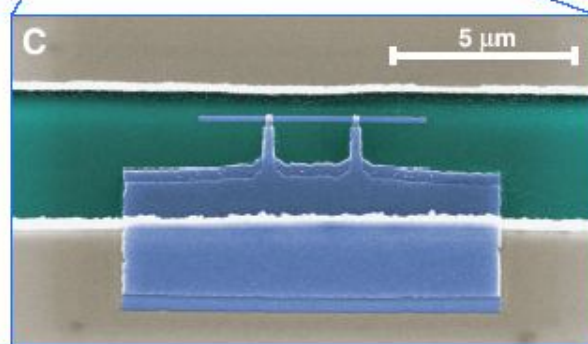
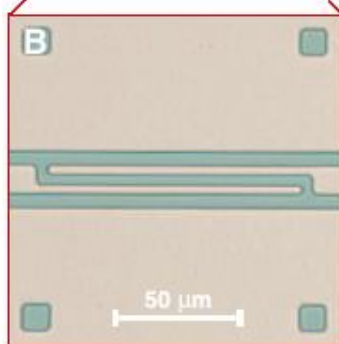
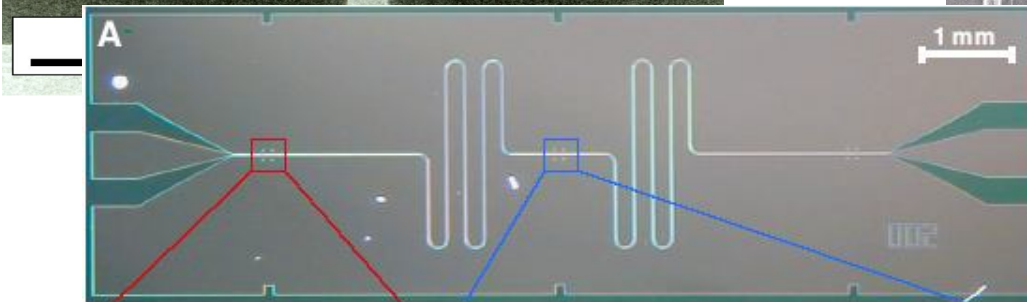
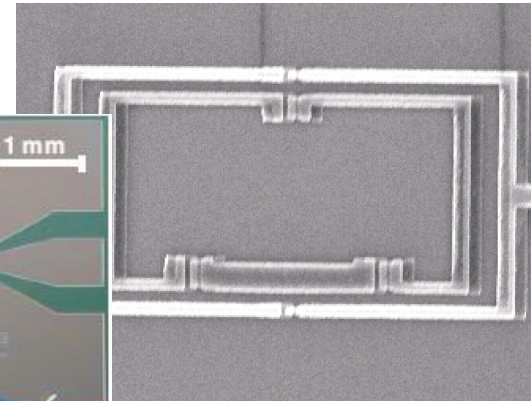
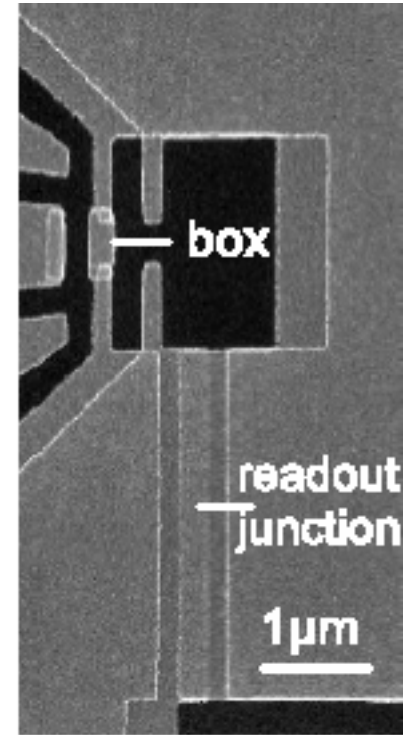
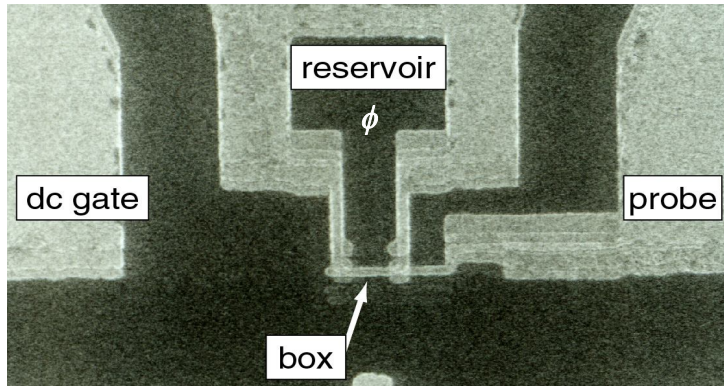
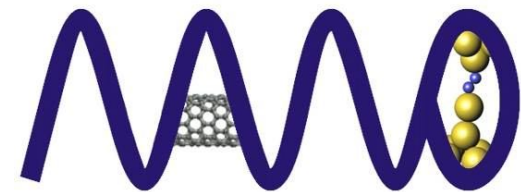


# Superconducting quantum bits



Péter Makk





## Qubit = quantum mechanical two level system

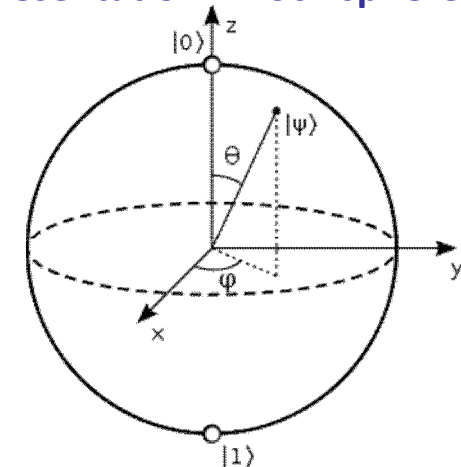
DiVincenzo criteria for quantum computation:

1. Register of 2-level systems (qubits),  $n = 2^N$  states: eg.  $|101..01\rangle$  (N qubits)
2. Initialization: e.g. setting it to  $|000..00\rangle$
3. Tools for manipulation: 1- and 2-qubit gates, e.g. Hadamard gates:  $U_H|0\rangle = (|0\rangle + |1\rangle)/2$ , and CNOT gates to create entangled states,  $U_{\text{CNOT}}U_H|00\rangle = (|00\rangle + |11\rangle)/2$
4. Read-out :  $|\psi\rangle = a|0\rangle + be^{i\Phi}|1\rangle \rightarrow a, b$
5. Long decoherence times:  $> 10^4$  2-qubit gate operations needed for error correction to maintain coherence "forever".
6. Transport qubits and to transfer entanglement between different coherent systems (quantum-quantum interfaces).
7. Create classical-quantum interfaces for control, readout and information storage.

## Two level system (quasi-spin)

$$\left. \begin{aligned} |1\rangle &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ |0\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{aligned} \right\} \text{basis, general state by rotation:}$$
$$|\psi\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)e^{i\Phi}|1\rangle$$

## Representation : Bloch sphere





## N-qubit states

$$|\psi\rangle = |\psi_1\rangle|\psi_2\rangle \dots |\psi_N\rangle = |\psi_1\psi_2\dots\psi_N\rangle$$

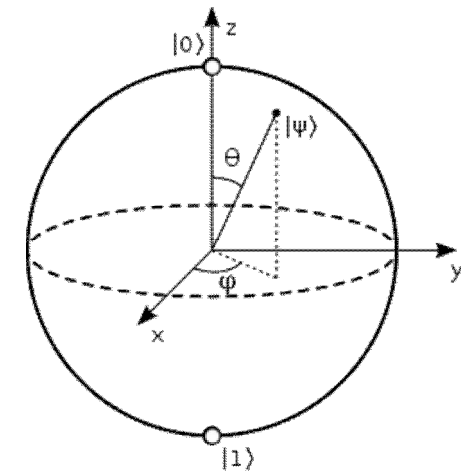
$$|\psi\rangle = c_1|0\dots 00\rangle + c_2|0\dots 01\rangle + \dots + c_n|1\dots 11\rangle$$

## Gates: Unitary operators

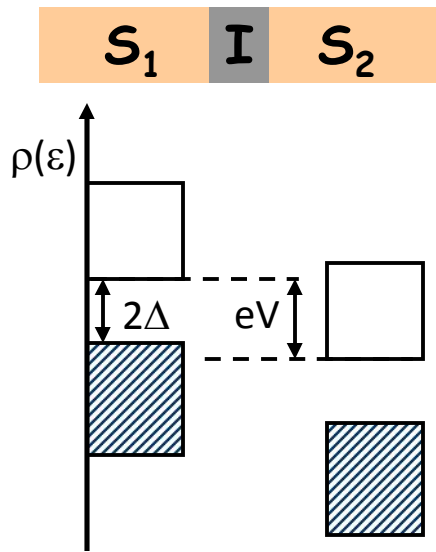
All operations are possible with one-qubit and one two-qubit gates  
e.g. CNOT: first control bit, second not

$$\begin{aligned} |0\rangle |0\rangle &\rightarrow |0\rangle |0\rangle \\ |0\rangle |1\rangle &\rightarrow |0\rangle |1\rangle \\ |1\rangle |0\rangle &\rightarrow |1\rangle |1\rangle \\ |1\rangle |1\rangle &\rightarrow |1\rangle |0\rangle. \end{aligned}$$

$$U_{\text{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$



# Josephson junctions



$$I = I_c \sin(\delta)$$

$$V = \frac{\Phi_0}{2\pi} \frac{d\delta}{dt}$$

Josephson equations

$$\Phi_0 = \frac{h}{2e}$$

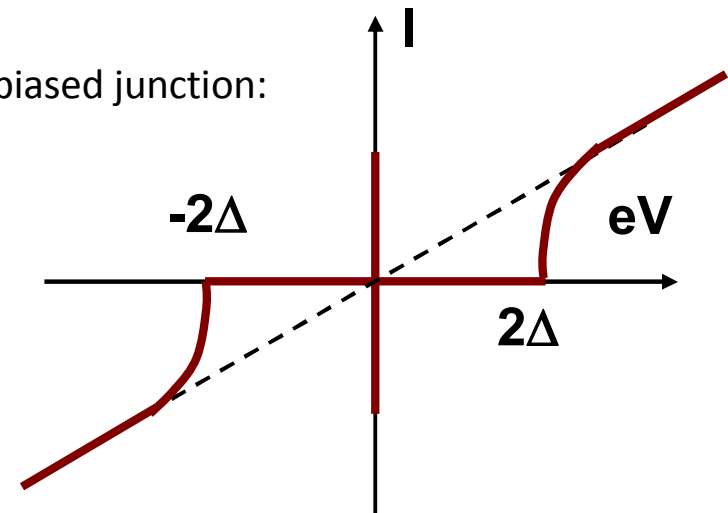
$$I_c = \frac{\pi\Delta}{2eR_n} \tanh(\Delta/2T)$$

Applying a constant bias voltage:

$$I = I_c \sin\left(\delta_0 + \frac{2eVt}{\hbar}\right)$$

An AC current with  $\omega=2eV/\hbar$  is flowing.  
The DC current averages to zero.

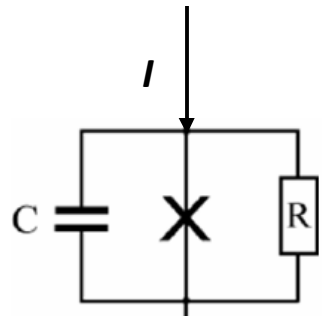
Voltage biased junction:



# Josephson junctions



Equation of motion



$$I = C \frac{dV}{dt} + \frac{V}{R} + I_c \sin(\delta)$$

$$V = \frac{\Phi_0}{2\pi} \frac{d\delta}{dt}$$

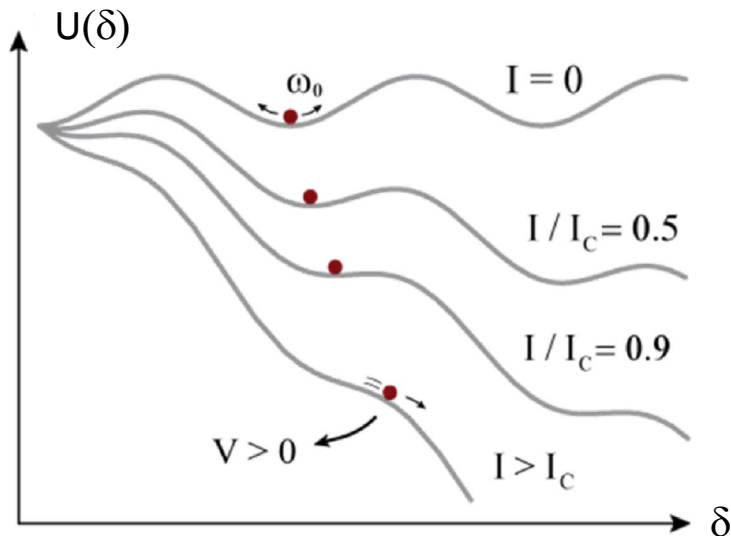
$$\left. \begin{aligned} 0 &= C \frac{\Phi_0}{2\pi} \frac{d^2\delta}{dt^2} + \frac{\Phi_0}{2\pi} \frac{1}{R} \frac{d\delta}{dt} - \frac{d}{d\delta} [I_c \cos(\delta + I\delta)] \\ &\leftrightarrow \\ F(t) &= m \frac{d^2x}{dt^2} + 2\gamma \frac{dx}{dt} - \frac{dU(x)}{dx} \end{aligned} \right\}$$

$$U(\delta) = -\frac{\Phi_0}{2\pi} [I_c \cos(\delta) + I\delta] \quad \text{potential}$$

$$\omega_p = \left( \frac{2\pi I_c}{C\Phi_0} \right)^2 [1 - (I/I_c)^2]^{1/4}$$

$$Q = \omega_p RC$$

„washboard potential”



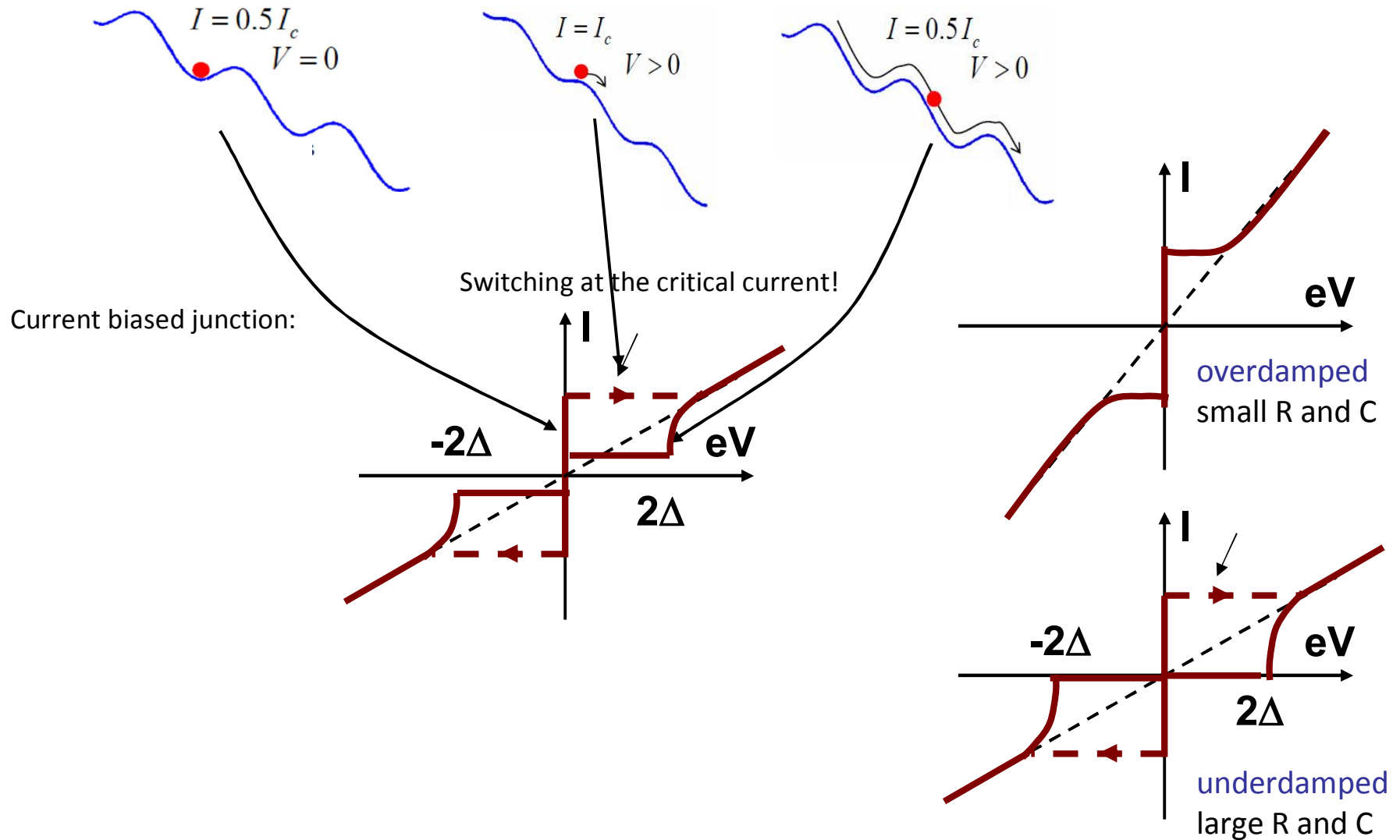
plasma oscillations  
for  $I > I_c$  DC voltage appears

# Josephson junctions

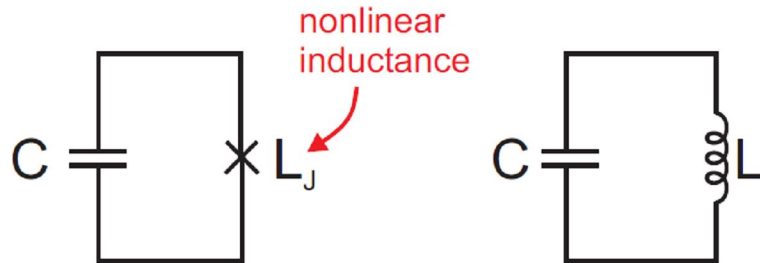


$S_1$  |  $I$  |  $S_2$

keeps sliding due to kinetic energy



# Josephson junctions



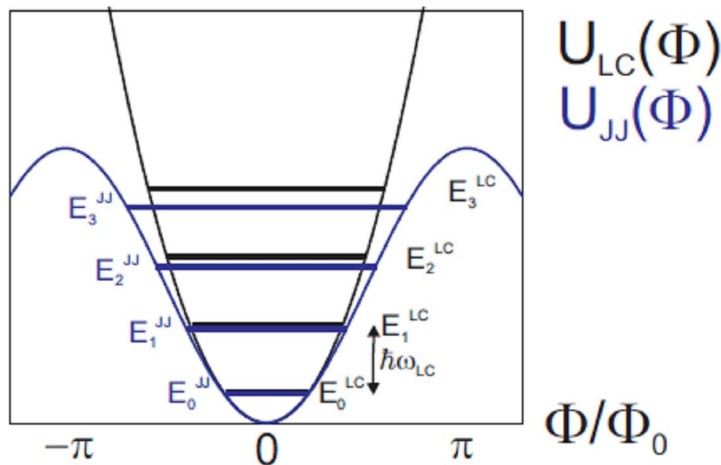
$$H = \frac{1}{2}CV^2 + \frac{1}{2}LI^2 \quad \text{resonant circuit}$$

$$V = L \frac{dI}{dt} \longrightarrow \Phi = LI \quad P_\Phi = CV = C\dot{\Phi}$$

$$H = \frac{1}{2C}p_\Phi^2 + \frac{1}{2L}\Phi^2$$

Flux quantization phase difference  $\leftrightarrow$  flux

Josephson junctions is a *non-linear inductance*  
the energy spectra is anharmonic



$$I = I_c \sin(2\pi\Phi/\Phi_0)$$

$$L_J = \frac{\Phi_0}{2\pi I_c}$$

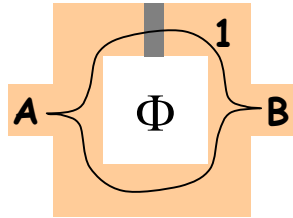
$$I = \frac{\Phi_0}{2\pi L_J} \sin(2\pi\Phi/\Phi_0)$$

for small  $\Phi$

$$I \simeq \frac{\Phi}{L_J}$$

$$H = \frac{1}{2} \frac{1}{C} p_\Phi^2 + \frac{(\Phi_0/(2\pi))^2}{L_J} \left( 1 - \cos 2\pi \frac{\Phi}{\Phi_0} \right)$$

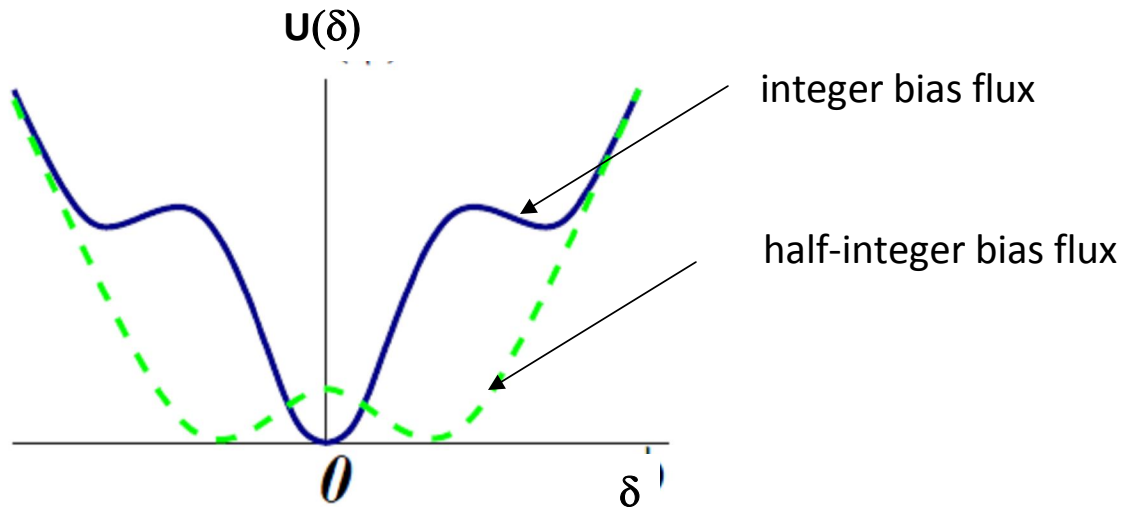
$\longrightarrow$  qubit is separable



Equation of motion

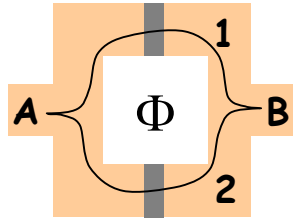
$$0 = C \frac{\Phi_0}{2\pi} \frac{d^2\delta}{dt^2} + \frac{\Phi_0}{2\pi} \frac{1}{R} \frac{d\delta}{dt} - I_c \sin(\delta) - \frac{1}{L}(\delta - \Phi)$$

$$U(\delta) = -\frac{\Phi_0}{2\pi} I_c \cos(\delta) + \frac{1}{2L}(\delta - \Phi)^2 \quad \text{potential}$$



For half integer quantum, two minima:  
two persistent current states, circulating in different direction





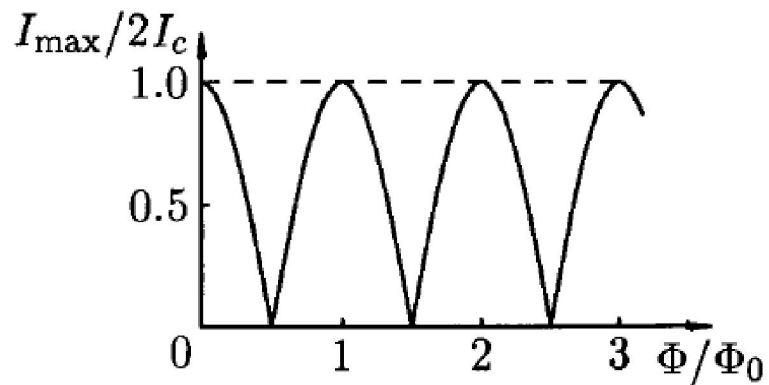
$$I = I_1 + I_2 = I_c [\sin(\delta_0 + 2\Phi / \hbar) + \sin(\delta_0 - 2\Phi / \hbar)] = 2I_c \sin \delta_0 \cos(e\Phi / \hbar)$$

If the critical current of the two qubits are the same

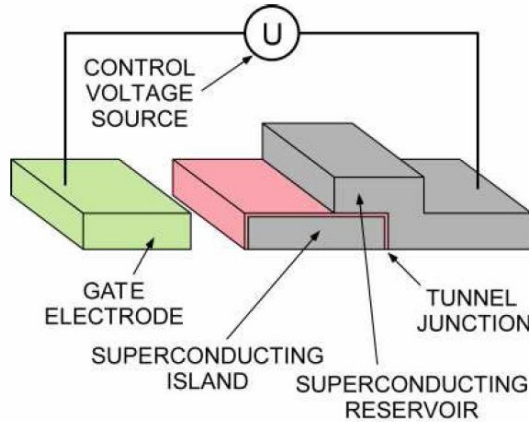
The maximal value of the critical current is tuned by the magnetic flux:  $I_{\max} = 2I_c |\cos(e\Phi / \hbar)|$

Dependence of the maximum supercurrent (critical current) through the two-junction interferometer on the total magnetic flux through its interior.

The strong dependence of  $I_{\max}$  on flux makes the DC-SQUID an **extremely sensitive flux detector**.



# Cooper pair box



Electrostatic energy (like for a Qdot):

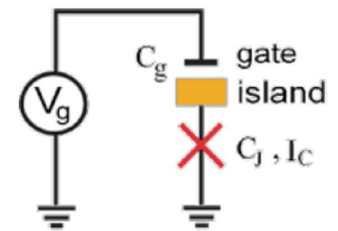
$$H_{el} = E_C (N - n_g)^2$$

Josephson energy:

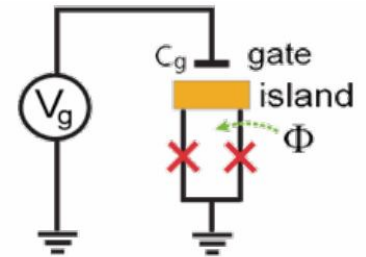
$$H_J = -E_J \cos \delta$$

$$E_C = \frac{e^2}{2(C + C_g)}$$

normal CPB



split CPB

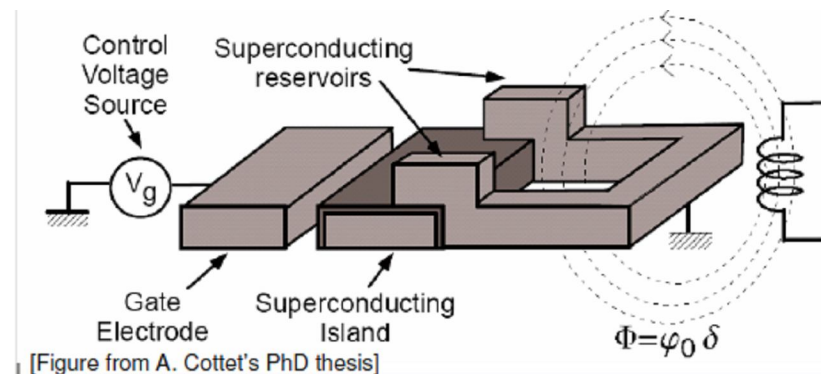


For two Josephson junctions:

The phase difference can be tuned by applying a magnetic field.

Thus the Josephson energy can be tuned

$$E_J^{\text{eff}}(\Phi) = \sqrt{\left(E_J^{(1)}\right)^2 + \left(E_J^{(2)}\right)^2 + 2 \cos(2\pi\Phi/\Phi_0) E_J^{(1)} E_J^{(2)}}$$



[Figure from A. Cottet's PhD thesis]

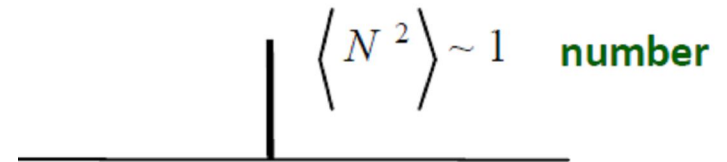
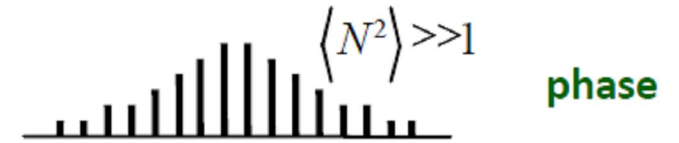
# Second quantization



Commutation relation

$$[\hat{N}, \hat{\delta}] = -2i$$

$$\hat{H} = E_J \frac{\hat{\delta}^2}{2} + E_C \frac{\hat{N}^2}{2}$$



**Phase and number are conjugated variables!**

*Phase representation* (analogue: coordinate representation)

N operator and N eigenstates:  $-2i\partial/\partial\delta$  and  $N \rightarrow e^{iN\delta/2}$

$$e^{i\delta}|N\rangle = |N+2\rangle$$

$$\cos(\delta) = \sum_N |N\rangle\langle N+2| + |N+2\rangle\langle N|$$

*in the other representation*

$$\hat{N}|N\rangle = N|N\rangle$$

$$H_{el} = E_C \sum_N (N - N_g)^2 |N\rangle\langle N|$$

$$H_J = -\frac{E_J}{2} \sum_N (|N\rangle\langle N+2| + |N+2\rangle\langle N|)$$

**transfer of Cooper pairs**

# Charge qubit



For  $E_J \ll E_C$

The charging energy states are degenerate at the crossing point:

$E_J$  makes a transition and lifts the degeneracy:

The charge qubit near the degeneracy point

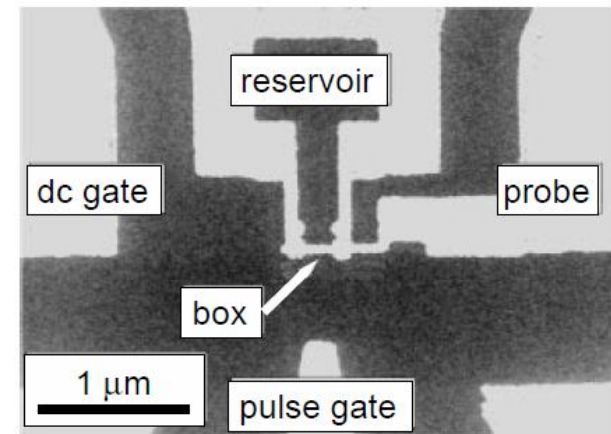
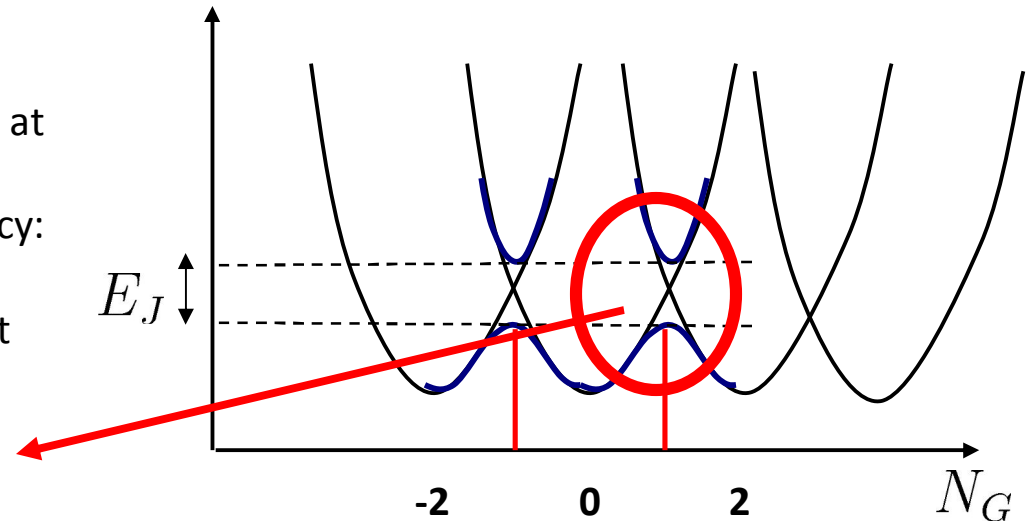
$$\hat{H} = E_C + \begin{pmatrix} \varepsilon & E_J/2 \\ E_J/2 & -\varepsilon \end{pmatrix}$$

where :  $\varepsilon = 2E_C(1 - N_G)$

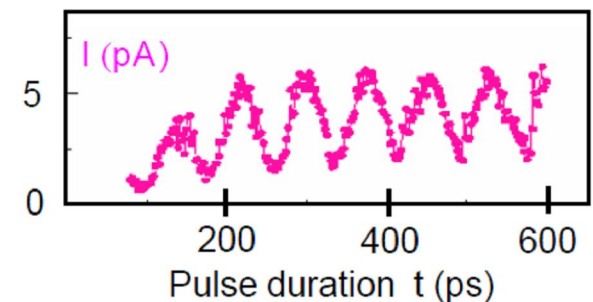
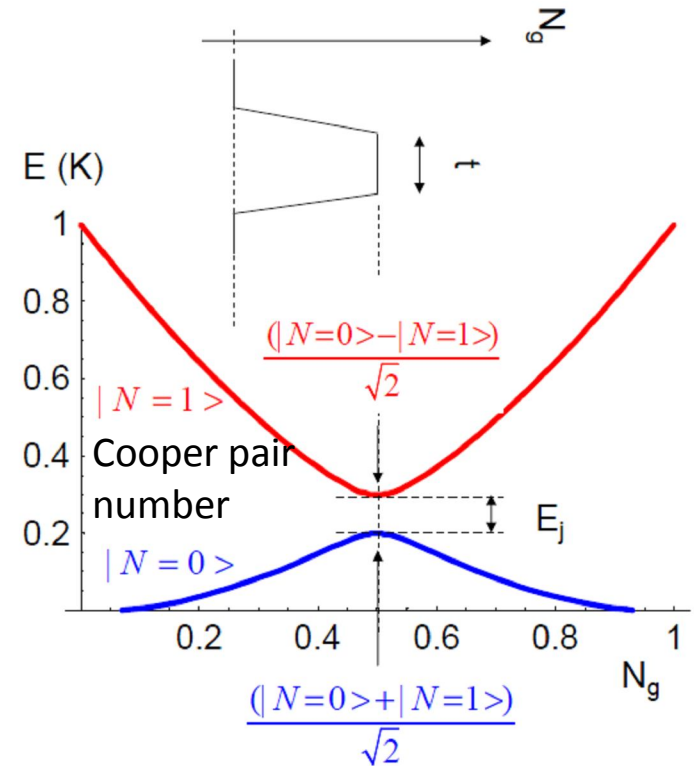
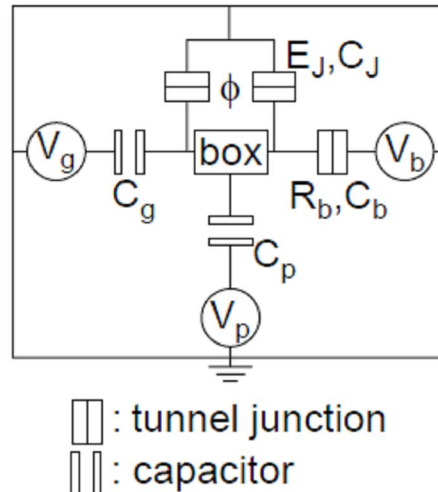
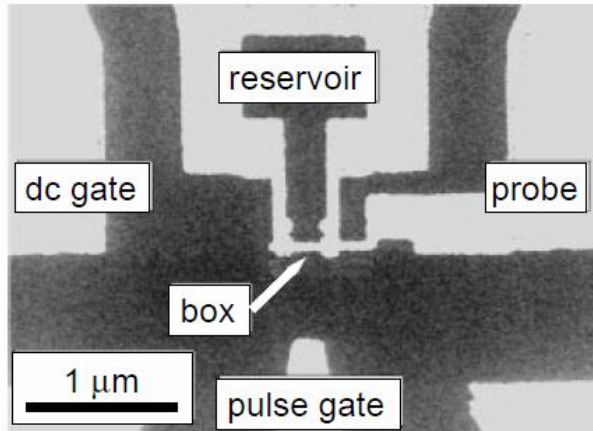
At the degeneracy point the eigenstates are trivial:  $|\pm\rangle = (|0\rangle \pm |2\rangle)/\sqrt{2}$

Far from the degeneracy points the original states are good eigenstates.

Higher order coupling through EJ are negligible



# Charge qubit



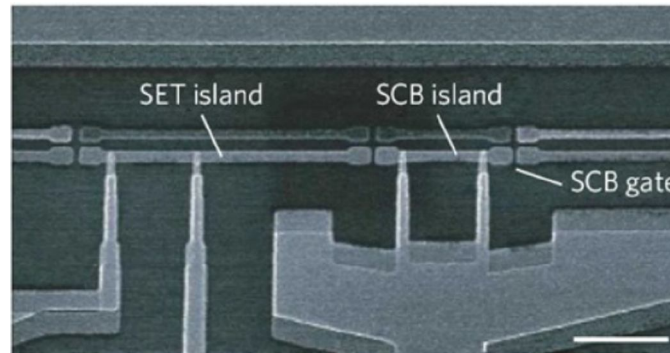
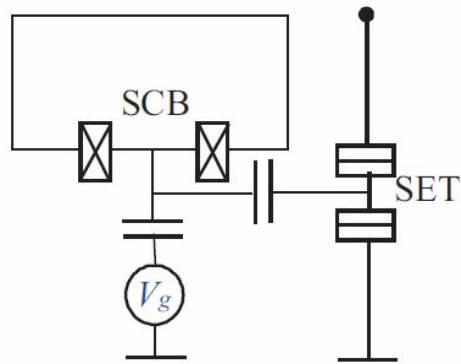
- First the qubit is prepared in state  $|0\rangle$
- Fast DC pulse to the gate  $\rightarrow$  not adiabatic, it remains in  $|0\rangle$
- It starts Rabi-oscillating, and evolves during the pulse length ( $t$ )
- If after the pulse, the qubit is in  $|1\rangle$  it will relax back and the current is detected (high repetition frequency) – **single shot readout**
- Detection: working point for state 1 is above delta in energy  $\rightarrow$  decay to quasi particles
- By adjusting  $t$ , the length of the pulse Rabi oscillation is seen
- DC gate is need to calibrate the pulse gate
- Relaxation  $<10\text{ns}$ , probably due to charge fluctuations

# Rabi oscillations

BUTE, Low temperature  
solid state physics laboratory



Other readout for charge qubit: with SET



# Relaxation - $T_1$ and $T_2$



## Larmor precession

$$\frac{d\mu}{dt} = \mu \times (\gamma H_0) \quad \text{where the larmor frequency is: } \omega_L = \gamma H_0$$

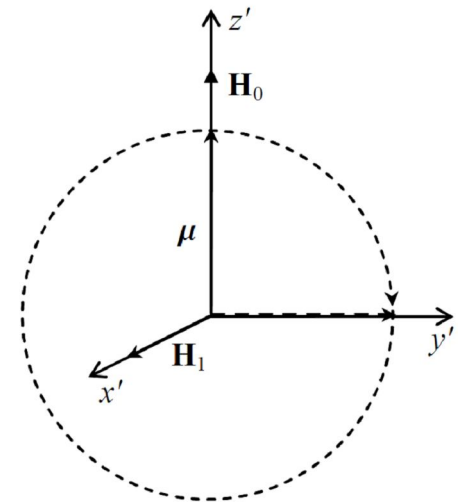
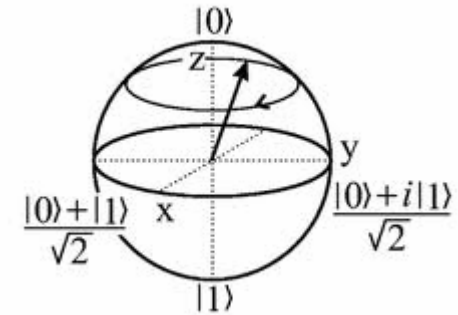
## Adding a small modulating field

$$H(t) = (H_1 \cos(\omega t), H_1 \sin(\omega t), H_0)$$

In rotating frame with angular momentum  $(0,0,\omega)$ ,  $H_1$  is static:  $H_1' = (H_1, 0, 0)$

$$\frac{\delta\mu}{\delta t} = \mu \times \gamma [(H_0 + \omega/\gamma)\hat{z}' + H_1\hat{x}']$$

If  $\omega = \omega_L$ , then the magnetic moment will precess around  $H_1'$   
 With short pulses, the moment can be rotated with arbitrary angle  
 E.g.  $\pi/2$  pulse rotates to  $y'$  axis, a precession in the  $x$ - $y$  plane in the lab frame



# Relaxation - $T_1$ and $T_2$

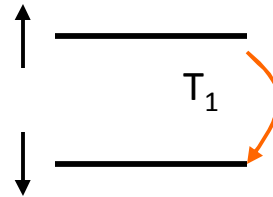


## Relaxation – Bloch equations

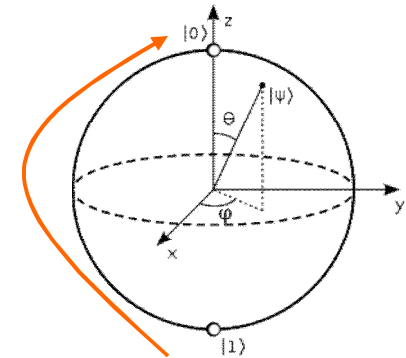
$$\frac{dM_{x,y}}{dt} = \gamma[\mu \times H_0]_{x,y} - \frac{M_{x,y}}{T_2} \quad \omega_L = \gamma H_0$$

$$\frac{dM_z}{dt} = \gamma[\mu \times H_0]_z - \frac{M_0 - M_z}{T_1}$$

$T_1$  (spin-lattice in NMR)



$T_2$  – decoherence time (spin-spin in NMR)



The time until a superposition preserves its phase

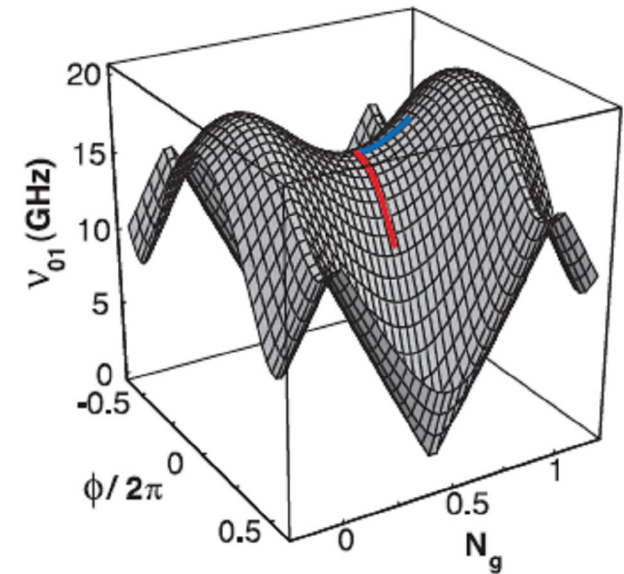
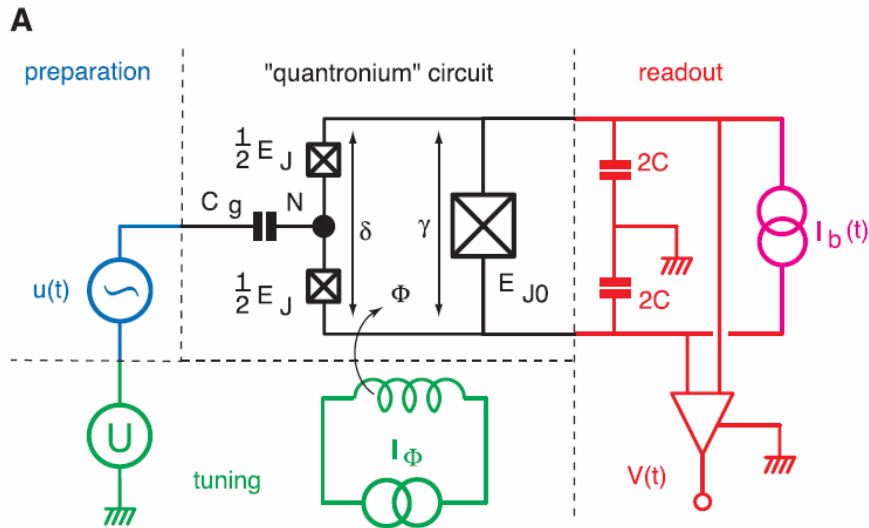
In NMR lot of spins, which precess with different larmor frequency, and the net spin disappears



# Quantronium



- charge qubits operated at the degeneracy point are not sensitive to *charge* fluctuations (2nd order) but to *phase* fluctuations
- for charge-flux qubit there is also a sweet spot in the flux-charge plain (**Quantronium**)



Quantronium is operated at the sweet point, where  $I_b=0$ .

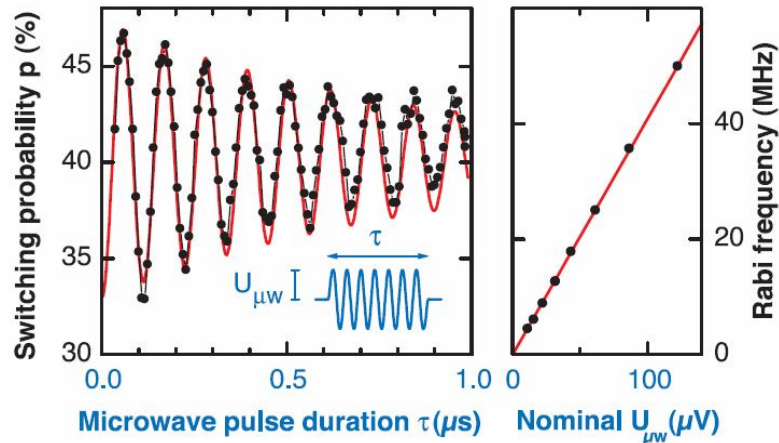
Except the sweet point a net supercurrent flows, the direction depending on the state

Another, more transparent Josephson junction, with large  $E_j$  is added

For readout  $I_b$  is applied such, that  $I_b+I_0 < I_c$ , but  $I_b+I_1 > I_c$ , and finite voltage is measured

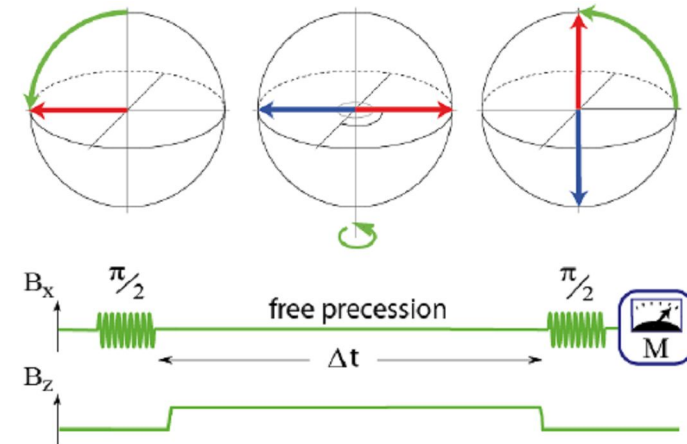
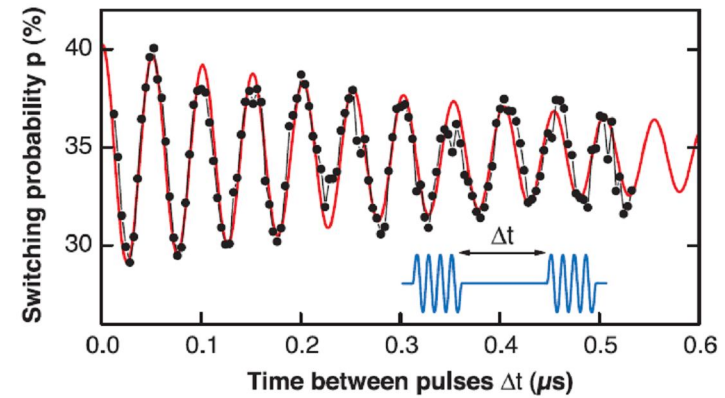


## Rabi oscillations



Rotation for  $\tau$  time, drive between the two states  
 $\rightarrow$  after the rotation measurement  
 Decay of oscillations go with  $T_{\text{Rabi}}$   
 Frequency depends on the microwave power

## Ramsey oscillations



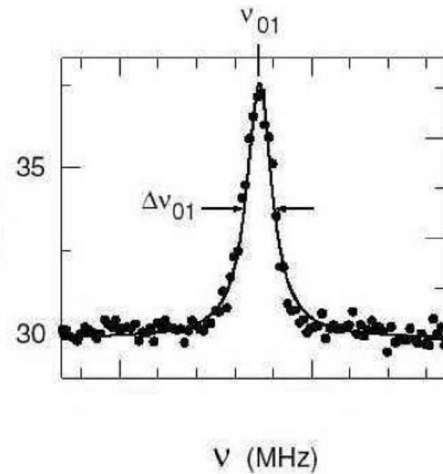
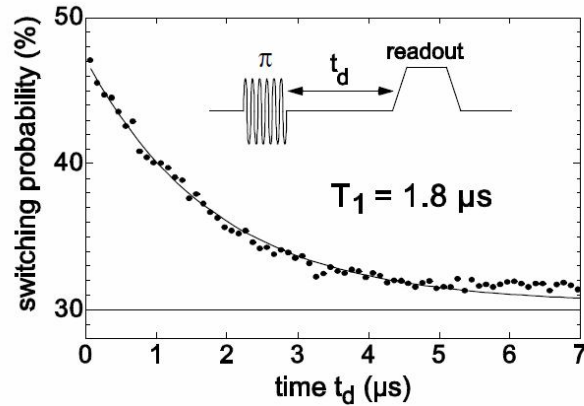
after free precession,  $\pi/2$  rotates up or down, depending on the acquired phase  
 $T_2$  follows from the decay of the oscillation

# Quantrium

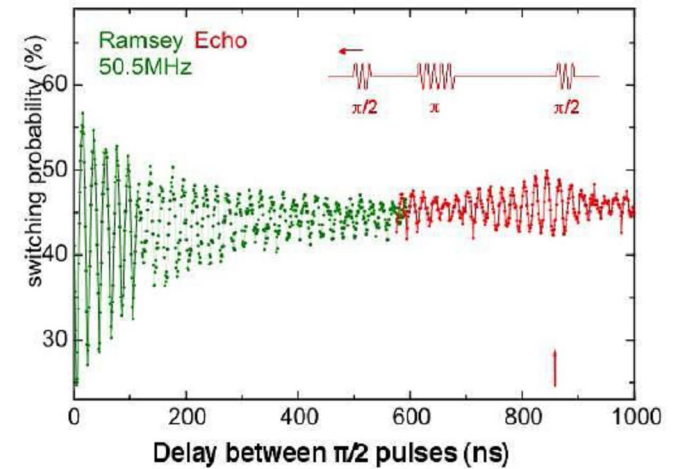
BUTE, Low temperature  
solid state physics laboratory



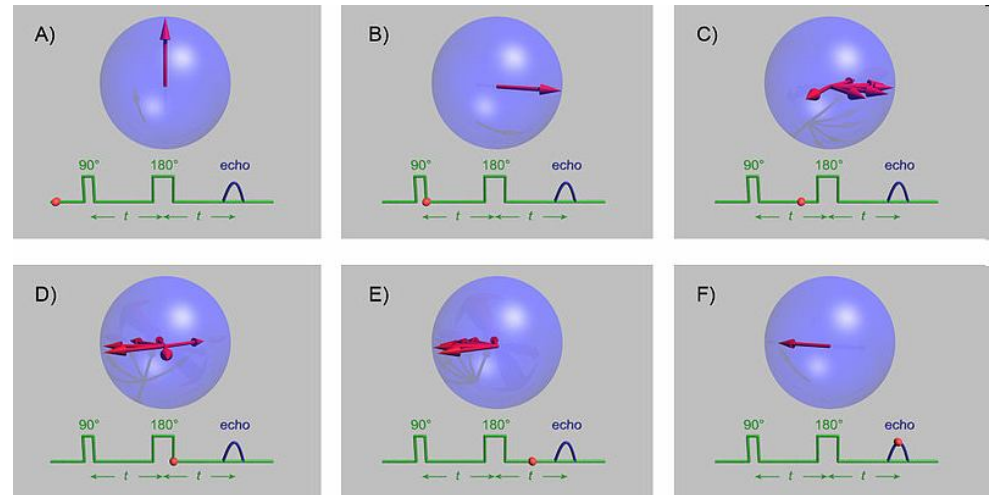
## Measurement of $T_1$



## Spin-echo



- $T_1$  is the longest scale and it has also contribution to  $T_2$
- With echo technique stationary inhomogenities can be filtered out
- from the resonance signal  $\sim T_2$  can be deduced

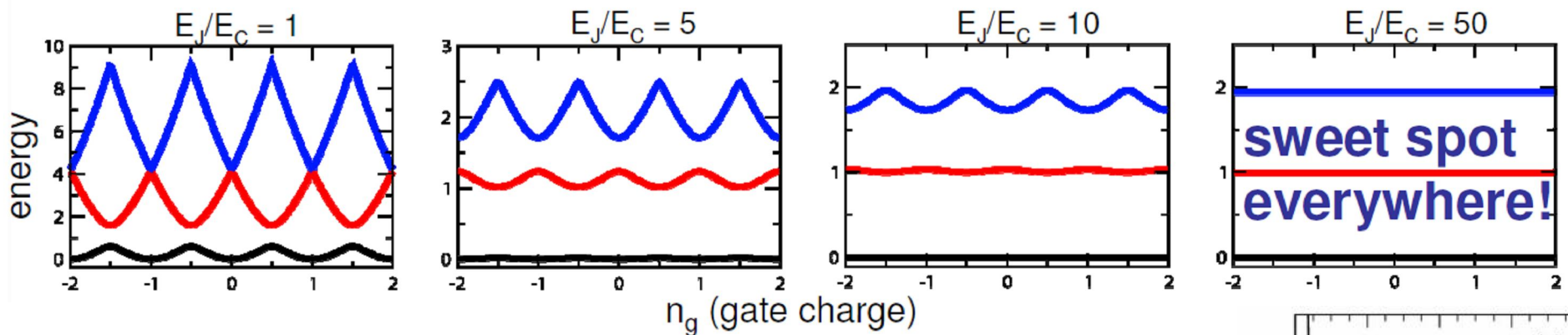


# Transmon

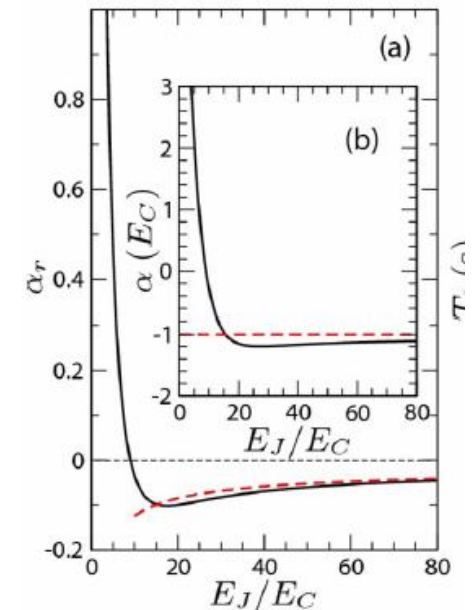


Problem with charge qubit: sensitivity to charge noise, fluctuations of the gate electrode  
The degeneracy point is a sweet point, sensitivity to charge fluctuations is second order

By increasing the  $E_J/E_C$  ratio:



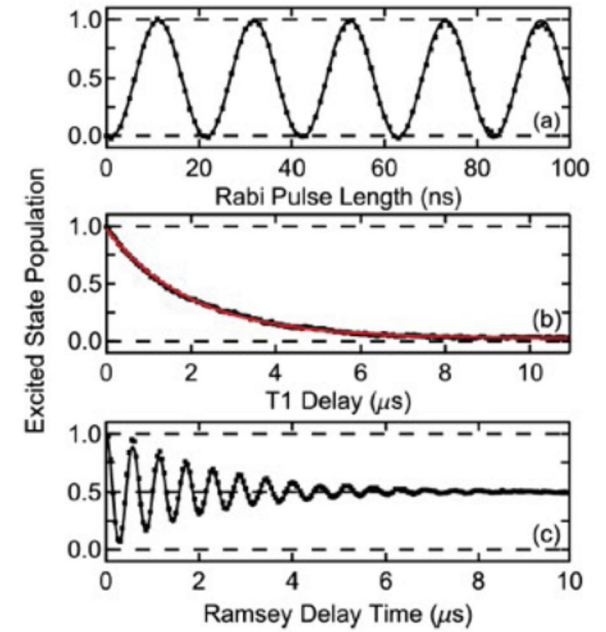
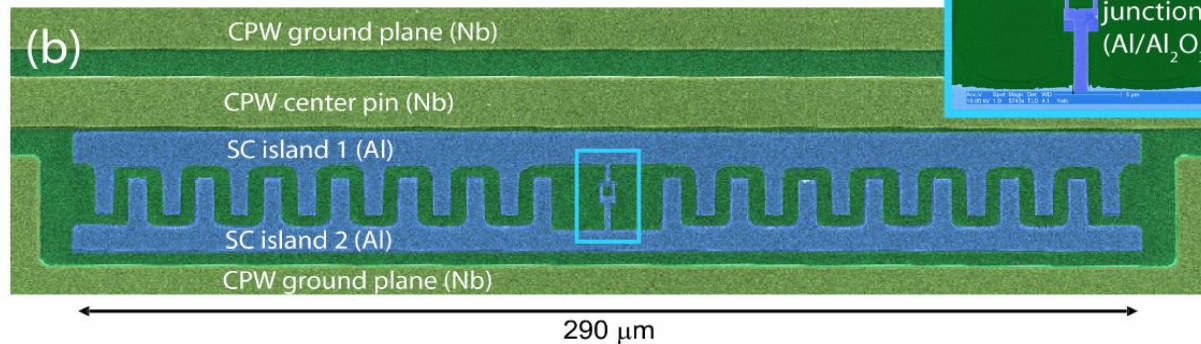
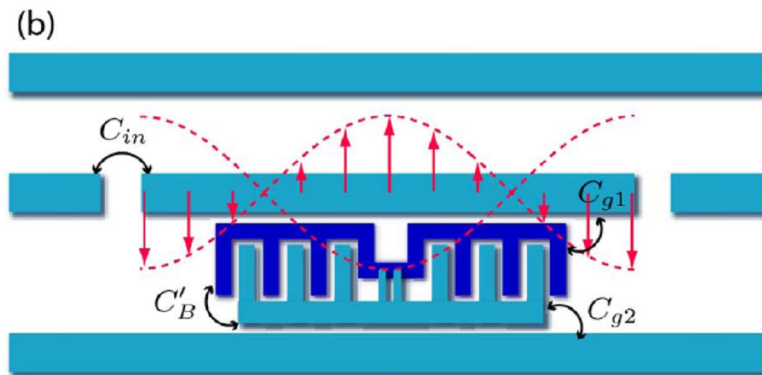
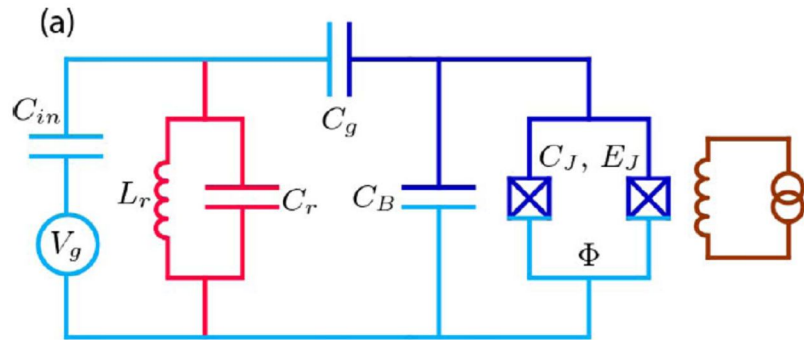
- *good news*: the dispersion of the bands disappear exponentially
- *bad news*: the anharmonicity decreases (even changes sign), but just linearly, still enough to separate qubit



# Transmon



$E_j/E_S$  increased by a shunting capacitance ( $E_C$  decreased): design parameter

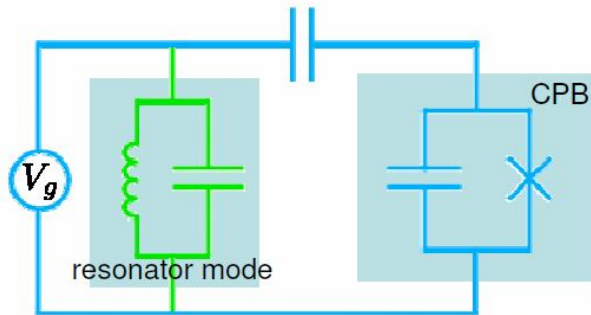


$T_1 \sim \mu\text{s}$   
 $T_\phi \sim 10\mu\text{s}$

# Transmon



## Coupling to resonator



$$V_g \rightarrow V_g + V_{RMS}(\hat{a} + \hat{a}^\dagger)$$

Coupling:

$$g = dE/\hbar$$

d and E higher than in 3d cavity

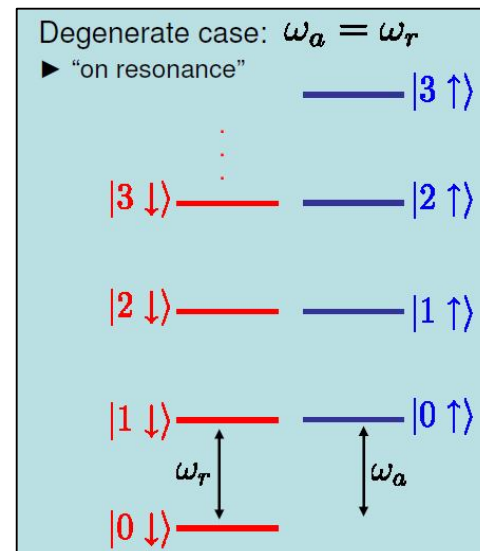
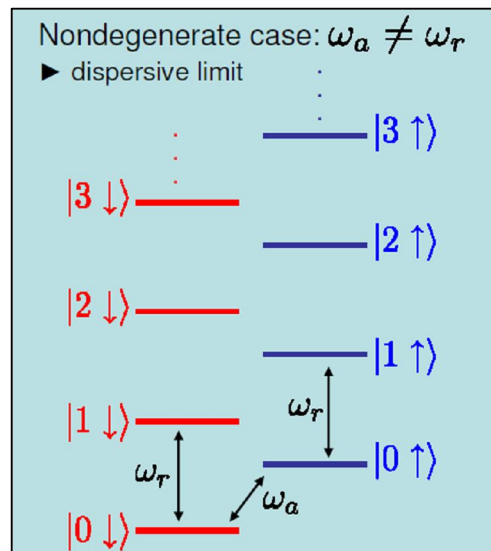
easy to arrive at strong coupling regime

$$H_c = 4/e * E_c C_G V_{RMS}(\hat{n}(\hat{a} + \hat{a}^\dagger))$$

Jaynes-Cummings Hamiltonian ↙ RWA

$$H = \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2) + \frac{\hbar\omega_a}{2}\sigma_z + \hbar g(\hat{a}\sigma_+ + \hat{a}^\dagger\sigma_-)$$

without coupling  
(g=0)

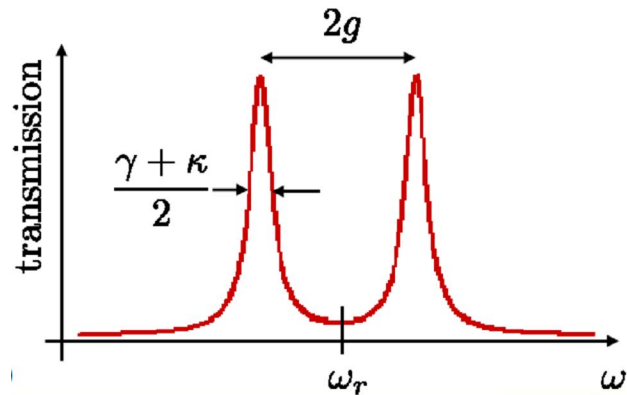
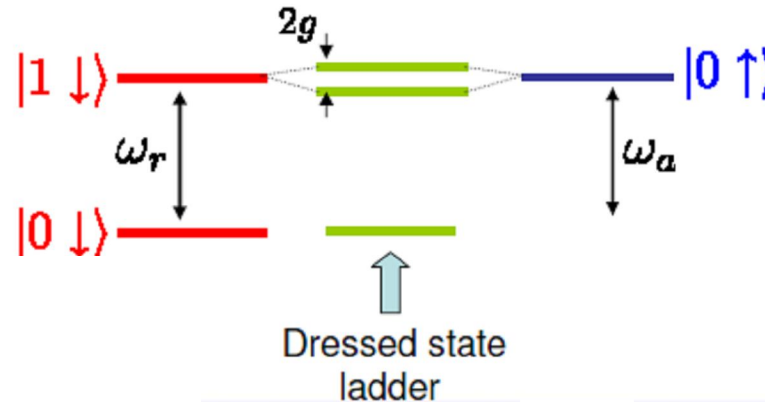


# Transmon

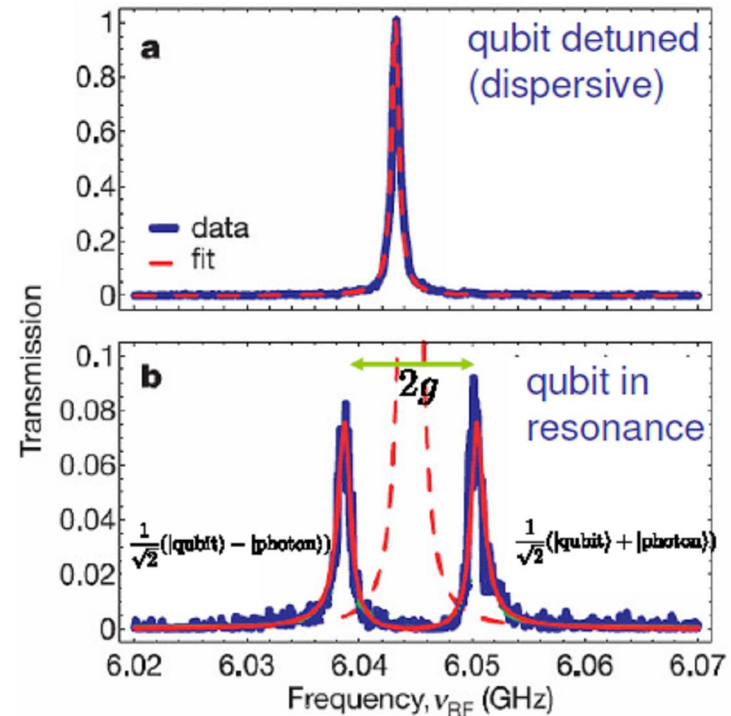


Resonant case,  $g \neq 0$

$$E_{n\pm} = n\hbar\omega \pm \hbar g\sqrt{n+1}$$



Vacuum Rabi splitting can be seen by measuring the transmission of the cavity



# Transmon



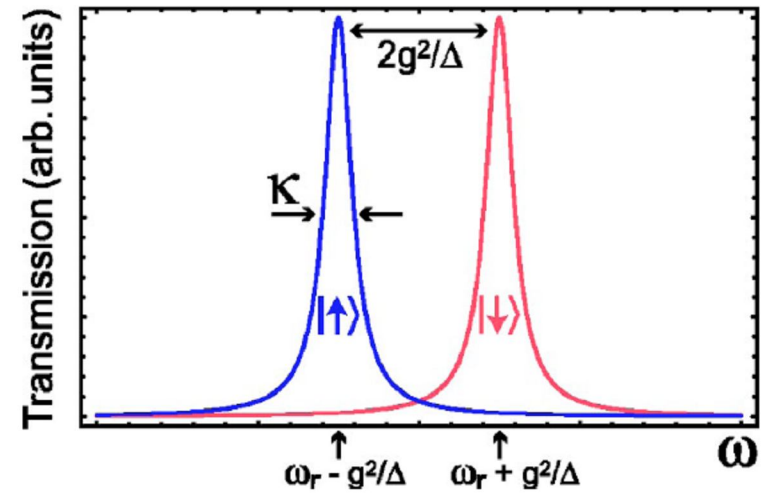
**Non resonant case,  $g \neq 0$**   
**dispersive regime:  $\omega_r - \omega_a \gg g$**

- State dependent polarizability of 'atom' pulls the cavity frequency

$$H_{eff} = \hbar \left( \omega_r + \frac{g^2}{\Delta} \sigma_z \right) + \frac{1}{2} \left( \omega_a + \frac{g^2}{\Delta} \sigma_z \right)$$

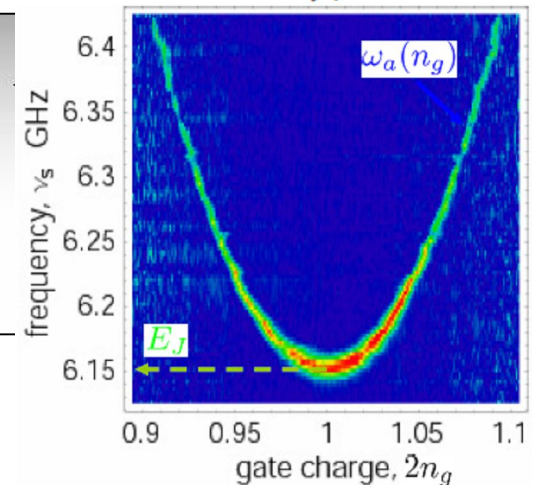
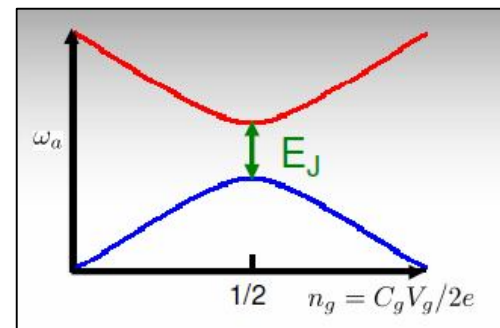
↙
↘

qubit ac Stark shift  
and cavity shift
Lamb shift



Example: **spectroscopy of CPB charge qubit**

The transition energies mapped with sophisticated setup...



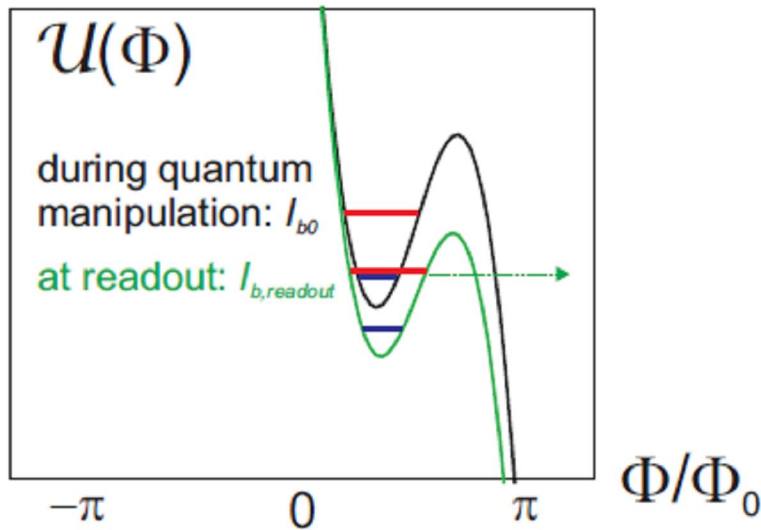


# Phase qubit



Tune the potential of a single JJ (washboard potential), such that it is asymmetric and only houses 2-3 levels

$$E_c \ll E_J, I < I_c$$



Problems: always connected to readout, quasiparticles shunt the junction, long recondensation time...

## a bit different design (flux controlled phase qubit)

flux bias  
dc squid readout

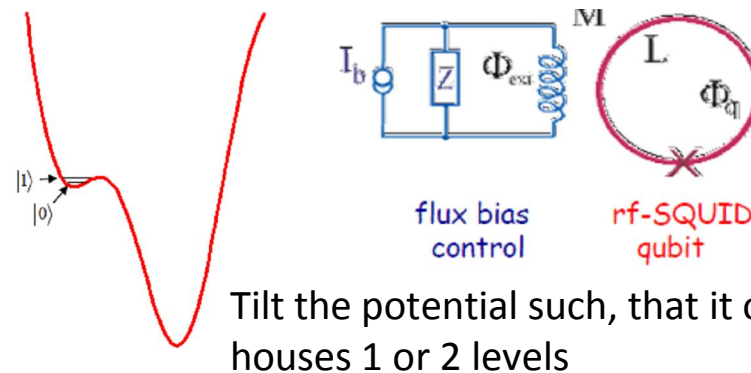
### Operation:

- Prepare eg. to state 1

### Readout:

- change  $I_c$ , such that 1 can tunnel out  $\rightarrow$  finite voltage appears on the junction
- or swap 1-2, and 2 can tunnel out

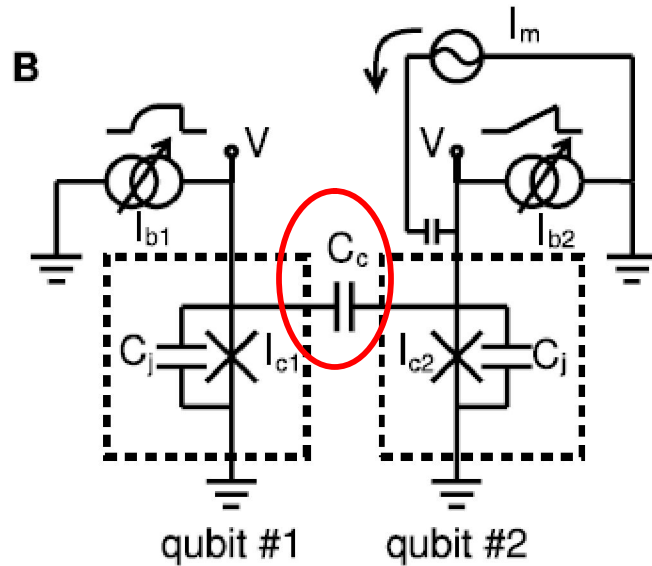
Rabi oscillations etc...



Tilt the potential such, that it only houses 1 or 2 levels



## Coupled phase qubits



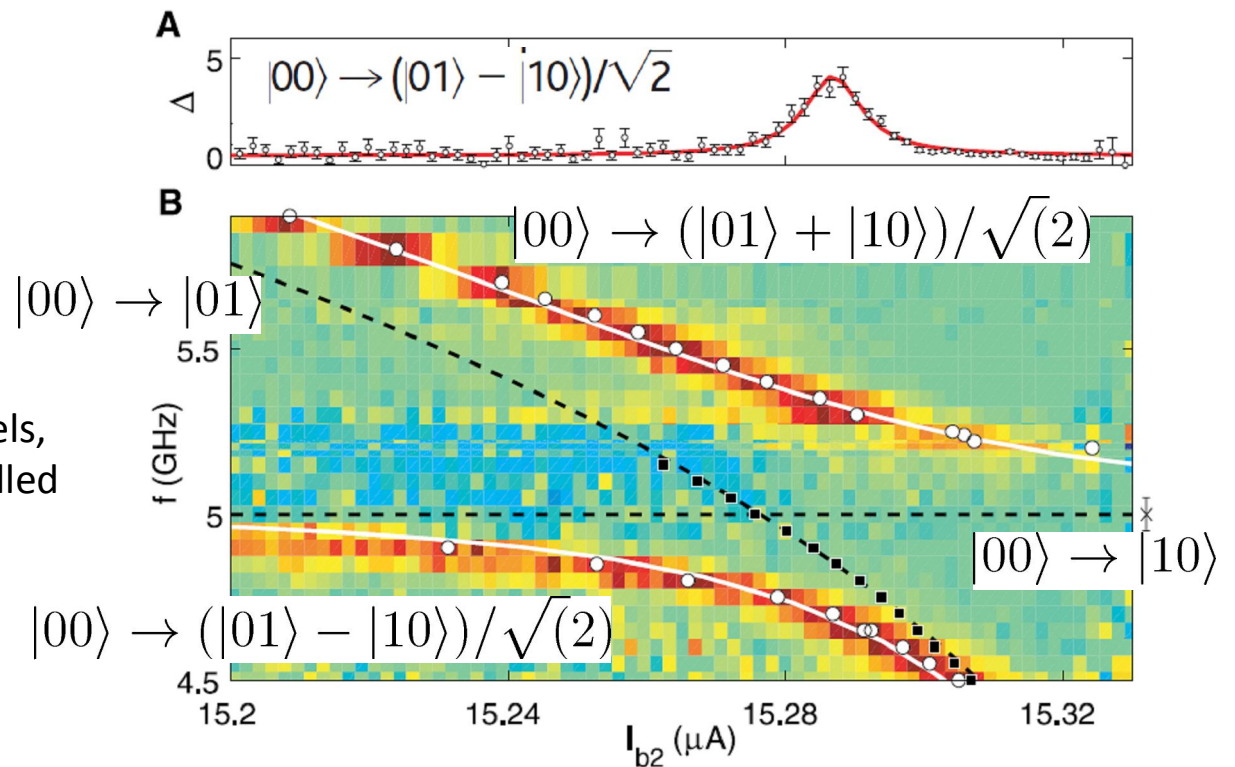
## Capacitive coupling

- The levels (splitting) of the single qubits are tunable
- If the level spacing is nearly the same than the states of the two qubits hybridize
- First three levels  $|00\rangle$ ,  $|01\rangle - |10\rangle$  and  $|01\rangle + |10\rangle$

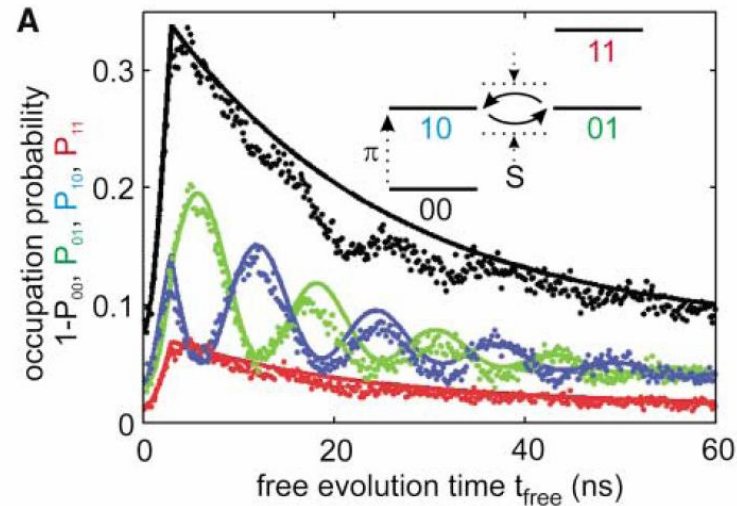
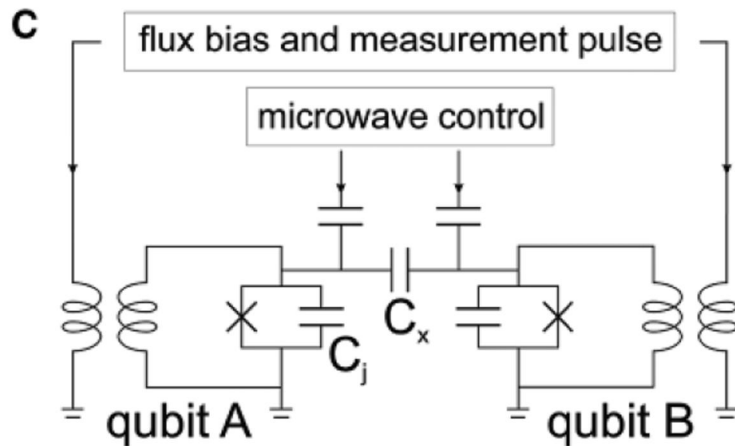
Escape probability is measured

$I_{b1}$  is fix,  $f$  is fix and  $I_{b2}$  is tuned  
For fix  $f$ ,  $I_{b2}$  will tune the coupled levels,  
and for some  $I_{b2}$  the condition is fulfilled

On the  $f$ - $I_{b2}$  map avoided crossing is  
seen  $\rightarrow$  coupling



# Phase qubit



Readout of both qubits at the same time  
Qubits are brought to resonance  
At resonance  $|01\rangle$  and  $|10\rangle$  are not eigenstates  
Prepare  $|00\rangle$ , then excite qubit 1  $\rightarrow |10\rangle$   
This is not an eigenstate and start to precess  
Measurement of the two qubits is anticorrelated  
Coherence time is the same as for single qubit

$$|\psi(t)\rangle = \cos(St/2\hbar)|10\rangle + i \sin(St/2\hbar)|01\rangle$$

$$H_{int} = S/2(|10\rangle\langle 01| + |01\rangle\langle 10|)$$

At  $t = \hbar/2S$ ,  $|10\rangle$  goes to  $i|01\rangle \rightarrow i\text{Swap}$

$$U(\hbar/2S) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Density matrix

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

If the whole system is entangled with the environment, then the reduced matrix is not pure  
Decoherence: offdiagonal elements

For a single qubit

$$\rho = \begin{bmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{bmatrix} = \frac{1}{2} \sum_{k=0,x,y,z} r_k \sigma_k \quad \text{and}$$

$$r_0 = \rho_{00} + \rho_{11} = 1$$

$$r_x = \rho_{01} + \rho_{10}$$

$$r_y = i(\rho_{01} - \rho_{10})$$

$$r_z = \rho_{00} - \rho_{11}$$

By projective measurement (one Stern-Gerlach measurement)

$$P_0 = \text{Tr}(\rho |0\rangle\langle 0|) = \frac{1}{2}(\sigma_0 + \sigma_z) = \frac{1}{2}(1 + r_z)$$

Other components, eg.  $r_x$  comes from the rotation around  $y$  by  $\pi/2$  so, that  $x$  and  $z$  are interchanged.  
More precisely by rotating with  $\theta$  angle and measuring fitting the angle dependence,  $r_x$  is obtained.

$$P_0 = \frac{1}{2}[1 + (\rho_{00} - \rho_{11}) \cos \theta - (\rho_{01} + \rho_{10}) \sin \theta]$$

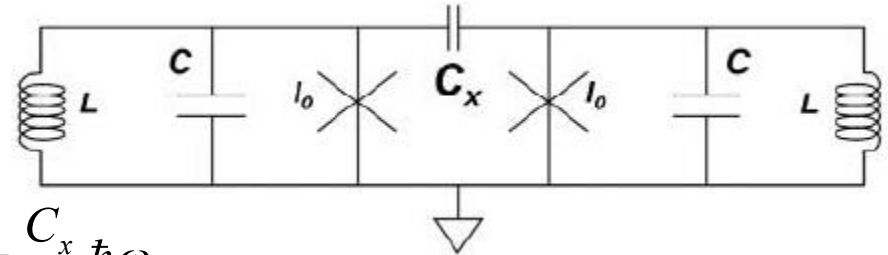
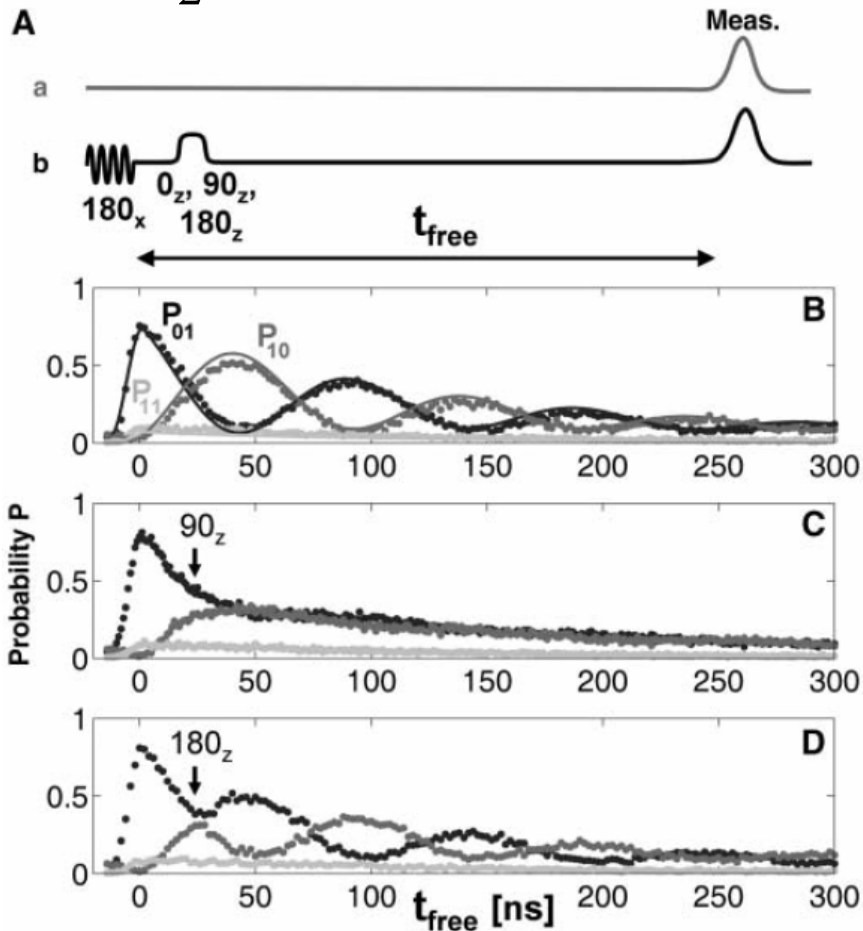
# State tomography



## Coupled phase qubits

At resonance: oscillation with  $S/\hbar=10\text{MHz}$   
freq between  $|01\rangle$  and  $|10\rangle$  2-qubit operation

$$H_{\text{int}} = \frac{S}{2} (|01\rangle\langle 10| + |10\rangle\langle 01|) \quad \text{where} \quad S = \frac{C_x}{C} \hbar \omega_{10}$$



$|00\rangle \rightarrow |01\rangle$  not eigenstate, starts to evolve

$$|\psi(t)\rangle = \cos\left(\frac{St}{2\hbar}\right) |01\rangle + i \sin\left(\frac{St}{2\hbar}\right) |10\rangle$$

-  $t_{\text{free}}=25$  ns: entangled state:

$$|\psi_1\rangle = \frac{1}{\sqrt{2}} (|01\rangle - i |10\rangle)$$

But: (pulse length not negligible)

- after  $t_{\text{free}}=16$  ns  $90_z$  rotation:

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)$$

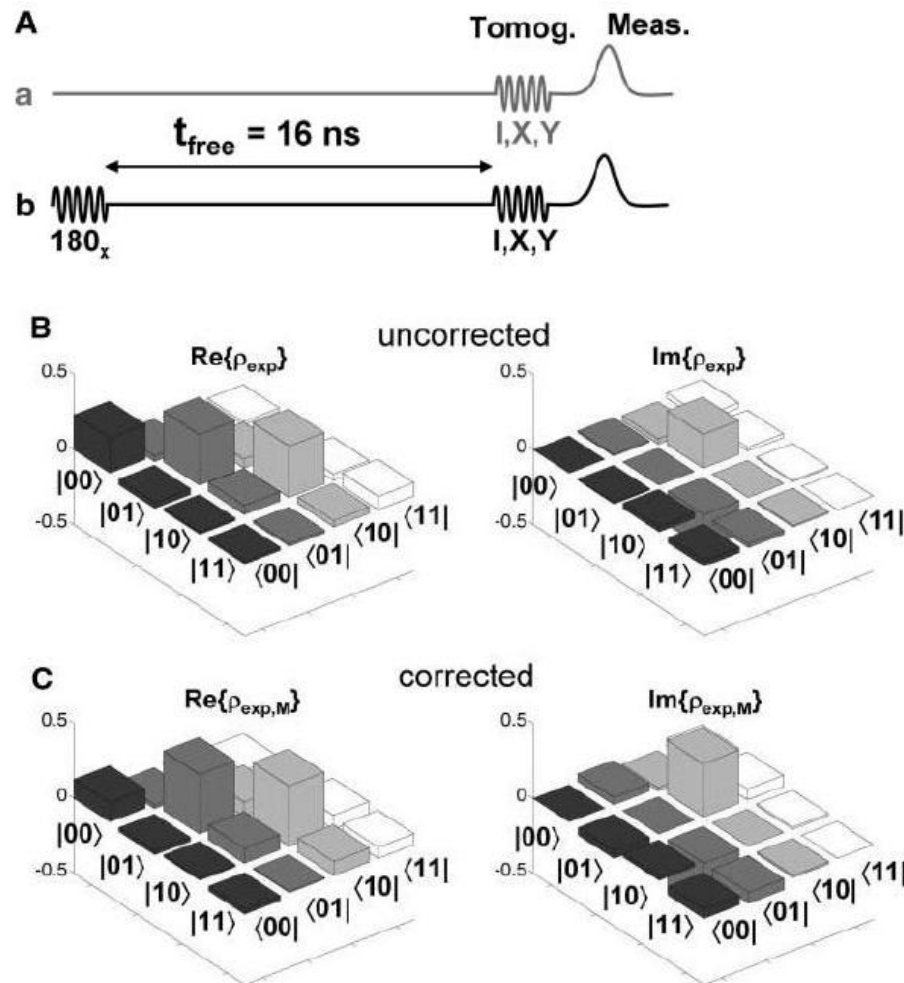
eigenstate, does not show any oscillations

To show it is not the destruction of coherence:

$\rightarrow 180_z$  pulse, and starts oscillating again

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|01\rangle + i |10\rangle)$$

# State tomography



State tomography on the state:

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|01\rangle - i|10\rangle)$$

Single qubit fidelities:

$$F_0=0.95, F_1=0.85$$

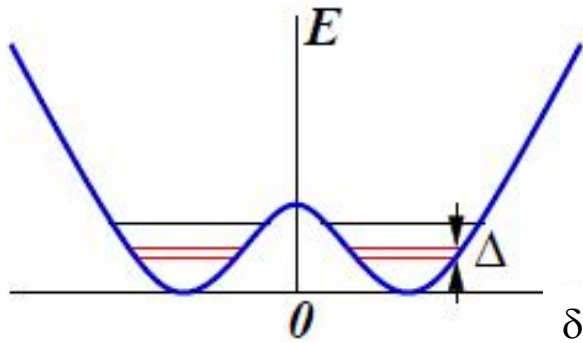
Fidelity for  $|\psi_1\rangle$   $F=0.75$

After correction with single qubit fidelities:  $F=0.87$

Estimated maximal fidelity:  $F=0.89$

Cause of fidelity loss:

- single qubit decoherence



potential for half integer flux bias

$$U(\delta) = E_J(1 - \cos(\delta)) + E_L(\delta - \Phi)^2/2$$

$$\sim E_L(-\epsilon\tilde{\delta}^2/2 - f\tilde{\delta} + (1 + \epsilon)/24\tilde{\delta}^4)$$

where  $\tilde{\delta} = \delta - \pi$   $f = \Phi - \pi$   
barrier height

$$\hat{H} = -1/2(\epsilon\sigma_z + \Delta\sigma_x) \quad \epsilon = E_J/E_L - 1 \ll 1$$

$$\epsilon = E_r - E_l$$

assymetry by flux

Two wells → two levels – for symmetric potential degenerate flux states

The two states have opposite persistent current

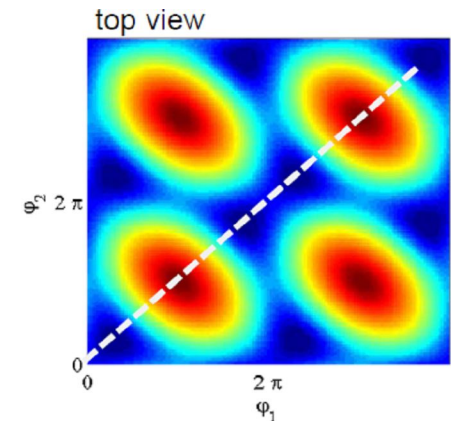
If tunneling is switched on ( $\Delta$ ), states hybridize and the macroscopic tunneling determines the separation

Hard to fabricate, big loop is needed for inductance matching (big decoherence) → 3 JJ-s qbit

### 3-junction flux qubit

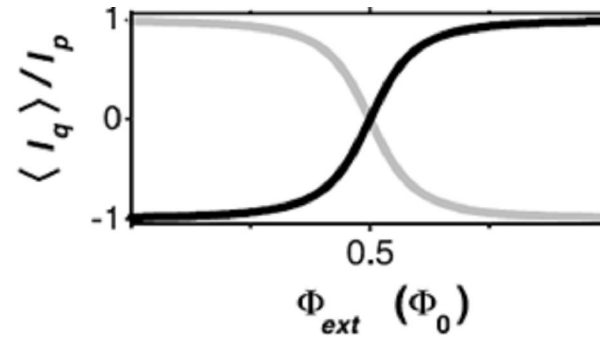
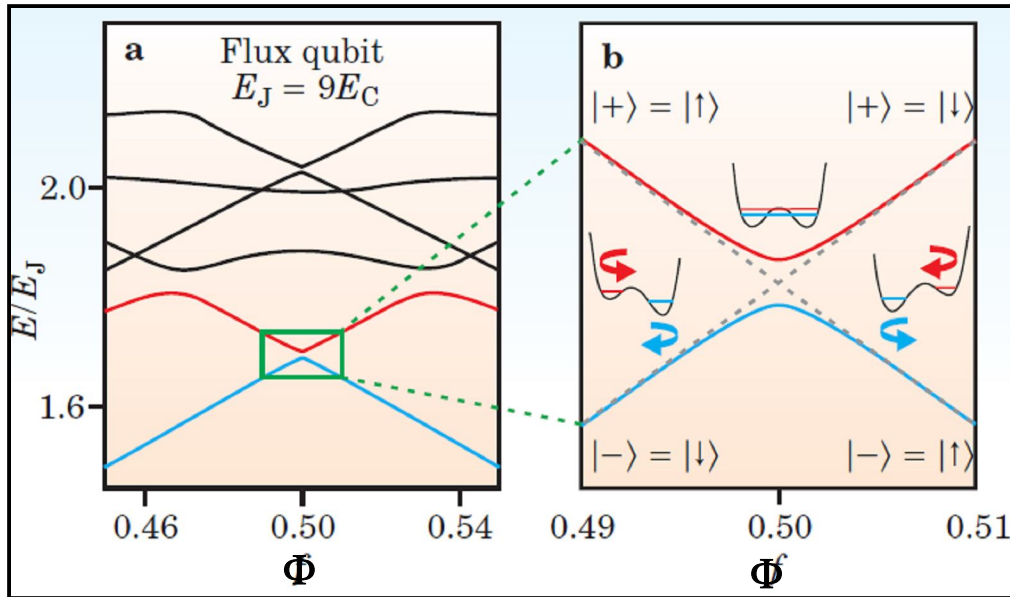
$$\varphi_1 + \varphi_2 + \varphi_3 + 2\pi\Phi/\Phi_0 = 2\pi n$$

$$U = -E_j [\cos(\varphi_1) + \cos(\varphi_2) + \alpha \cos(\varphi_1 - \varphi_2 - 2\pi\Phi_{ext}/\Phi_0)]$$



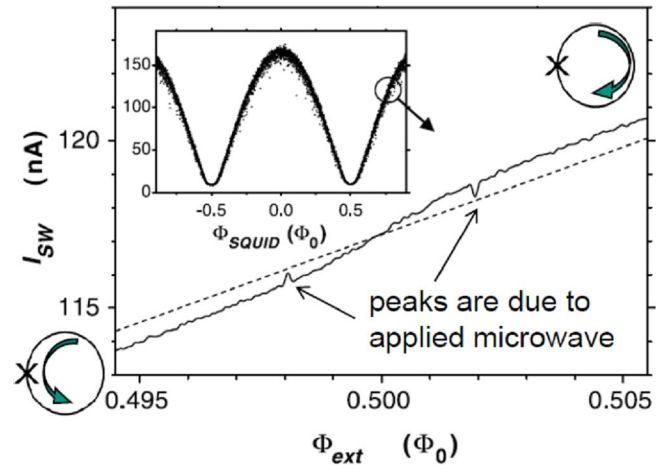
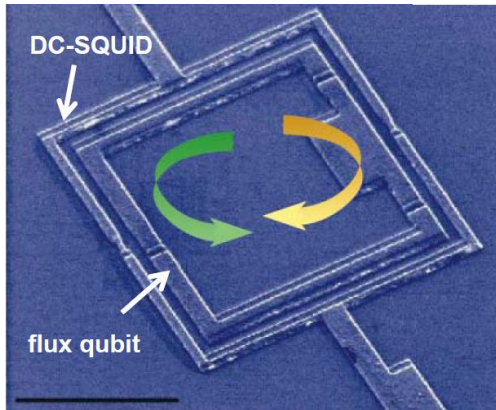
the potential is parabolic on the white intersection

# Flux qubit



The current as a function of the flux  
Away from half flux quanta, pure flux states

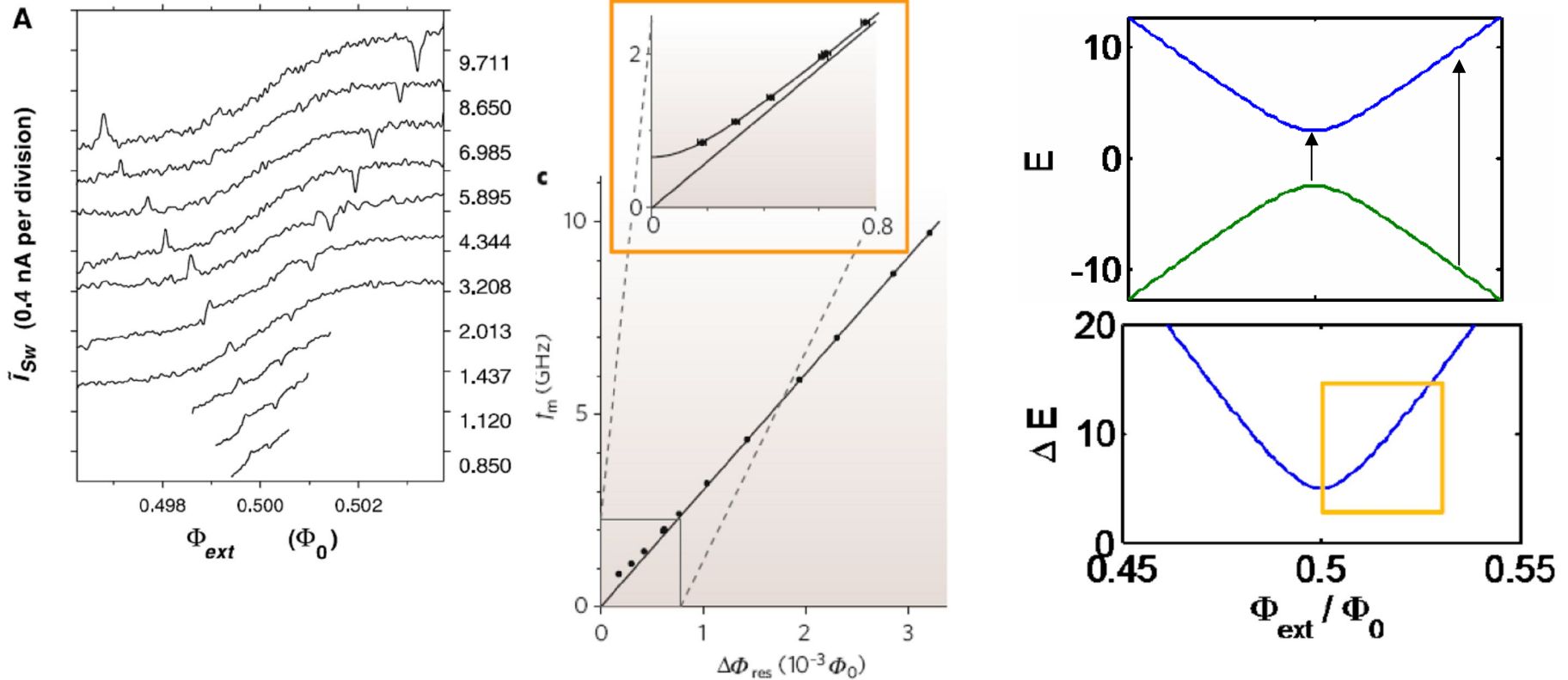
Detection – by DC squid measuring the opposite supercurrents



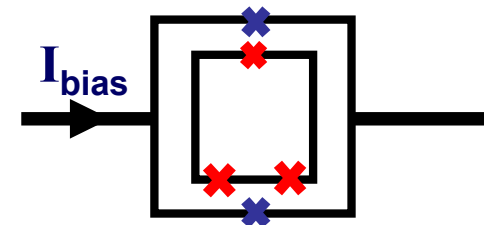
A small change in the squid signal  
shows the persistent current states



# Flux qbit



- During the sweeping of the magnetic field, microwave applied
- the resonance seen for different frequencies at different flux points
  - peaks indicate switching between flux states
  - the spectra is nicely reproduced



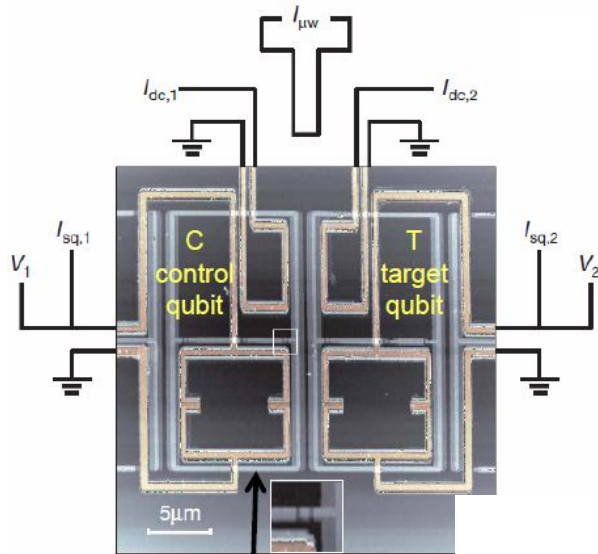
# C-NOT with coupled flux qubits



$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

If control qubit is 1, it flips the target qubit  
here, the first qubit is the control qubit,  $|10\rangle \rightarrow |11\rangle$  and  $|11\rangle \rightarrow |10\rangle$

$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$



Qubits are operated far from the degeneracy point

**Main idea:** If one of the qubits manipulated, e.g.. qubit 1 is excited, the change of the persistent current will change the resonance frequency of qubit 2

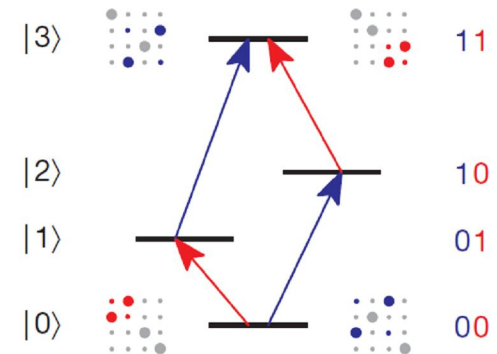
Qubit1= Control, Qubit2=Target

states  $0_C 0_T, 0_C 1_T, 1_C 0_T, 1_C 1_T$

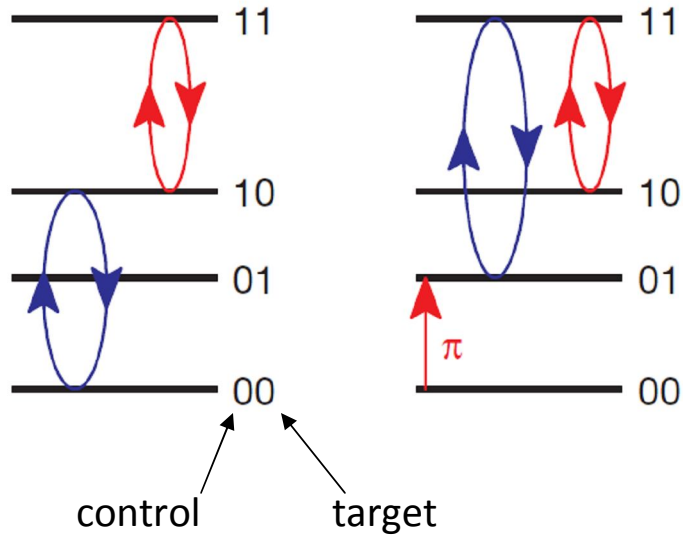
Using microwave pulses for rotations (its phase determines the axes)

With this rotation qubit 1 controlled CNOT is realized (almost, but phase can be easily shifted)

$$\hat{R}_{1_C 0_T - 1_C 1_T}(\omega, \tau) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \frac{\omega\tau}{2} & i \sin \frac{\omega\tau}{2} \\ 0 & 0 & i \sin \frac{\omega\tau}{2} & \cos \frac{\omega\tau}{2} \end{pmatrix}$$



# C-NOT with coupled flux qubits



- preparation of state
- **CNOT**
- measurement of qubit states

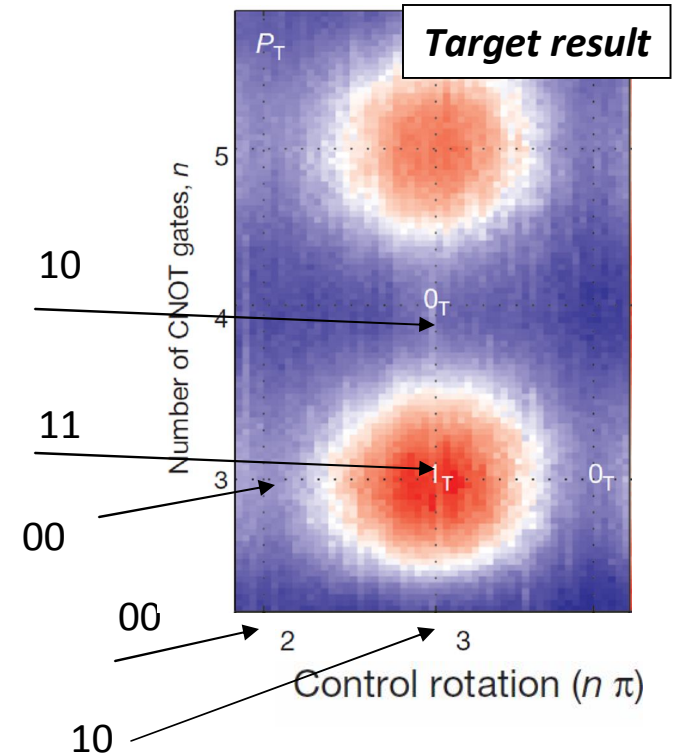
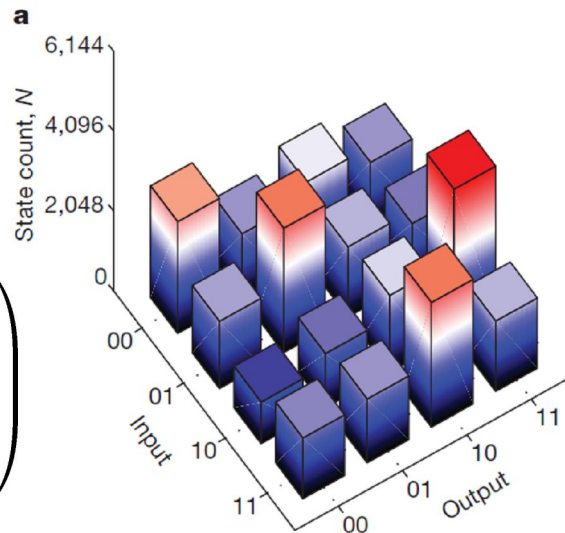
e.g. first rotation prepares from  $|00\rangle \rightarrow a|00\rangle + b|10\rangle$   
 than the CNOT is only resonant for  $|10\rangle$

$|10\rangle \rightarrow |11\rangle$

$|00\rangle \rightarrow |00\rangle$

With other preparation sequence other 2 relations  
 From readout  $P_{00}$  and  $P_{10}$  ... can be determined

$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



# Summary



|            | Charge        | Charge-flux   | Flux         | Phase         |
|------------|---------------|---------------|--------------|---------------|
| $E_j/E_c$  | 0.1           | 1             | 10           | $10^6$        |
| $\nu_{01}$ | 10 GHz        | 20 GHz        | 10 GHz       | 10 GHz        |
| $T_1$      | 1–10 $\mu$ s  | 1–10 $\mu$ s  | 1–10 $\mu$ s | 1–10 $\mu$ s  |
| $T_2$      | 0.1–1 $\mu$ s | 0.1–1 $\mu$ s | 1–10 $\mu$ s | 0.1–1 $\mu$ s |

Superconducting Circuits and Quantum Information

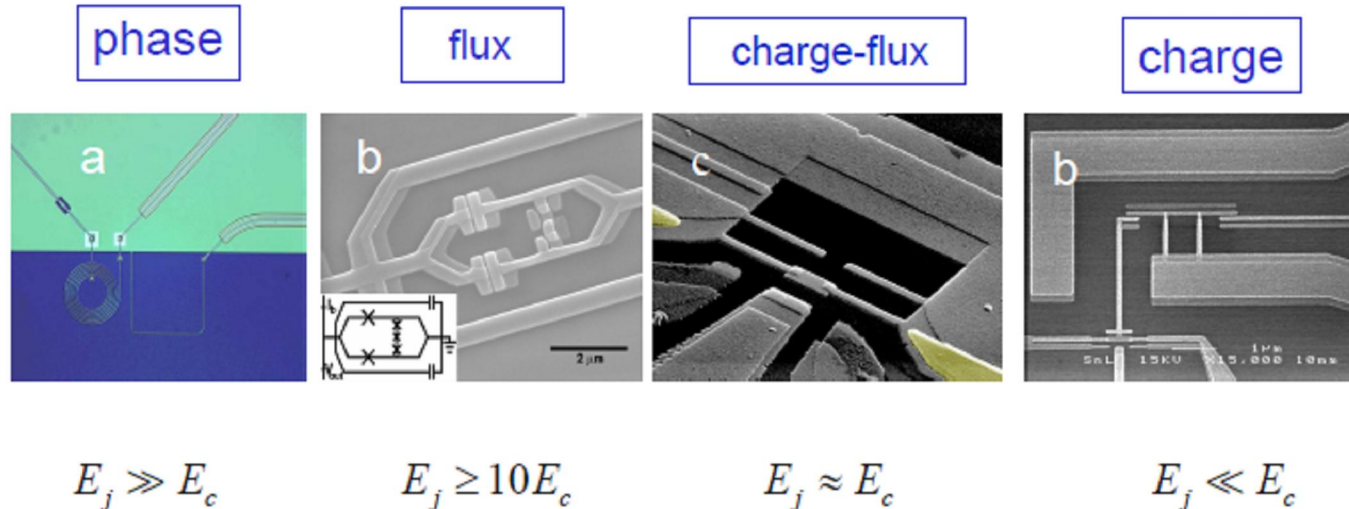
J. Q. You and Franco Nori

Phys. Today 58(11), 42 (2005)

Superconducting Quantum Circuits, Qubits and Computing

G. Wendin and V.S. Shumeiko

[arXiv:cond-mat/0508729v1](https://arxiv.org/abs/cond-mat/0508729v1)





## Superconductivity-Based Qubit Research Groups

|               |  |   |
|---------------|--|---|
| Theory        | Averin & Likharev (SUNY StonyBrook)<br>Bruder (Basel)<br>Choi (Korea)<br>Falci (Catania)<br>Fazio (Pisa)   | Levitov (MIT)<br>Lloyd (MIT)<br>Nori (Michigan and RIKEN)<br>Schön, Schnirman & Makhlin (Karlsruhe)<br>Wilhelm (Münich)   |
| Charge qubits | Buisson (Grenoble)<br>Delsing (Chalmers)<br>Devoret (Yale)<br>Echternach (JPL)<br>Esteve (Saclay)  | Kouwenhoven (Delft)<br>Likharev (SUNY StonyBrook)<br>Manheimer (LPS Maryland)<br><u>Nakamura</u> (NEC)<br>Schoelkopf (Yale)   |
| Phase qubits  | Han (Kansas)<br><u>Martinis</u> (UCSB)   | Simmonds (NIST)<br>Wellstood, Anderson & Lobb (Maryland)  |
| Flux qubits   | Berggren (MIT)<br><u>Clarke</u> (Berkeley)<br>Cosmelli (Rome)<br>Feldman & Bocko (Rochester)<br>Han (Kansas)<br>Koch (IBM)<br>Ladizinski (TRW)<br><u>Lukens, Likharev &amp; Semenov</u> (StonyBrook) | <u>Mooij</u> (Delft)<br>Oliver & Gouker (MIT)<br>Orlando (MIT)<br>Silvestrini (Naples)<br>Tanaka (NTT)<br>Ustinov (Erlangen)<br>Van Harlingen (Illinois)<br>Wellstood, Anderson & Lobb (Maryland) |