Introduction to cQED

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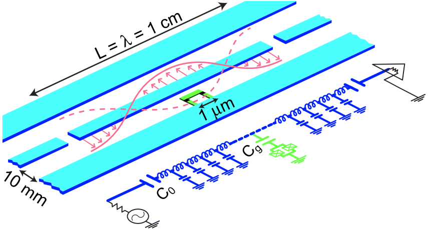
BME Physics Department, RF seminar

Outline

- Review of cavity QED
- Circuit implementation of cavity QED
- Dressed states
- Dispersive limit
- Experimental realization with a Cooper pair box



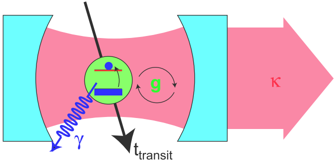
Sources

A. Blais et al., Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation, Phys. Rev. A 69, 062320 (2004)

A. Wallraff et al., Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics, Nature 431, 162–167 (2004)

Review of cavity QED

- Interaction between atoms and EM modes
- Optical cavity driven by laser
- Transmission properties → atom state



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_{\mathrm{r}}\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

- ω_{r}: cavity resonance frequency (bare resonance frequency)
- Ω: atom transition frequency (qubit frequency)
- H_κ: cavity decay (decay rate $\kappa = \omega_{\mathrm{r}}/Q$)
- H_γ: atom decay (decay rate γ)
- g: atom-cavity coupling strength, $g = \mathcal{E}_{\mathrm{rms}} d/\hbar$

Strong coupling limit: $g \gg \kappa, \gamma$

Review of cavity QED: typical parameter values

parameter	symbol	3D optical	3D microwave	1D circuit
resonance/transition frequency	$\omega_{\mathrm{r}}/2\pi$, $\Omega/2\pi$	$350\,\mathrm{THz}$	$51\,\mathrm{GHz}$	$10\,\mathrm{GHz}$
vacuum Rabi frequency	g/π, g/ω_{r}	$220\,\mathrm{MHz}$, 3×10^{-7}	$47\,\mathrm{kHz}$, 1×10^{-7}	$100\,\mathrm{MHz}$, 5×10^{-3}
transition dipole	d/ea_0	~ 1	1×10^{3}	2×10^{4}
cavity lifetime	$1/\kappa$, Q	$10\,\mathrm{ns}$, 3×10^{7}	$1\,\mathrm{ms}$, 3×10^{8}	$160\,\mathrm{ns}$, 10^{4}
atom lifetime	$1/\gamma$	$61\,\mathrm{ns}$	$30\,\mathrm{ms}$	$2\,\mu\mathrm{s}$
atom transit time	t_{transit}	$\geq 50\,\mu\mathrm{s}$	$100\,\mu\mathrm{s}$	∞
critical atom number	$N_0 = 2\gamma\kappa/g^2$	6×10^{-3}	3×10^{-6}	$\leq 6 \times 10^{-5}$
critical photon number	$m_0 = \gamma^2/2g^2$	3×10^{-4}	3×10^{-8}	$\leq 1 \times 10^{-6}$
# of vacuum Rabi flops	$n_{\mathrm{Rabi}} = 2g/(\kappa + \gamma)$	~ 10	~ 5	$\sim 10^{2}$

Jaynes-Cummings Hamiltonian

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Dressed states

Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_{\mathrm{r}}\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

Without cavity and atom decay,

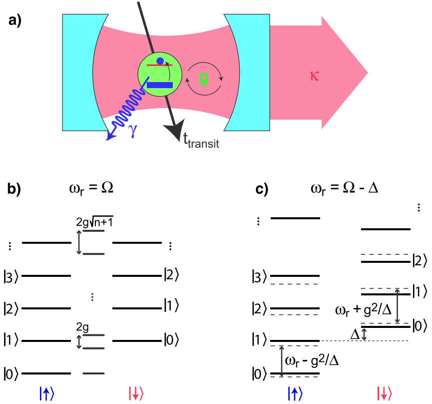
$$\begin{aligned} |\overline{+,n}\rangle &= \cos\theta_n\,|\downarrow, n\rangle + \sin\theta_n\,|\uparrow, n+1\rangle \\ |\overline{-,n}\rangle &= -\sin\theta_n\,|\downarrow, n\rangle + \cos\theta_n\,|\uparrow, n+1\rangle \end{aligned}$$

Eigenenergies:

$$\begin{aligned} E_{\overline{\pm,n}} &= (n+1)\hbar\omega_{\mathrm{r}} \pm \frac{\hbar}{2}\sqrt{4g^2(n+1)+\Delta^2} \\ E_{\uparrow,0} &= -\frac{\hbar\Delta}{2} \end{aligned}$$

$$\theta_n = \frac{1}{2}\tan^{-1}\left(\frac{2g\sqrt{n+1}}{\Delta}\right)$$

$\Delta \equiv \Omega - \omega_{\mathrm{r}}$: atom-cavity detuning



Dressed states

Jaynes-Cummings Hamiltonian

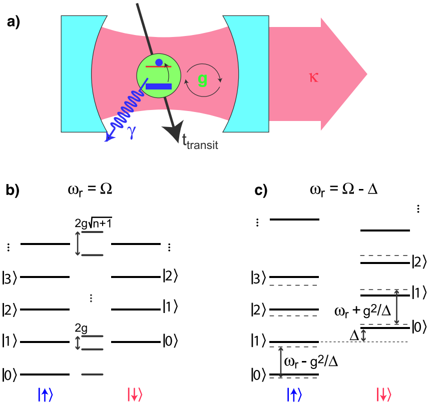
$$H = \hbar\omega_{\mathrm{r}}\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

Zero detuning, $\Delta = 0$

$|\overline{+, n}\rangle \leftrightarrow |\overline{-, n}\rangle$ lifted $\rightarrow$ splitting: $2g\sqrt{n+1}$

Single-excitation manifold

- state: $|\overline{\pm, 0}\rangle = (|\uparrow, 1\rangle \pm |\downarrow, 0\rangle)/\sqrt{2}$
- energy: $E_\pm = \hbar(\omega_r \pm g)$
- Vacuum Rabi oscillation
  $|\downarrow, 0\rangle \rightarrow |\uparrow, 1\rangle \rightarrow |\downarrow, 0\rangle$
- oscillation frequency: g/π
- decay rate: $(\kappa + \gamma)/2$



Dressed states

Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_{\mathrm{r}}\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

Large detuning, $g/\Delta \ll 1$ (dispersive regime)

Unitary transformation:

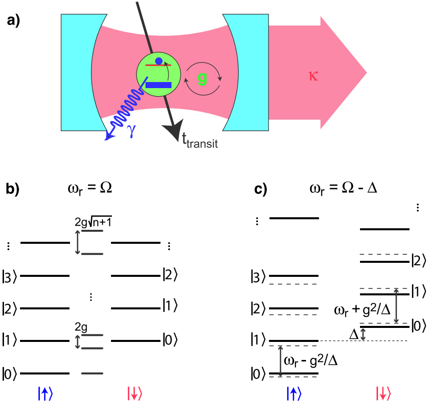
$$U = \exp\left[\frac{g}{\Delta}(a\sigma^+ - a^\dagger\sigma^-)\right]$$

$$UHU^\dagger \approx \hbar\left[\omega_{\mathrm{r}} + \frac{g^2}{\Delta}\sigma^z\right]a^\dagger a + \frac{\hbar}{2}\left[\Omega + \frac{g^2}{\Delta}\right]\sigma^z$$

$$UHU^\dagger \approx \frac{\hbar}{2}\left[2\frac{g^2}{\Delta}\left(a^\dagger a + \frac{1}{2}\right) + \Omega\right]\sigma^z + \hbar\omega_{\mathrm{r}}a^\dagger a$$

Interpretations

- shift of the resonance frequency by $\pm\chi = \pm(g^2/\Delta)$
  Lamb shift of atomic transition by g^2/Δ
- ac-Stark shift of atomic transition by
  $2(g^2/\Delta)(n + 1/2) \rightarrow$ backaction



Dressed states

Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_{\mathrm{r}}\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

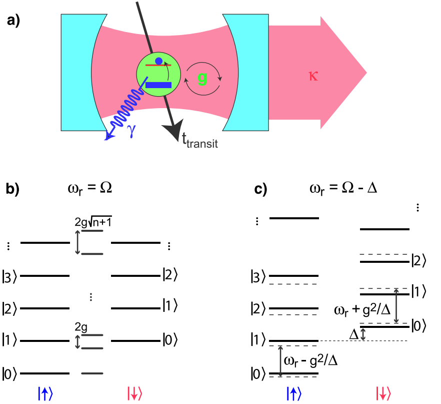
Large detuning, $g/\Delta \ll 1$ (dispersive regime)

Single-excitation manifold:

$$\begin{aligned}
\left|\overline{-,0}\right\rangle &\sim -(g/\Delta)\left|\downarrow,0\right\rangle + \left|\uparrow,1\right\rangle \\
\left|\overline{+,0}\right\rangle &\sim \left|\downarrow,0\right\rangle + (g/\Delta)\left|\uparrow,1\right\rangle .
\end{aligned}$$

Decay rates:

$$\begin{aligned}
\Gamma_{\overline{-,0}} &\simeq (g/\Delta)^2\gamma + \kappa \\
\Gamma_{\overline{+,0}} &\simeq \gamma + (g/\Delta)^2\kappa .
\end{aligned}$$



Dispersive readout

Cavity drive

- drive at $\omega_{\mu\mathrm{w}}$

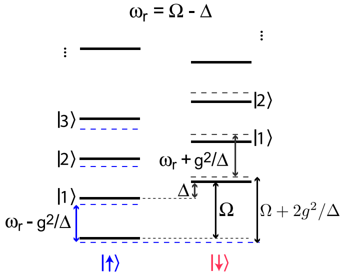
$$H_{\mu w}(t) = \hbar\varepsilon(t)(a^\dagger e^{-i\omega_{\mu\mathrm{w}}} + a e^{+i\omega_{\mu\mathrm{w}}})$$

- initial state: ground state $|\uparrow, 0\rangle$
- go to rotating frame
- transition at $\omega_\mathrm{r} - g^2/\Delta$ with weight

$$\langle\uparrow, 0|\, H_{\mu w}\, |\overline{-, n}\rangle \sim \varepsilon$$

- transition at $\Omega + 2g^2/\Delta$ with weight

$$\langle\uparrow, 0|\, H_{\mu w}\, |\overline{+, n}\rangle \sim \frac{\varepsilon g}{\Delta}$$



Dispersive Hamiltonian

$$UHU^\dagger \approx \hbar\left[\omega_\mathrm{r} + \frac{g^2}{\Delta}\sigma^z\right] a^\dagger a + \frac{\hbar}{2}\left[\Omega + \frac{g^2}{\Delta}\right]\sigma^z$$

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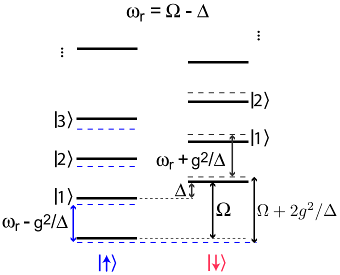


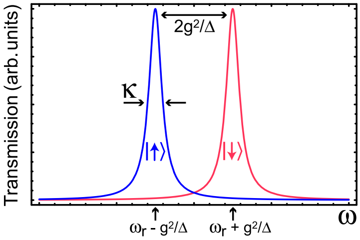
Dispersive Hamiltonian

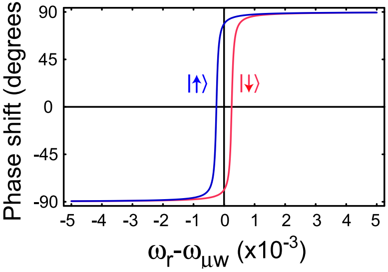
$$UHU^\dagger \approx \frac{\hbar}{2}\left[2\frac{g^2}{\Delta}\left(a^\dagger a + \frac{1}{2}\right) + \Omega\right]\sigma^z + \hbar\omega_\mathrm{r} a^\dagger a$$

Dispersive readout: $|T|$ or phase

Transmission spectrum

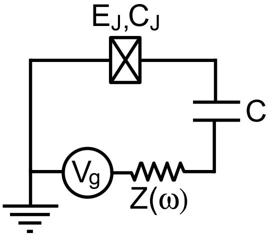
- measuring at $\omega_r \pm g^2/\Delta$: information is in $|T|$
- measuring at ω_r: information is in the phase
- photons become entangled with the qubit
  $\rightarrow$ coupling of qubits through a cavity
  $\rightarrow$ quantum communication

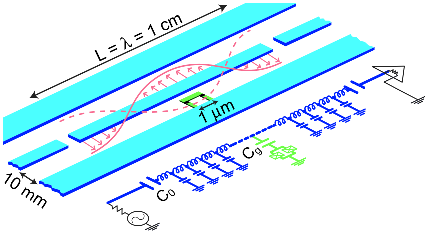


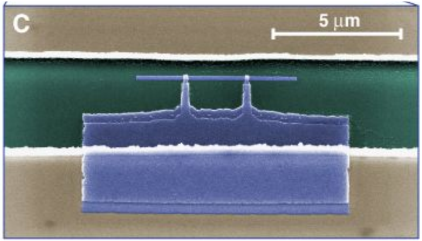


Realization of cQED with the Cooper pair box

- Cavity $\to$ 1D coplanar waveguide resonator
  Typical values:
  $f_r \sim 10\,\mathrm{GHz}$ ($hf_r/k_B \sim 0.5\,\mathrm{K}$)
  $V^0_{\mathrm{rms}} \sim \sqrt{\hbar\omega_{\mathrm{r}}/cL} \sim 2\,\mu\mathrm{V}$
  $\mathcal{E}_{\mathrm{rms}} \sim 0.2\,\mathrm{V/m}$
  $Q \sim 10^6$
- Atom $\to$ Cooper pair box







10.1103/PhysRevA.69.062320

Realization of cQED with the Cooper pair box

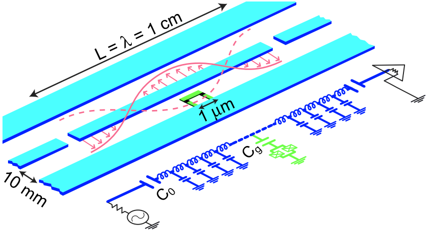
- Cavity → 1D coplanar waveguide resonator
- Atom → Cooper pair box

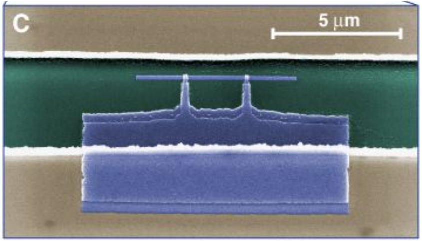
$$
\begin{aligned}
H_Q &= 4E_c \sum_N (N - N_g)^2 \left|N\right\rangle \left\langle N\right| \\
&\quad - \frac{E_J}{2} \sum_N \left(\left|N+1\right\rangle \left\langle N\right| + h.c.\right)
\end{aligned}
$$

For $4E_c \gg E_J$ and $N_g \in [0, 1]$,

$$H_Q = -\frac{E_{\mathrm{el}}}{2}\bar{\sigma}^z - \frac{E_J}{2}\bar{\sigma}^x$$

with $E_{\mathrm{el}} = 4E_C(1 - 2N_g)$
Double JJ → tunability: $E_J \cos(\pi\Phi_{\mathrm{ext}}/\Phi_0)/2$





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Coupled resonator-CPB system

Resonator $\rightarrow$ ac gate voltage $v = V^{\mathbf{0}}_{\mathrm{rms}}(a^\dagger + a)$

$$H_Q = -2E_C(1-2N_g^{\mathrm{dc}})\bar{\sigma}^z - \frac{E_J}{2}\bar{\sigma}^x - e\frac{C_g}{C_\Sigma}\sqrt{\frac{\hbar\omega_{\mathrm{r}}}{Lc}}(a^\dagger + a)(1-2N_g-\bar{\sigma}^z)$$

Total Hamiltonian

$$H = \hbar\omega_{\mathrm{r}}\left(a^\dagger a + \frac{1}{2}\right) + \frac{\Omega}{2}\sigma^z - e\frac{C_g}{C_\Sigma}\sqrt{\frac{\hbar\omega_{\mathrm{r}}}{Lc}}(a^\dagger + a)(1-2N_g - \cos(\theta)\sigma^z + \sin(\theta)\sigma^x)$$

With $\theta = \arctan[E_J/4E_C(1-2N_g^{\mathrm{dc}})]$ and $\Omega = \sqrt{E_J^2 + [4E_C(1-2N_g^{\mathrm{dc}})]^2}$

Special case:

- For $N_{\mathrm{g}} = 1/2$, neglecting rapidly oscillating terms $\rightarrow \theta = \pi/2, \Omega = E_J$ and $g = \frac{C_g e}{C_\Sigma \hbar}\sqrt{\frac{\hbar\omega_{\mathrm{r}}}{cL}}$
- Away from $N_{\mathrm{g}} = 1/2$: coupling reduced by $\sin\theta$ and an extra term $\propto (a^\dagger + a)$

Experimental realization

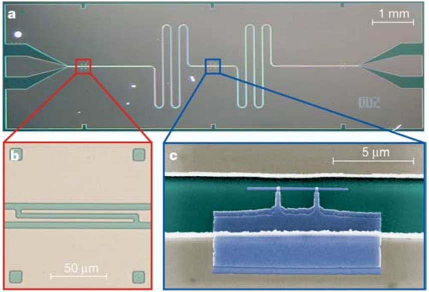
A. Wallraff et al., Nature 431, 162–167 (2004)

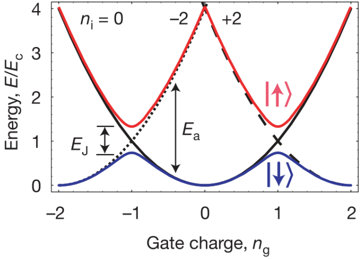
Resonator

- $\nu_\mathrm{r} = 6.04\,\mathrm{GHz}$
- $T^\star = \hbar\omega_\mathrm{r}/k_B \approx 300\,\mathrm{mK}$
  $T < 100\,\mathrm{mK} \to n < 0.06$
- $V_\mathrm{rms} = \sqrt{\hbar\omega_\mathrm{r}/2C} \approx 1\,\mu\mathrm{V}$, $E_\mathrm{rms} \approx 0.2\,\mathrm{V/m}$
- $Q_\mathrm{int} \approx 10^6$
  $T_r = 1/\kappa > 100\,\mathrm{ns}$ even for $Q \approx 10^4$

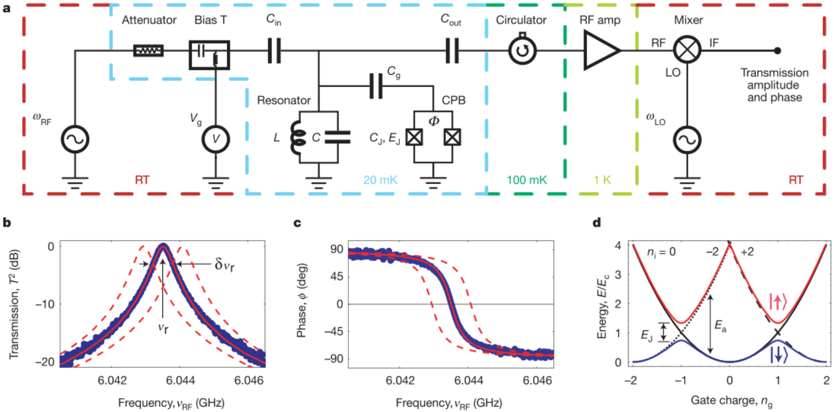
Qubit

- Cooper pair box
- qubit energy
  $E_a = \hbar\omega_a = \sqrt{E^2_{J,\mathrm{max}}\cos^2(\pi\Phi_b) + 16E_C^2\,(1-n_g)^2}$
- gate $\to n_g$
- external coil $\to$ flux bias Φ_b





Experimental realization: measurement setup



$$H \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right) a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z \qquad \Delta = \omega_r - \omega_a$$

Experimental realization: read-out

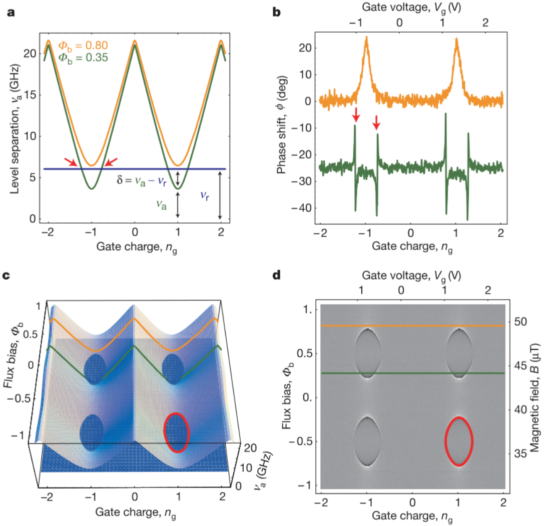
Read-out

- continuous wave with low drive
- fixed read-out tone at ω_r
- phase shift $\Delta\phi = \tan^{-1}\left(2g^2/\kappa\Delta\right)$

Parameter extraction

- periodicity $\rightarrow$ calibration of n_g, Φ_b
- cavity-qubit resonance on a set of $[n_g, \Phi_b]$ (red)
  $E_{J,\mathrm{max}} = 8.0\,(\pm 0.1)\,\mathrm{GHz}$
  $E_C = 5.2\,(\pm 0.1)\,\mathrm{GHz}$

$$E_a = \sqrt{E_{J,\mathrm{max}}^2 \cos^2(\pi\Phi_b) + 16E_C^2\,(1 - n_g)^2}$$



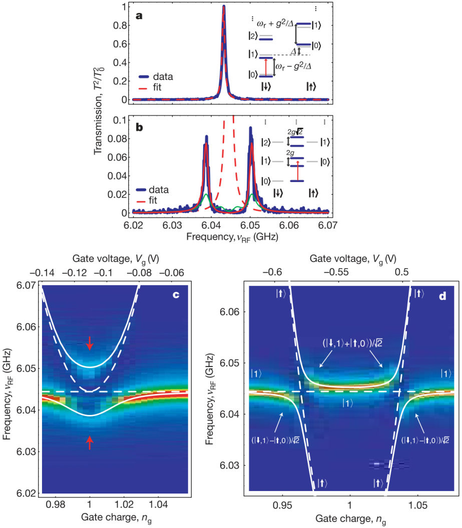
Experimental realization: vacuum Rabi oscillation

Swept read-out frequency

- spectrum at large detuning ($g^2/\Delta\kappa \ll 1$)
  $\kappa/2\pi = 0.8\,\mathrm{MHz}$
  $n \approx 1$ at resonance
- spectrum for $\Delta = 0$ at $n_g = 1$
  reduced the power by 5 dBm, $n \approx 0.5$
  $\nu_{\mathrm{Rabi}} \approx 11.6\,\mathrm{MHz}$

Swept read-out frequency and n_g

- $\Delta \neq 0$, dispersive shift
- $\Delta = 0$, avoided crossing
  Vacuum Rabi splitting



Lifetime: zero detuning

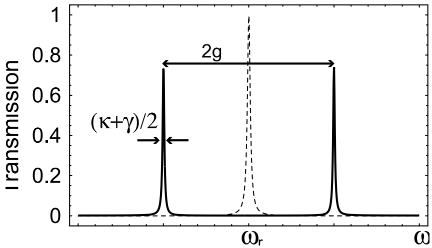
Strong coupling $g > \kappa, \gamma$

- decay rate: $(\kappa + \gamma)/2$
- Rabi splitting $\propto \sqrt{n+1}$
- measurement time estimate
  ν =10 GHz HEMT amplifier
  $n_{\mathrm{amp}} = k_B T_N / \hbar\omega \sim 100$ photons
  $t_{\mathrm{meas}} = 2 n_{\mathrm{amp}} / \langle n \rangle \kappa$,
  $\sim 32\,\mu\mathrm{s}$ for $\langle n \rangle \sim 1$

Low Q cavity ($g < \kappa$)

- qubit decay enhanced by a factor Q

Vacuum Rabi splitting



Lifetime: large detuning

Qubit outside cavity

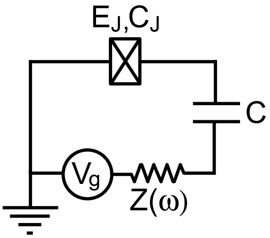
- voltage fluctuation $\to$ relaxation

$$\frac{1}{T_1} = \frac{E_\mathrm{J}^2}{E_\mathrm{J}^2 + E_\mathrm{el}^2} \left(\frac{e}{\hbar}\right)^2 \left(\frac{C_\mathrm{g}}{C_\Sigma}\right)^2 S_V(+\Omega)$$

- spectral density

$$S_V(+\Omega) = 2\hbar\Omega\,\mathrm{Re}[Z(\Omega)]$$

- estimate $T_1 \sim 1\,\mu s$



Qubit inside the cavity

- noise is filtered by the cavity
- lower $\mathrm{Re}[Z(\Omega)]$
- approximation:

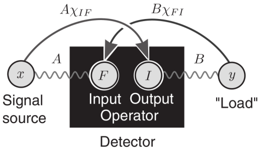
$$S_V(\Omega) = \frac{2\hbar\omega_\mathrm{r}}{Lc} \frac{\kappa/2}{\Delta^2 + (\kappa/2)^2}.$$

- estimate $1/T_1 \equiv \gamma_\kappa = (g/\Delta)^2\kappa \sim 1/(64\,\mu\mathrm{s})$
- large detuning $\to$ lifetime enhancement
- total decay rate
  $\gamma = \gamma_\kappa + \gamma_\perp + \gamma_\mathrm{NR}$
- 1D superconducting resonators: γ_κ dominates

Quantum non-demolition readout

Quantum non-demolition readout

Generic linear detector



Les Houches 2011, A. Clerk

- qubit Hamiltonian $\hat{H}_{\mathrm{qb}} = \frac{\hbar\Omega}{2}\hat{\sigma}_z$
- interaction Hamiltonian $\hat{H}_{\mathrm{int}} = A\hat{\sigma}_z\hat{F}$
- if $\left[\hat{H}_{\mathrm{int}}, \hat{H}_{\mathrm{qb}}\right] = 0$
  $\rightarrow$ non-demolition w.r.t. $\hat{\sigma}_z$
- $\left[\hat{H}_{\mathrm{tot}}, \hat{\sigma}_z\right] = \left[\hat{H}_{\mathrm{tot}}, \hat{F}\right] = 0 \rightarrow$ constants of motion
- allows continuous or repeated measurements

Dispersive Hamiltonian

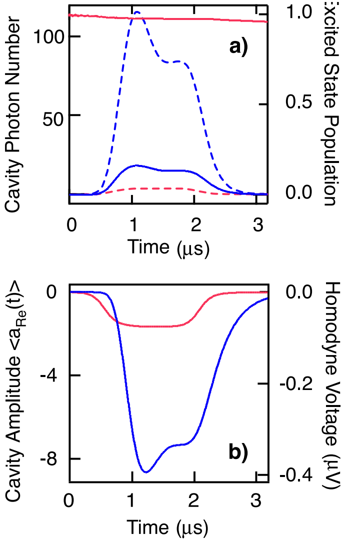
$$UHU^\dagger \approx \hbar\left[\omega_{\mathrm{r}} + \frac{g^2}{\Delta}\sigma^z\right] a^\dagger a + \frac{\hbar}{2}\left[\Omega + \frac{g^2}{\Delta}\right]\sigma^z$$

- Interaction H in the dispersive limit

$$H_{\mathrm{int}} = \hbar\frac{g^2}{\Delta}\sigma^z\hat{a}^\dagger\hat{a}$$

- $\hat{F} = \hat{a}^\dagger\hat{a} = \hat{n}$
- measuring n is QND

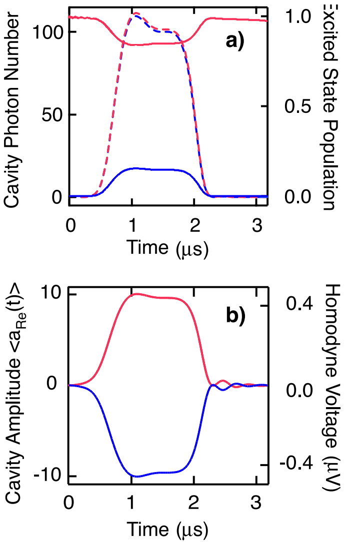
Measuring at $\omega_r + g^2/\Delta$



- pulse centered at $\omega_r + g^2/\Delta$
- pulse duration $\sim 15/\kappa$
- tanh rise and fall
- photon power
  $P = \langle n \rangle \hbar \omega_{\mathrm{r}} \kappa / 2 = \langle V_{\mathrm{out}} \rangle^2 / R$
- homodyne voltage
  $\langle V_{\mathrm{out}} \rangle = \sqrt{R \hbar \omega_{\mathrm{r}} \kappa} \langle a + a^\dagger \rangle / 2$
- simulation with quantum state diffusion method
- qubit states are mixed, but low transition probabilities
- information in the transmission (photon number)

Ground/excited: blue/red
Dashed: left scale

Measuring at ω_r



- pulse centered at ω_r
- pulse duration $\sim 15/\kappa$
- tanh rise and fall
- photon power
  $P = \langle n \rangle \hbar \omega_{\mathrm{r}} \kappa / 2 = \langle V_{\mathrm{out}} \rangle^2 / R$
- homodyne voltage
  $\langle V_{\mathrm{out}} \rangle = \sqrt{R \hbar \omega_{\mathrm{r}} \kappa} \langle a + a^\dagger \rangle / 2$
- simulation with quantum state diffusion method
- qubit states are mixed, but low transition probabilities
- information in the phase

Ground/excited: blue/red
Dashed: left scale

Measurement backaction

Dephasing

- changing n
- changing ac Stark shift $(g^2/\Delta)n\sigma^z$
- suppose we measure at ω_r and $\chi < \kappa$
- phase difference between g and e state

$$\varphi(t) = 2\frac{g^2}{\Delta}\int_0^t dt' n(t')$$

- mean phase

$$\langle\varphi\rangle = 2\theta_0 N$$

with $\theta_0 = 2g^2/\kappa\Delta$ and $N = \kappa\bar{n}t/2$

- weak coupling, long-time limit: Gaussian noise

- evaluate the correlator

$$\begin{aligned}\langle\sigma^+(t)\sigma^-(0)\rangle &= \langle e^{i\int_0^t dt'\varphi(t')}\rangle \\ &\simeq e^{-\frac{1}{2}\left(2\frac{g^2}{\Delta}\right)^2\int_0^t\int_0^t dt_1 dt_2 \langle n(t_1)n(t_2)\rangle}\end{aligned}$$

- dephasing rate

$$\Gamma_\varphi = 4\theta_0^2\frac{\kappa}{2}\bar{n}$$

- measurement time to resolve the phase change $\delta\theta = 2\theta_0$

$$T_\mathrm{m} = \frac{1}{2\kappa\bar{n}\theta_0^2} \rightarrow T_\mathrm{m}\Gamma_\varphi = 1$$

- quantum limit: $T_\mathrm{m}\Gamma_\varphi = 1/2$
- origin: reflected photons carry information

\+ **Mixing of states**
\+ **Driving transitions**

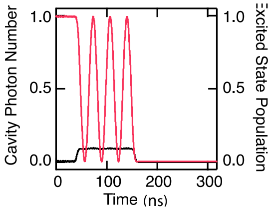
Coherent control

- readout: $\omega_{\mu\mathrm{w}}$ near ω_r
- control: $\omega_{\mu\mathrm{w}}$ near Ω
- dispersive limit, rotating frame $\rightarrow$

$$
\begin{aligned}
H_{1q} \quad = \quad & \frac{\hbar}{2}\left[\Omega + 2\frac{g^2}{\Delta}\left(a^\dagger a + \frac{1}{2}\right) - \omega_{\mu w}\right]\sigma^z \\
& + \hbar\frac{g\varepsilon(t)}{\Delta}\sigma^x \\
& + \hbar(\omega_\mathrm{r} - \omega_{\mu w})a^\dagger a + \hbar\varepsilon(t)(a^\dagger + a)
\end{aligned}
$$

- choice of $\omega_{\mu w} = \Omega + (2n+1)g^2/\Delta$
  $\rightarrow \sigma_x$ (rotation around x),
  with Rabi frequency $g\varepsilon/\Delta$
- choice of $\omega_{\mu w} = \Omega + (2n+1)g^2/\Delta - 2g\varepsilon/\Delta$
  with $t = \pi\Delta/2\sqrt{2}g\varepsilon$
  $\rightarrow$ Hadamard gate $\mathcal{H}$ (rotation of π around $x + z$)

- $\mathcal{H}\sigma_x\mathcal{H} = \sigma_z \rightarrow$ complete set



Remarks

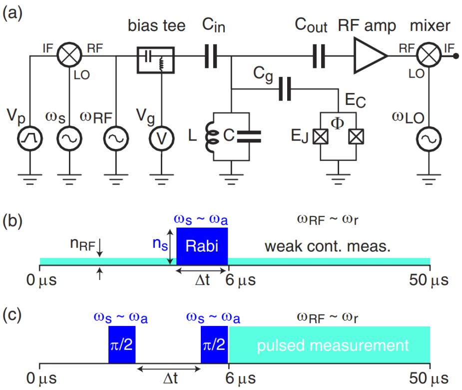
- n depends on ε
- cavity is only virtually populated with $\bar{n} \approx \varepsilon^2/\Delta^2 \sim 0.1$
- reflected photons carry no information about the qubit state (no entanglement)

Experiment

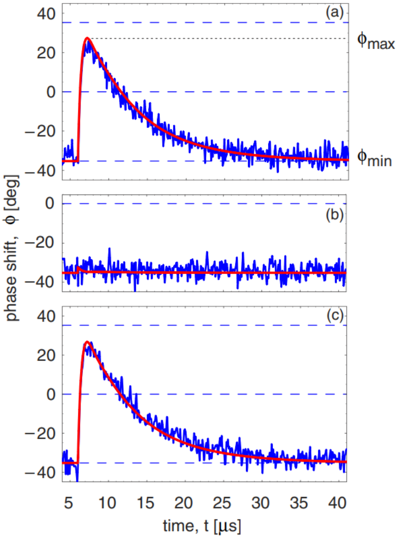
A. Wallraff et al., Phys. Rev. Lett. 95, 060501 (2005)

Setup

- qubit: Cooper pair box
- qubit frequency $\omega_{\mathrm{a}}/2\pi \approx 4.3\,\mathrm{GHz}$
- read-out at $\omega_{\mathrm{RF}}/2\pi = \omega_{\mathrm{r}}/2\pi \approx 5.4\,\mathrm{GHz}$
- phase shift $\phi = \tan^{-1}\left(2g^2/\kappa\Delta\right)\sigma_z$
- $Q \sim 0.7 \times 10^4 \leftrightarrow \kappa/2\pi = 0.73\,\mathrm{MHz}$
- large $\Delta \rightarrow$ defining $\phi = 0$
- prepare ground state $|{\downarrow}\rangle$
- work at the charge degeneracy point
- go to $\Delta/2\pi \approx -1.1\,\mathrm{GHz}$
- here $\phi_{|\downarrow\rangle} = -35.3\,\mathrm{deg} \leftrightarrow g/2\pi = 17\,\mathrm{MHz}$
- check: apply long microwave pulse, $P_{|\downarrow\rangle} = P_{|\uparrow\rangle} = 1/2$, get $\phi = 0$



Experiment



Measurement protocol

- continuous read-out
- prepare ground state
- pulse at $t = 6\mu s$: $\pi, 2\pi$ or 3π
- repeated every 50 μs, averaged 5×10^4 times
- risetime $2/\kappa \approx 400$ ns
- decay $T_1 \sim 7.3\,\mu$s
- contrast $C = (\phi_{\max} - \phi_{\min})/(\phi_{|\uparrow\rangle} - \phi_{|\downarrow\rangle}) \sim 85\,\%$

Summary

Future talks (hopefully)

- Resonator-qubit coupling: direction of $\mathcal{E}$ field, inductive coupling
- Correspondance between classical circuit description and the J-C Hamiltonian
- Purcell effect, Purcell filter
- Classical analogy: readout of polarizability; quantum capacitance?
- Input-output theory?
- Strong coupling, ultrastrong coupling
- Novel qubit types: Andreev level qubit, Andreev spin qubit, Andreev molecule
- Scalability
- SNR in dispersive readout

Further sources

Alexandre Blais - Quantum Computing with Superconducting Qubits - CSSQI 2012

`https://www.youtube.com/watch?v=t5nxusm_Umk` (Part 1)

`https://www.youtube.com/watch?v=K0ZCP1_DyDU` (Part 2)

Thank you for your attention!