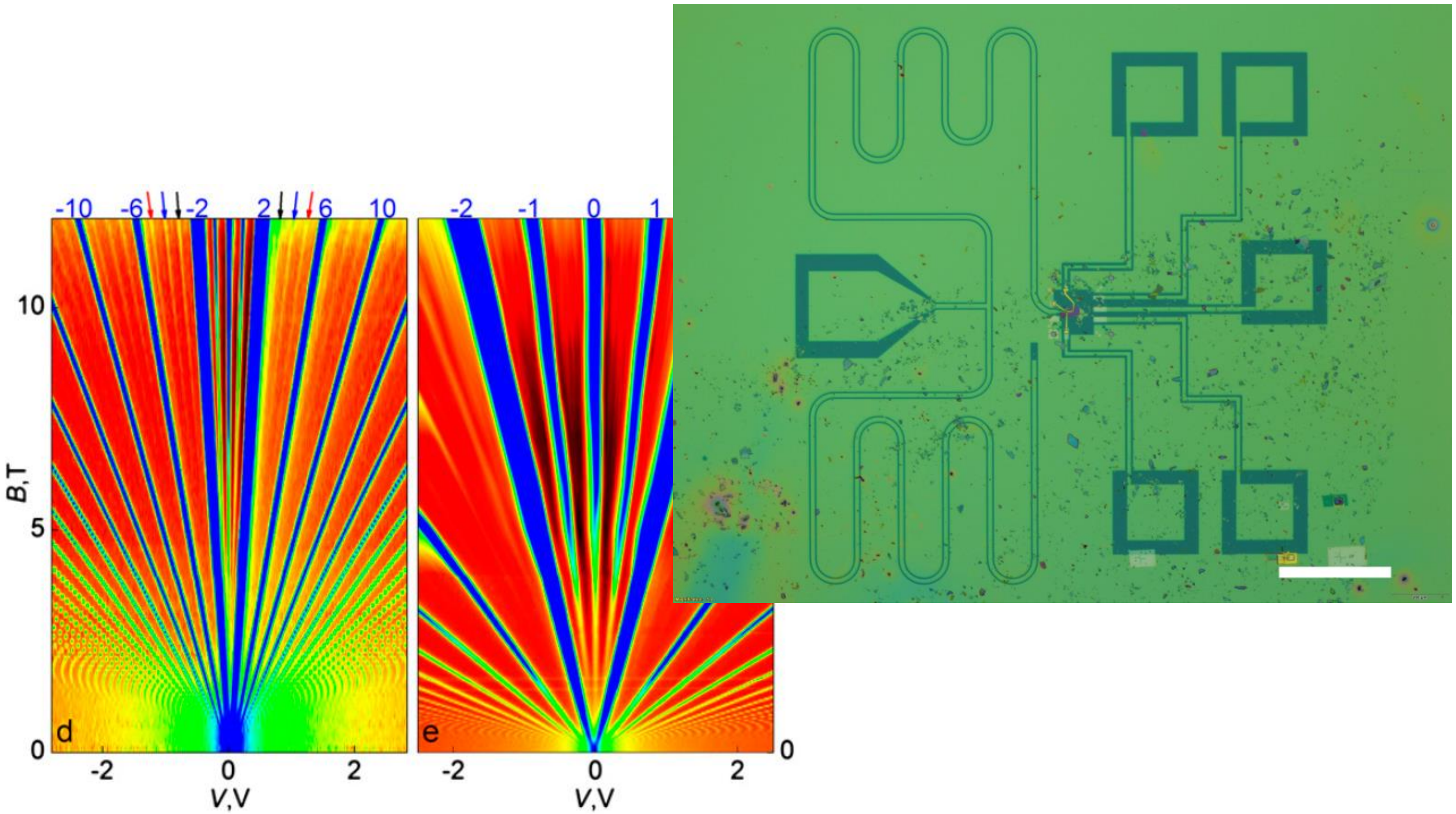
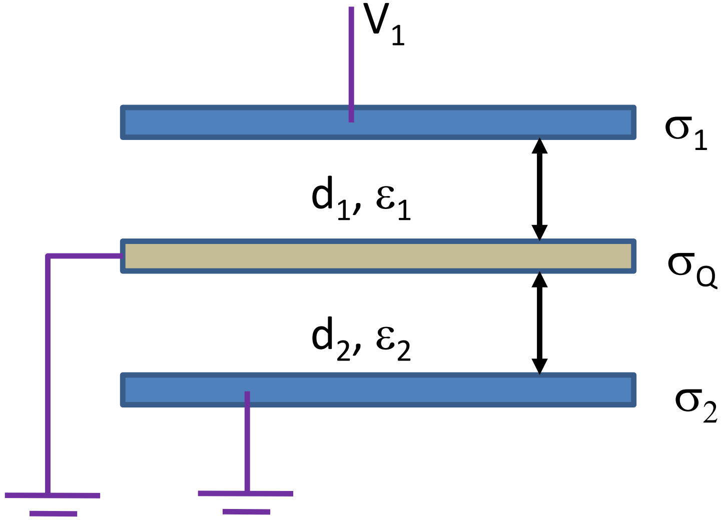
Quantum Capacitance

Budapest University of Technology and Economics 2019 spring



Quantum Capacitance – Double layer



$$C_i = \frac{\epsilon_i}{4\pi d_i}, \quad i = 1,2$$

$$\sigma_1 + \sigma_2 + \sigma_Q = 0$$

$$\sigma_2 = -\sigma_1 \sin^2(\phi),$$

$$\sigma_Q = -\sigma_1 \cos^2(\phi)$$

$$\tan^2(\phi) = \hbar^2\epsilon_2/4mg_v d_2 e^2 \equiv C_2/C_Q$$

$$C_Q = \frac{g_v m e^2}{\pi\hbar^2} \qquad \hbar \rightarrow 0,\ C_Q \rightarrow \infty$$

$$\sigma_2 = -\sigma_1[C_2/(C_2 + C_Q)]$$

Energy terms:

Electrostatic

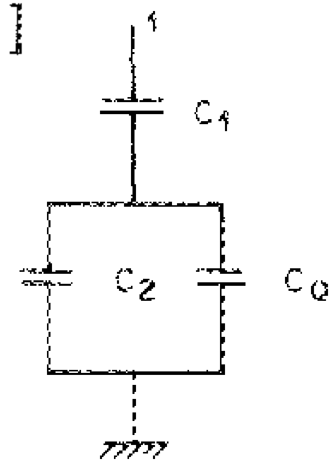
$$E_i = \int_0^{d_i} \frac{\epsilon_i F_i^2\, dx}{8\pi} = \frac{2\pi d_i \sigma_i^2}{\epsilon_i}, \quad i = 1,2.$$

$$F_1 = 4\pi\sigma_1/\epsilon_1 \text{ and } F_2 = -4\pi\sigma_2/\epsilon_2$$

Kinetic

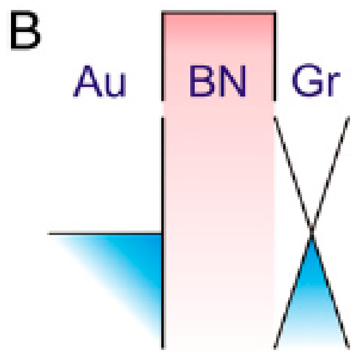
$$E_Q = \pi\hbar^2\sigma_Q^2/2g_v m e^2$$

$$\delta E_{tot}(\phi) = 0$$

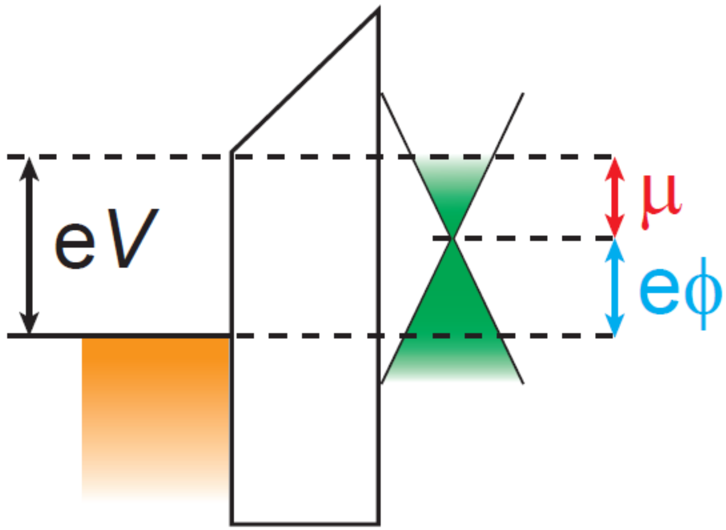


S: Luryi, Appl. Phys. Lett. ***52****, 501 (1988)*

Quantum Capacitance – Coupling to an electrode



2D material coupled to tunnel barrier via tunnel barrier



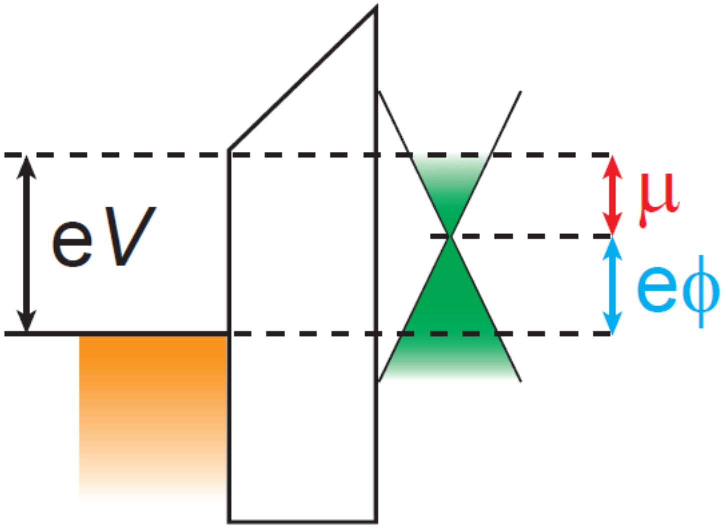
$$eV = e\phi + \mu$$

$$\frac{dV}{dQ} = \frac{d\phi}{dQ} + \frac{1}{Ae^2}\frac{d\mu}{dn}$$

$$\frac{1}{C} = \frac{1}{C_g} + \frac{1}{C_q}$$

Geometric capacitance

Quantum Capacitance $= e^2 \cdot DOS = e^2 \cdot D_0$



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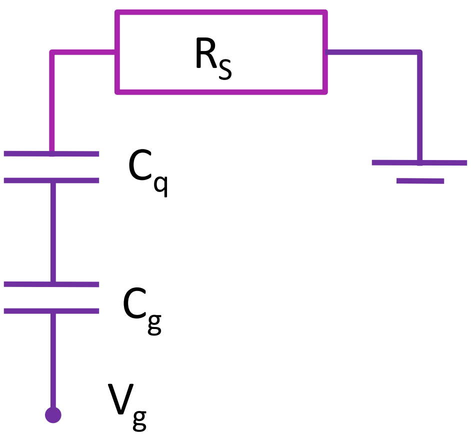
$$\frac{1}{C} = \frac{1}{C_g} + \frac{1}{C_q}$$

$$n = \frac{C_Q C_G}{C_Q + C_G} V_G$$



For a metal: Cq>>Cg, Fermi level does not move
n= $C_G V_G$

For a low DOS: Cq<<Cg, Fermi level moves substantially
n= $C_Q V_G$

Finite temperature:

$$C_Q = e^2 \int_{-\infty}^{+\infty} D(E)\left(-\frac{\partial f(E\text{-}E_F)}{\partial E}\right) dE$$

Quantum Capacitance

$$E = -qV_g + U_0 (N - N_0)$$

$$\frac{\partial E}{\partial(-qV_g)} = 1 + U_0 \frac{\partial N}{\partial(-qV_g)} =$$

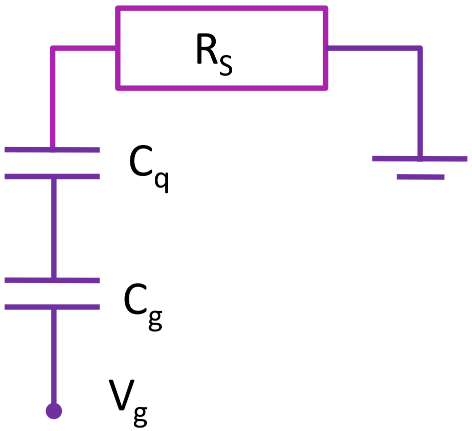
$$= 1 + U_0 \underbrace{\frac{\partial N}{\partial E}}_{-D_0} \frac{\partial E}{\partial(-qV_g)}$$

$$\frac{\partial E}{\partial(-qV_g)} = \frac{1}{1 + D_0 U_0}$$

using $U_0 = q^2 / C_E$

$$\frac{dE}{d(-qV_g)} = \frac{1}{1 + \frac{C_q}{q^2} \frac{q^2}{C_E}} = \frac{1}{1 + \frac{C_q}{C_E}} = \frac{C_E}{C_E + C_q}$$

$D_0 \gg 1$ Large DOS, Coulomb term dominates



$$\frac{dE}{d(-qV_g)} = \frac{C_G}{C_q}$$

This is the naïve picture one would use for graphene: calculate from Cg the density and than calculate E_f using the DOS of graphene

Not for a metal Cq→ inf, E_f does not move

$D_0 \ll 1$ Small DOS, Quantum capacitance dominates

$$\frac{dE}{d(-qV_g)} = 1 - C_q/C_G$$

extreme case: gapped system – C_q=0

Screening

$$\mathcal{H} = \sum_{\boldsymbol{k}\sigma} \varepsilon_{\boldsymbol{k}\sigma} c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} + \frac{1}{2V} \sum_{\substack{\boldsymbol{k}\boldsymbol{k}'\boldsymbol{q} \\ \sigma\sigma'}} U(\boldsymbol{q}) c^\dagger_{\boldsymbol{k}+\boldsymbol{q}\sigma} c^\dagger_{\boldsymbol{k}'-\boldsymbol{q}\sigma'} c_{\boldsymbol{k}'\sigma'} c_{\boldsymbol{k}\sigma}$$

General Hamiltonian:
Kinetic energy + Interaction

$$\rho(\boldsymbol{r},t) = \rho_{\text{ext}}(\boldsymbol{r},t) + \rho_{\text{ind}}(\boldsymbol{r},t)$$

As a reaction to external perturbation: density rearrangement

$$\epsilon_0 \operatorname{div} \boldsymbol{E}(\boldsymbol{r},t) = \rho(\boldsymbol{r},t)$$

Total density linked to E, external to D

potentials

$$\boldsymbol{E}(\boldsymbol{r},t) = -\operatorname{grad} \varphi(\boldsymbol{r},t)$$

$$\boldsymbol{D}(\boldsymbol{r},t) = -\epsilon_0 \operatorname{grad} \varphi_{\text{ext}}(\boldsymbol{r},t)$$

$$\epsilon_0 \boldsymbol{\nabla}^2 \varphi(\boldsymbol{r},t) = -\rho(\boldsymbol{r},t)\,,$$

$$\epsilon_0 \boldsymbol{\nabla}^2 \varphi_{\text{ext}}(\boldsymbol{r},t) = -\rho_{\text{ext}}(\boldsymbol{r},t)$$

Interactions

$$\mathcal{H}_1(t) = \frac{1}{V} \sum_{\boldsymbol{q}} V_{\text{ext}}(\boldsymbol{q},t) n(-\boldsymbol{q})$$

Response function: general susceptibility

$$n_{\text{ind}}(\boldsymbol{q},\omega) = \widetilde{\Pi}(\boldsymbol{q},\omega) V(\boldsymbol{q},\omega)$$

$$n_{\text{ind}}(\boldsymbol{q},\omega) = \Pi(\boldsymbol{q},\omega) V_{\text{ext}}(\boldsymbol{q},\omega)$$

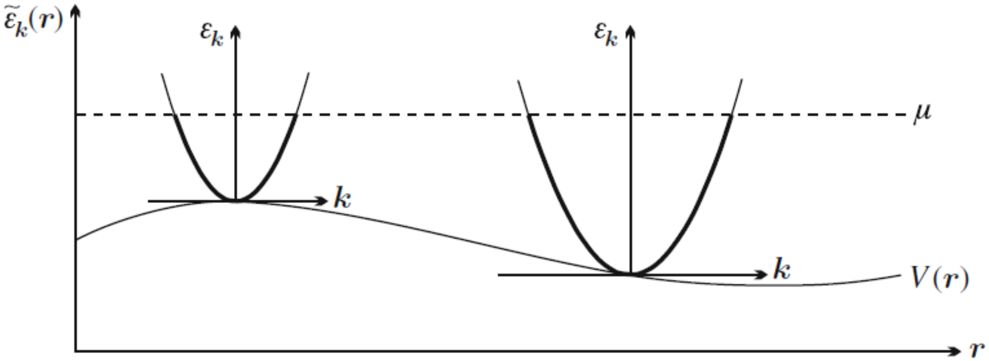
$$\Pi(\boldsymbol{r},\boldsymbol{r}',t-t') = -\frac{\mathrm{i}}{\hbar}\theta(t-t')\Big\langle \big[n(\boldsymbol{r},t), n(\boldsymbol{r}',t')\big]_- \Big\rangle$$

Self consistency

$$\Pi(\boldsymbol{q},\omega) = \frac{\widetilde{\Pi}(\boldsymbol{q},\omega)}{1 - U(q)\widetilde{\Pi}(\boldsymbol{q},\omega)}$$

Dielectric constant

$$\frac{1}{\epsilon_{\mathrm{r}}(\boldsymbol{q},\omega)} = 1 + \frac{4\pi\tilde{e}^2}{q^2} \Pi(\boldsymbol{q},\omega)$$



$$n_{\mathrm{ind}}(\boldsymbol{q}) = -\rho(\varepsilon_{\mathrm{F}})V(\boldsymbol{q})$$

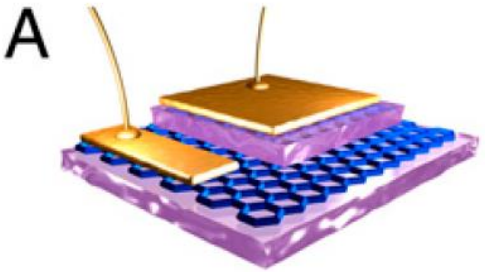
Quantum capacitance picture: takes into account long wavelength

$$\Pi_0(\boldsymbol{q},\omega) = \frac{2}{V}\sum_{\boldsymbol{k}} \frac{f_0(\varepsilon_{\boldsymbol{k}}) - f_0(\varepsilon_{\boldsymbol{k}+\boldsymbol{q}})}{\hbar\omega - \varepsilon_{\boldsymbol{k}+\boldsymbol{q}} + \varepsilon_{\boldsymbol{k}} + \mathrm{i}\delta}$$

Lindhard picture

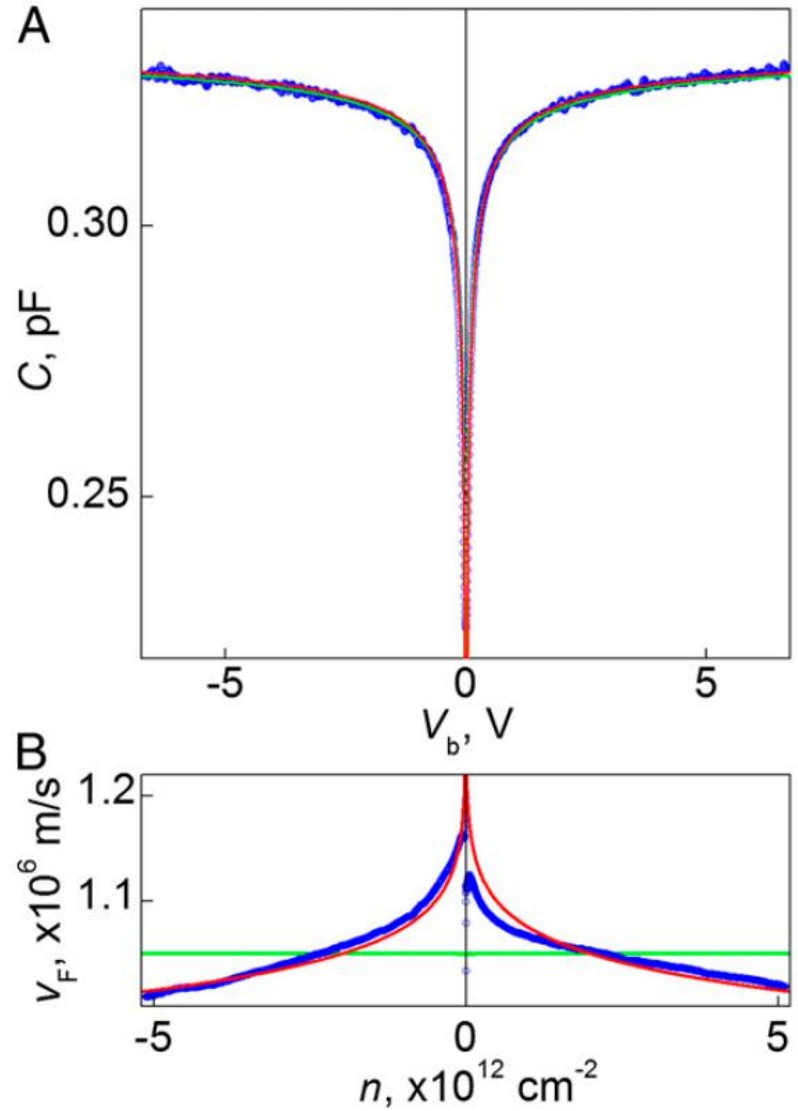
$$\Pi(\boldsymbol{q},\omega) = \frac{\widetilde{\Pi}(\boldsymbol{q},\omega)}{1 - U(\boldsymbol{q})\widetilde{\Pi}(\boldsymbol{q},\omega)}$$

$$n = C_G V_G = \frac{C_G \cdot C_Q}{C_G + C_Q} V_g = \frac{C_Q}{\frac{C_Q}{C_g} + 1}$$

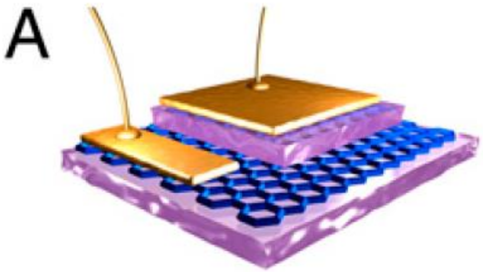


$$v_F(n) = v_F(n_0)\left[1 + e^2 \cdot \ln(n_0/n)/16h\varepsilon_0\varepsilon v_F(n_0)\right]$$

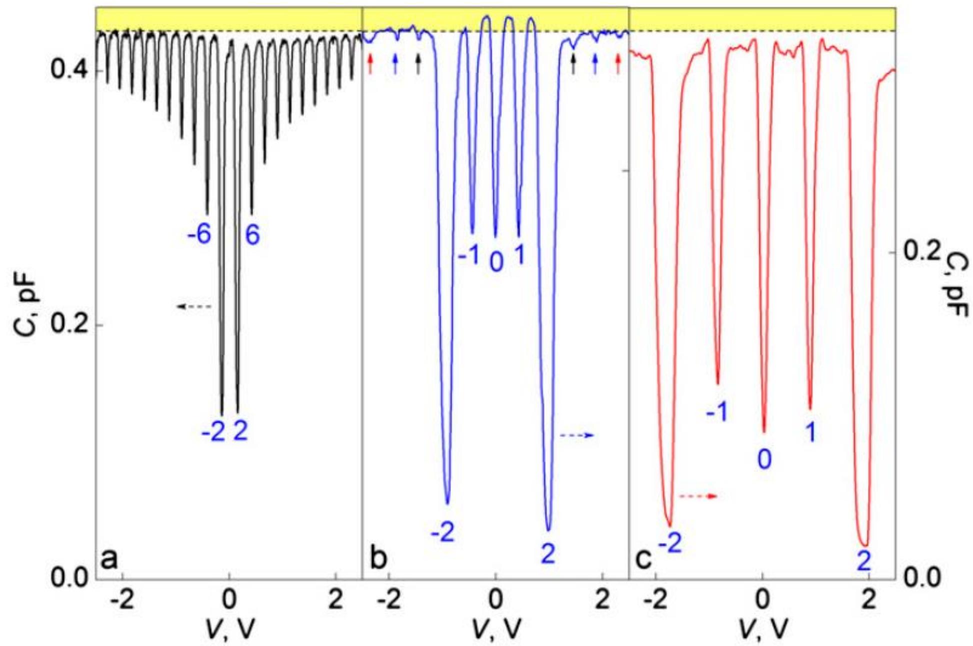
Electron-electron interaction renormalizes vf

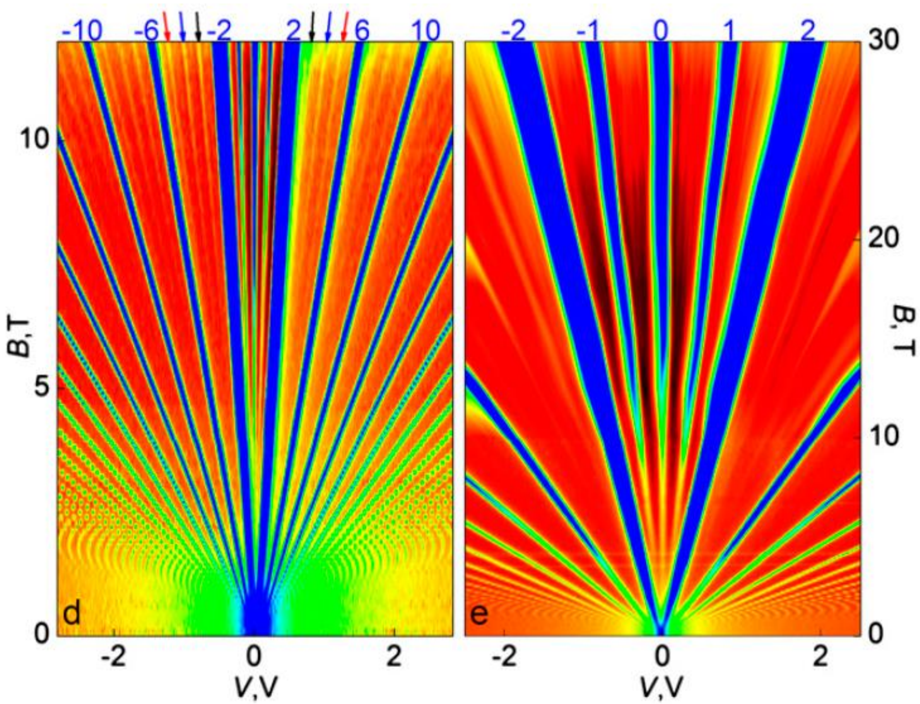


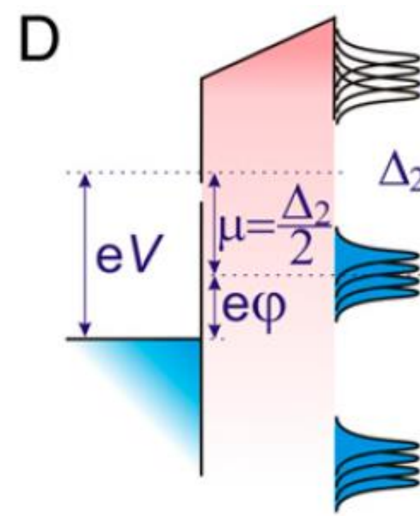
G. L. Yu et al., PNAS 110,3282 (2013)



Quantum Hall regime: when bulk is gapped capacitance is zero.
Can be tested if the bulk is gapped.



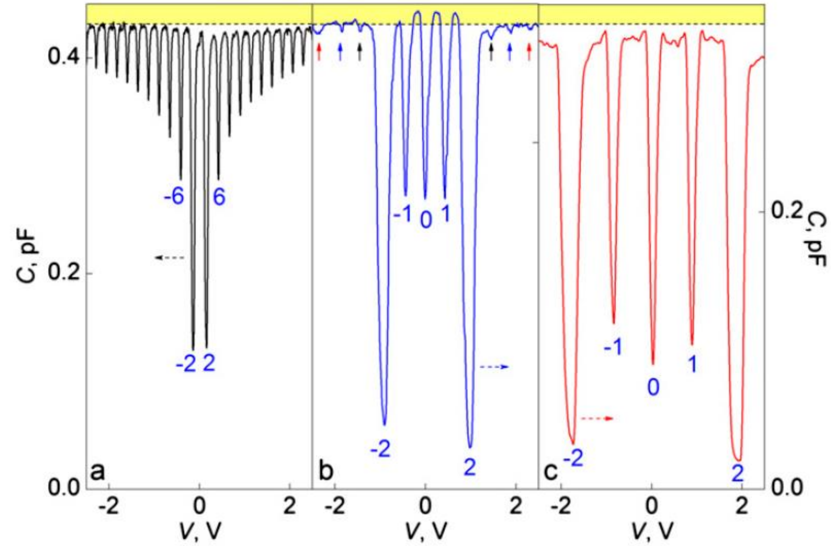




G. L. Yu et al., PNAS 110,3282 (2013)

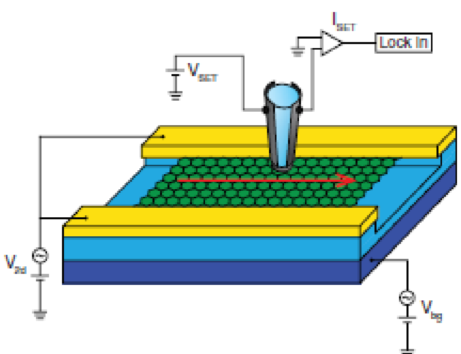
$$\kappa^{-1} = n^2 d\mu/dn$$ Inverse compressibility

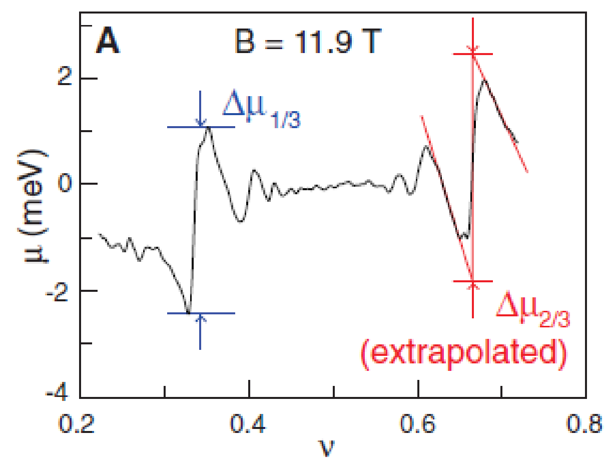
$\kappa \sim D_0$ compressibility



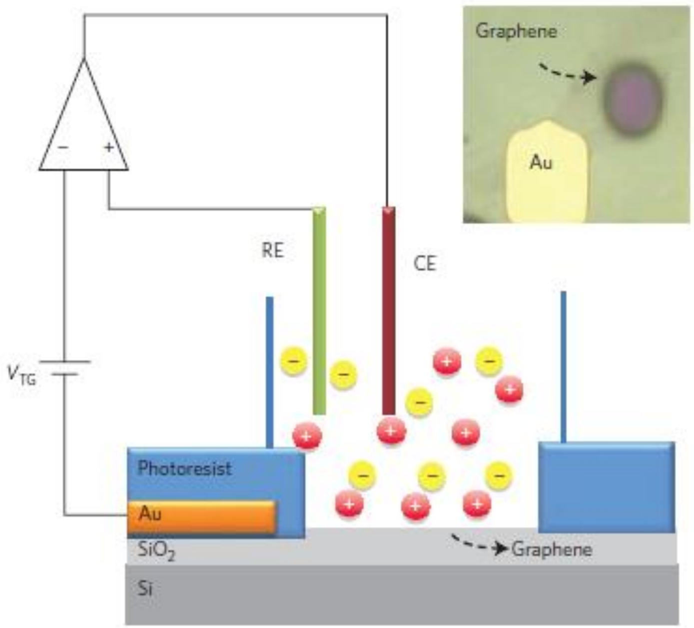
Dips: incompressible state
Gapped state:
Chemical potential can be increased without adding charges

Compressible state
On LL: you have to add charges to increase chemical potential



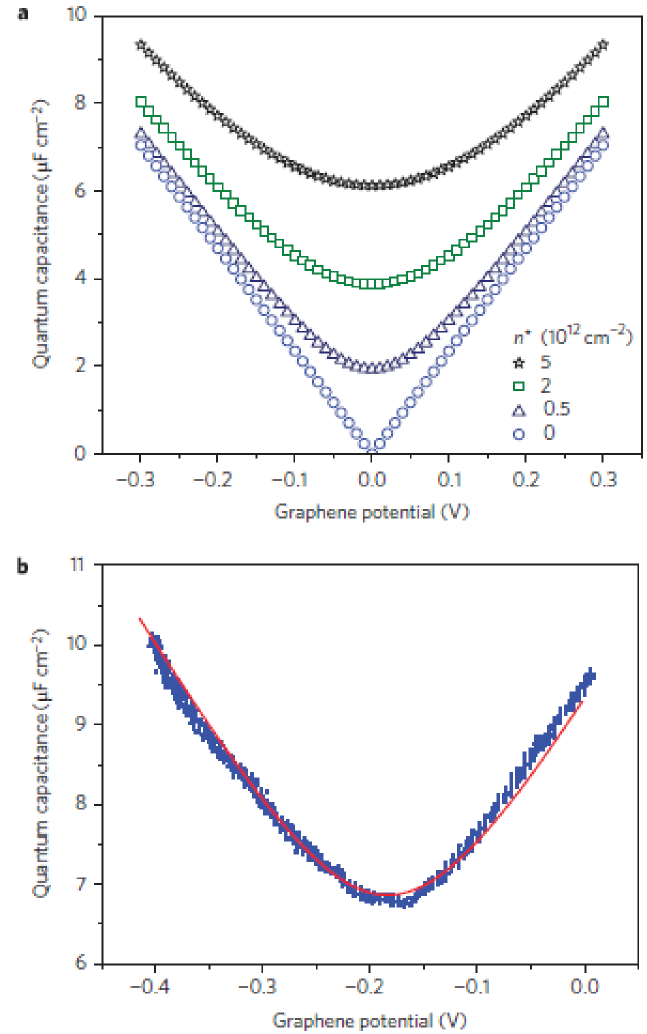


B. E. Feldman et al., Science 337 1196 (2012)

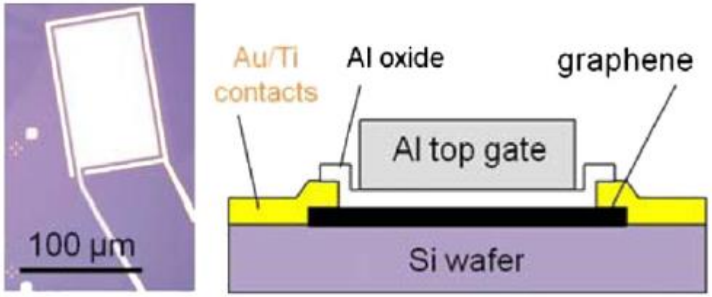


Using a liquid gate quantum capacitance dominates

$$C_Q = \frac{2e^2}{\hbar v_F \sqrt{\pi}} (|n_G| + |n^*|)^{1/2}$$



J. Xia et al., Nat. Nanotech. 4, 505 (2009)

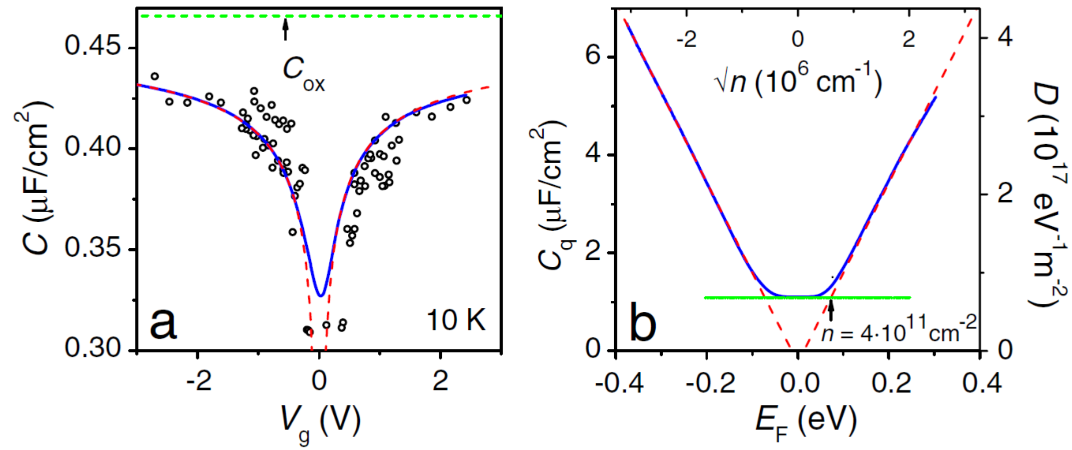


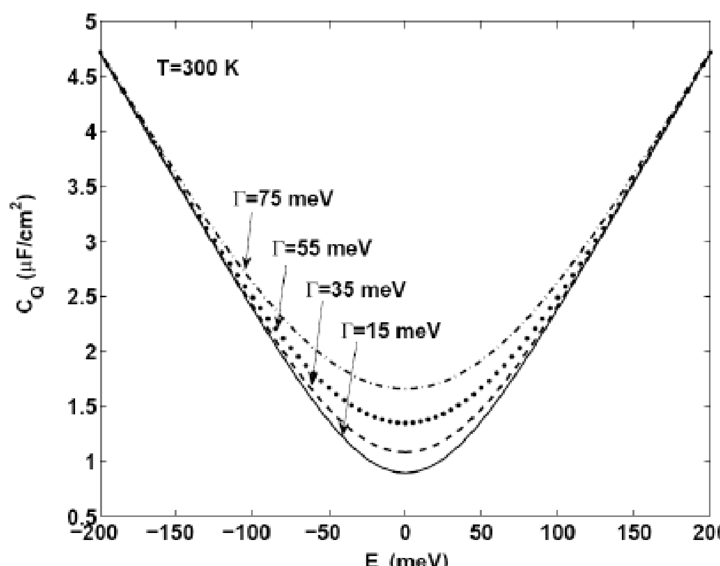
Using a topgate capacitance

$$C_Q = \frac{2e^2}{\hbar v_F \sqrt{\pi}} \left(|n_G| + |n^*|\right)^{1/2}$$

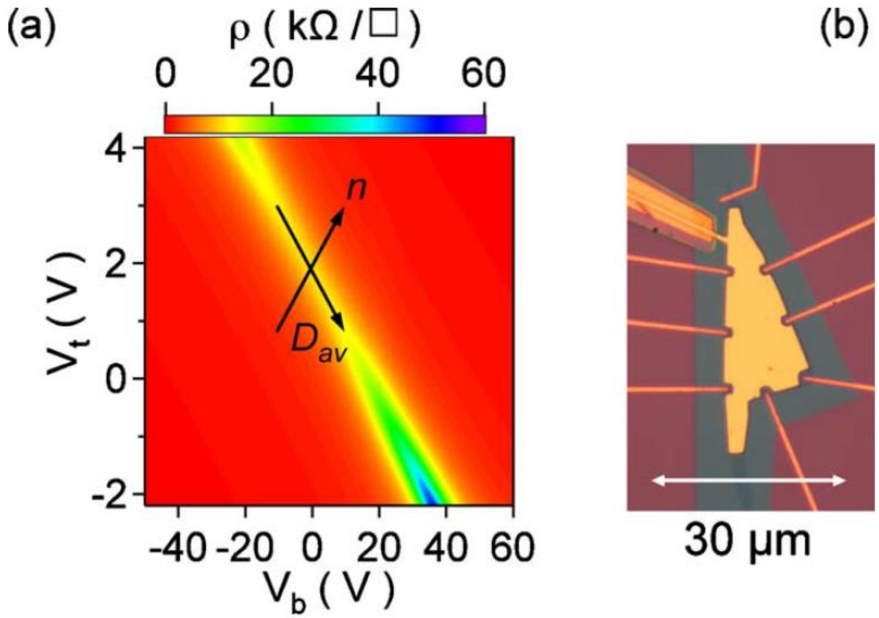
Sometimes smearing of DOS is introduced (e.g. puddles):

$$D(E) = \frac{1}{\sqrt{2\pi}\,\Gamma} \int_{-\infty}^{+\infty} \exp\left(-\frac{(\varepsilon - E)^2}{2\Gamma^2}\right) g(\varepsilon)\, d\varepsilon$$





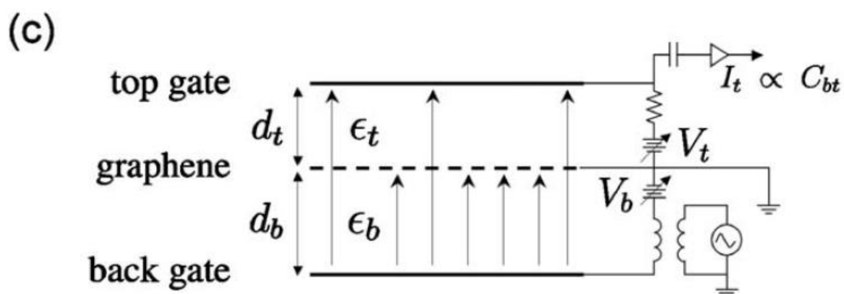
L. A. Ponomarenko et al., PRL 105, 136801 (2010)

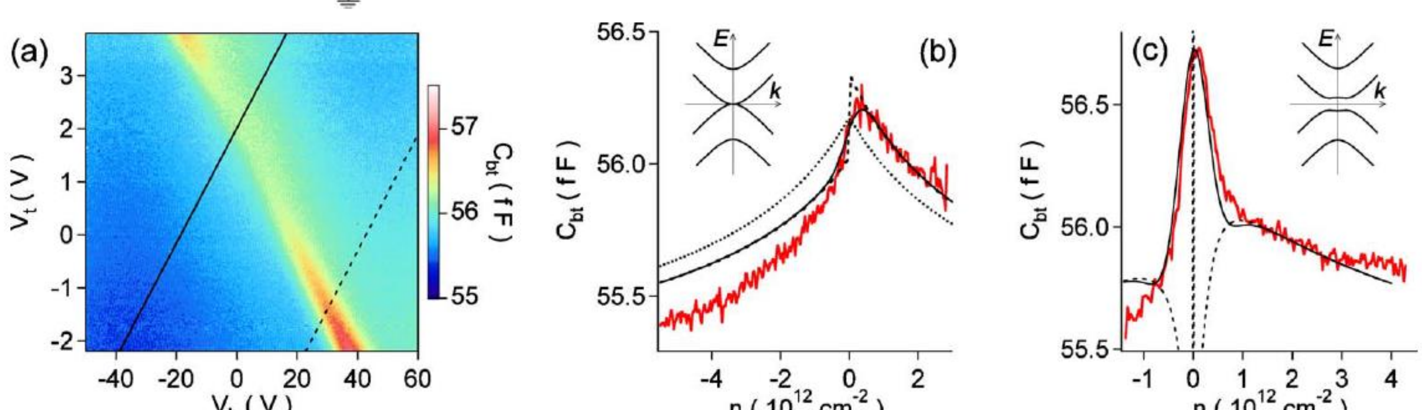


$$C_{bt} = \frac{A_t d_g \epsilon_b \epsilon_t}{d_g(\epsilon_t d_b + \epsilon_b d_t) + \epsilon_0 d_b d_t} + C_s.$$

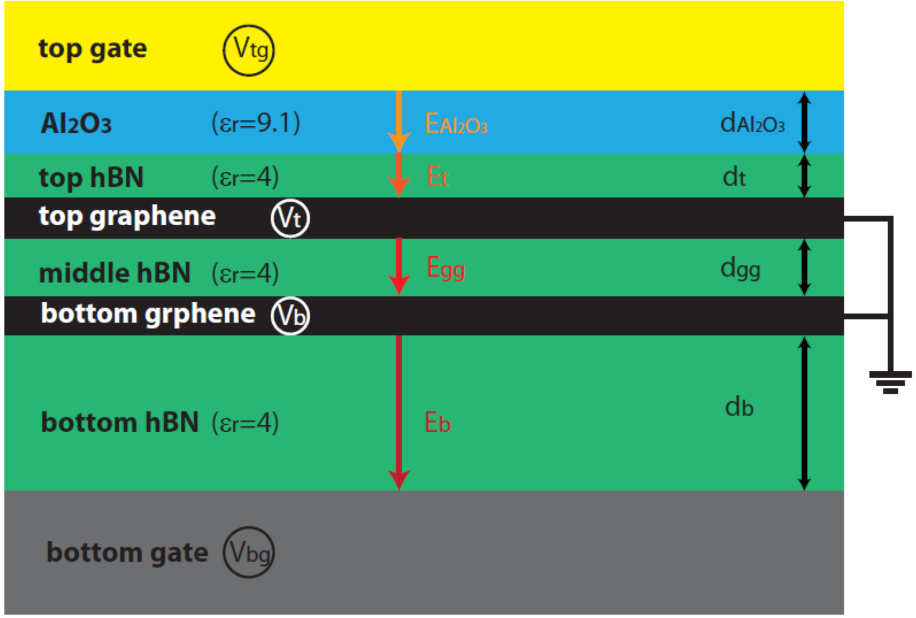
$$d_g = (\epsilon_0 / e^2)\, \partial \mu / \partial n,$$

Bilayer graphene – measure penetration capacitance:





E. A. Henriksen et al., PRB 82, 041412 (R) (2010)



$$\mu_c^t + eV_t = \mu_c^b + eV_b = 0,$$

$$\mu_c = sng(n)\hbar v_F \sqrt{\pi |n|},$$

$$en_t = \epsilon_0 \epsilon_r^{hBN}(E_{gg} - E_t)$$

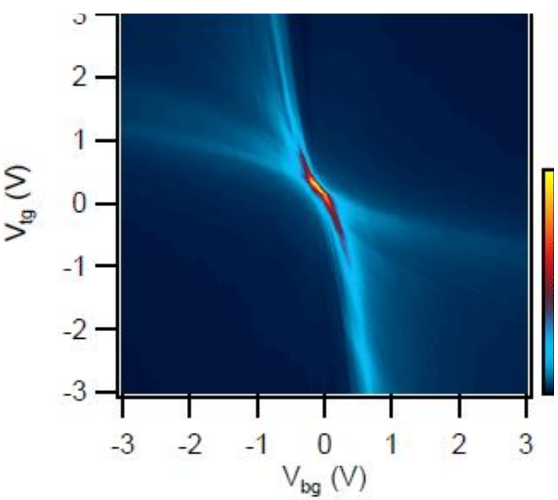
$$en_b = \epsilon_0 \epsilon_r^{hBN}(E_b - E_{gg})$$

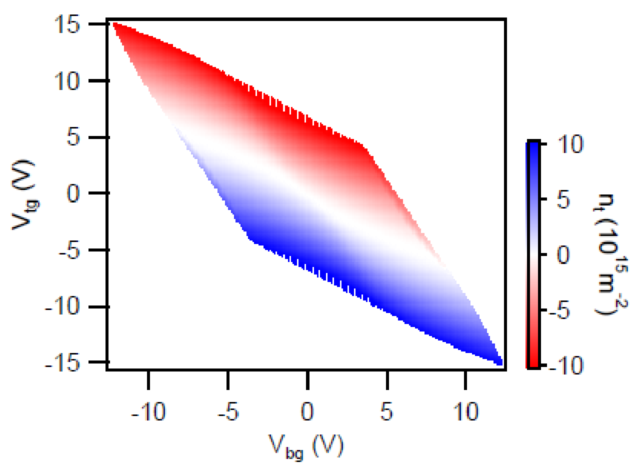
$$E_b d_b = V_b - V_{bg}$$

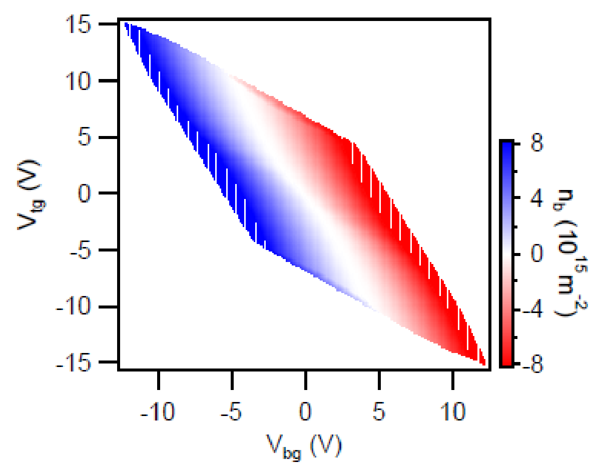
$$E_{gg} d_{gg} = V_t - V_b$$

$$E_t d_t + E_{Al_2O_3} d_{Al_2O_3} = V_{tg} - V_t$$

$$V_{tg} = -\frac{sgn(n_t)\sqrt{\pi}\hbar v_F}{e}\sqrt{|n_t|} - \frac{en_t d_t^{eff}}{\epsilon_0 \epsilon_r^{hBN}} - \frac{\sqrt{\pi}\hbar v_F d_t^{eff}}{ed_{gg}}\left(sgn(n_t)\sqrt{|n_t|} - sgn(n_b)\sqrt{|n_b|}\right).$$

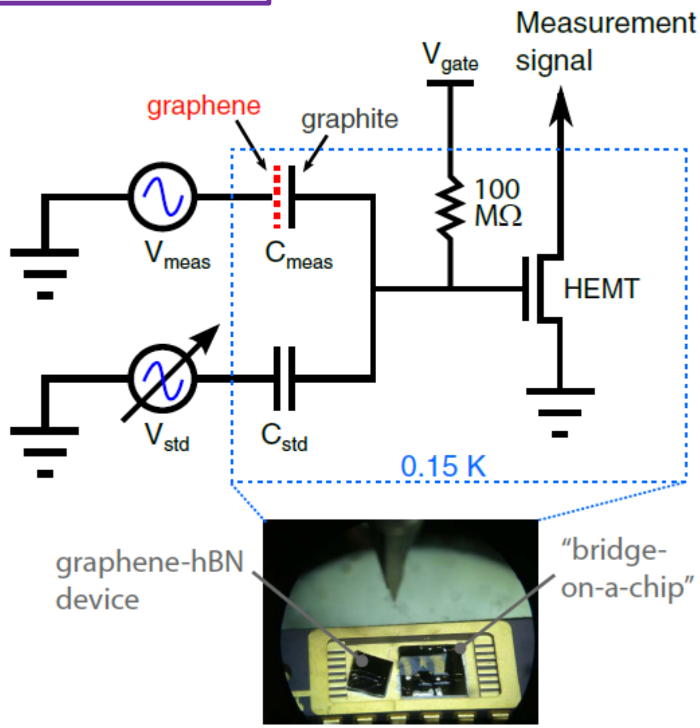






D. Indolese

Methods

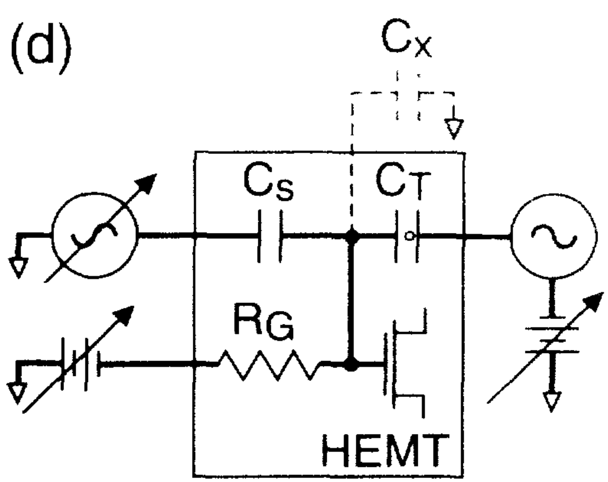


Idea: Capacitance bridge
Measure only the change in capacitance.

Capacitance change is sensed by the HEMT (in AC).
First at a particular gate voltage the bridge is balanced by changing V_{std}:

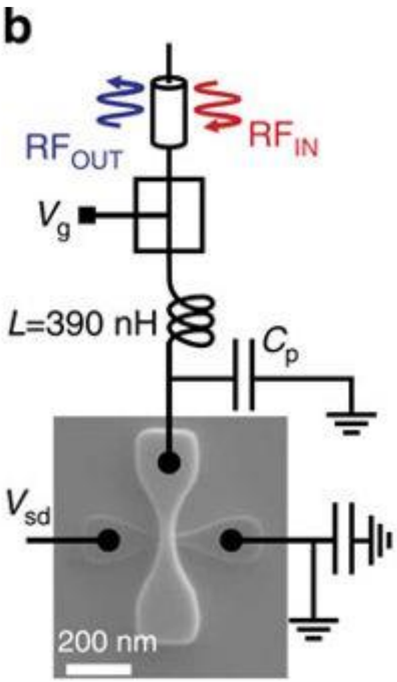
$$C_{meas} = (V_{std}/V_{meas})\, C_{std}$$

Also care has to be taken, since if the sample gaps time constants become long….



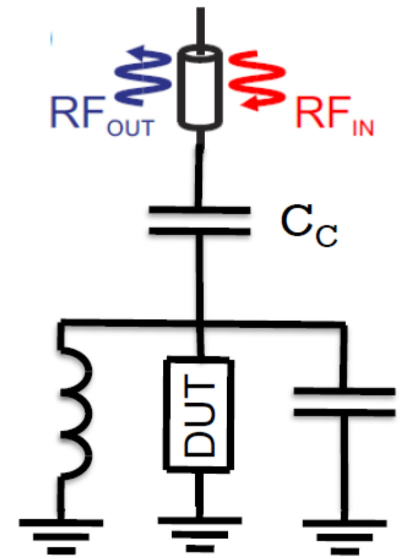
B. Hunt et al., Science 340, 1427 (2013)

Methods



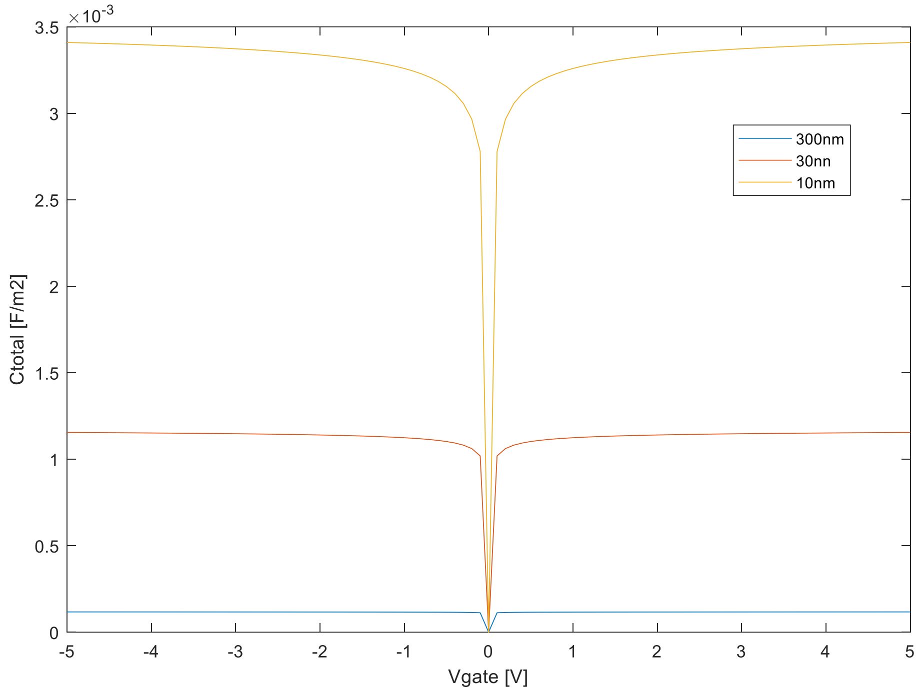
Reflectometry? Not often used for 2D systems
Gate reflectometry
Challenge:
Readuce parallel capacitance

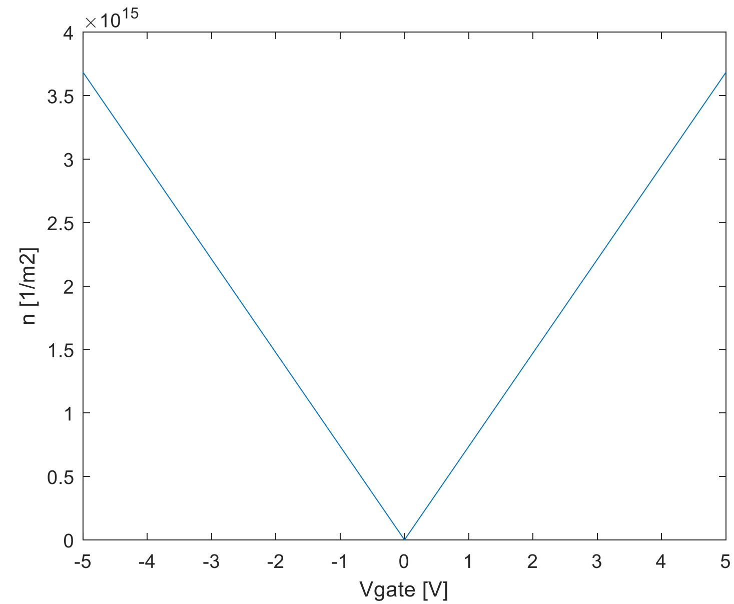
- Parallel resonator



M.F. Gonzalez-Zalba et al., Nature Comm. 6, 6084 (2015)

Some calculation SLG





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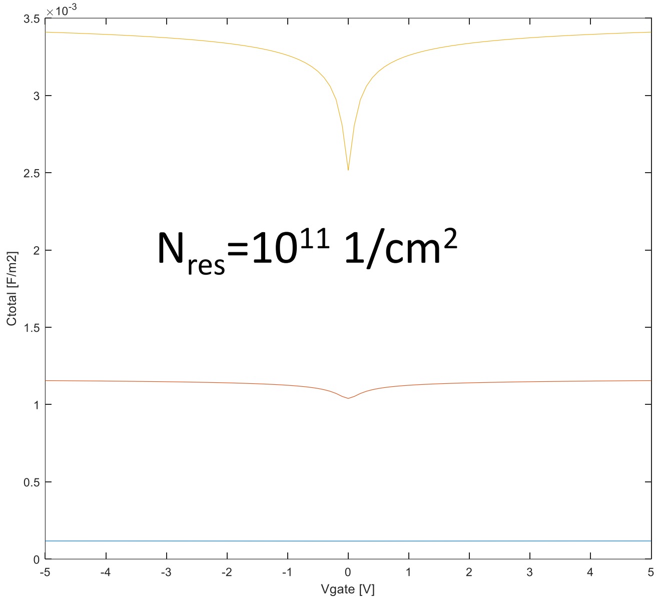

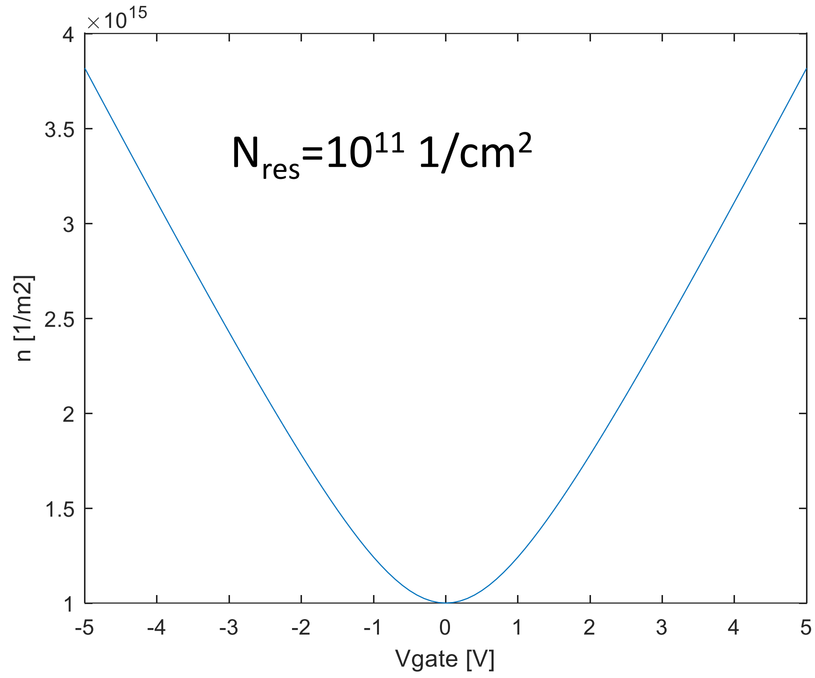

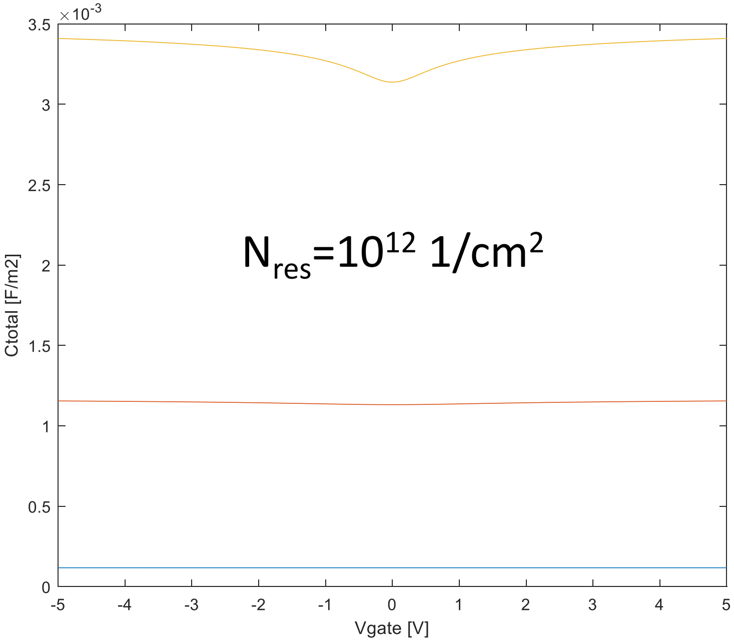

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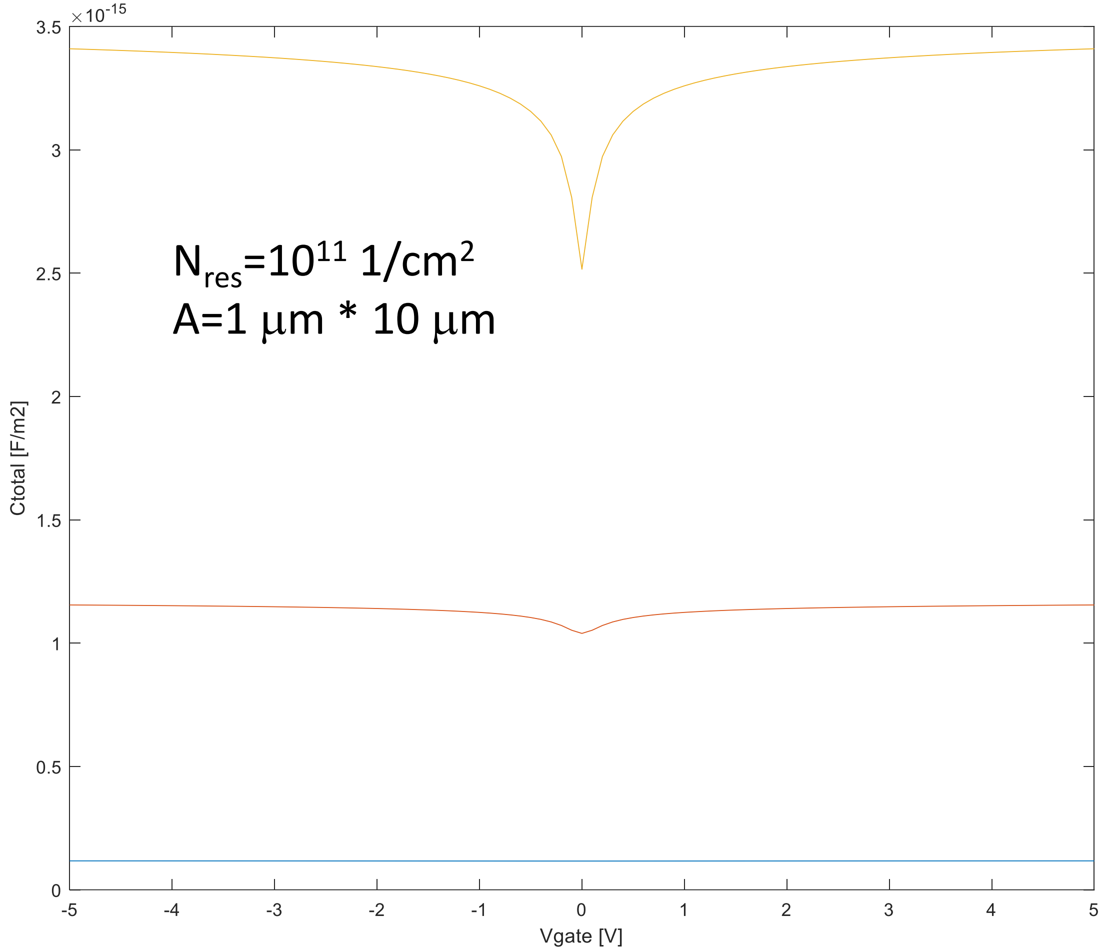
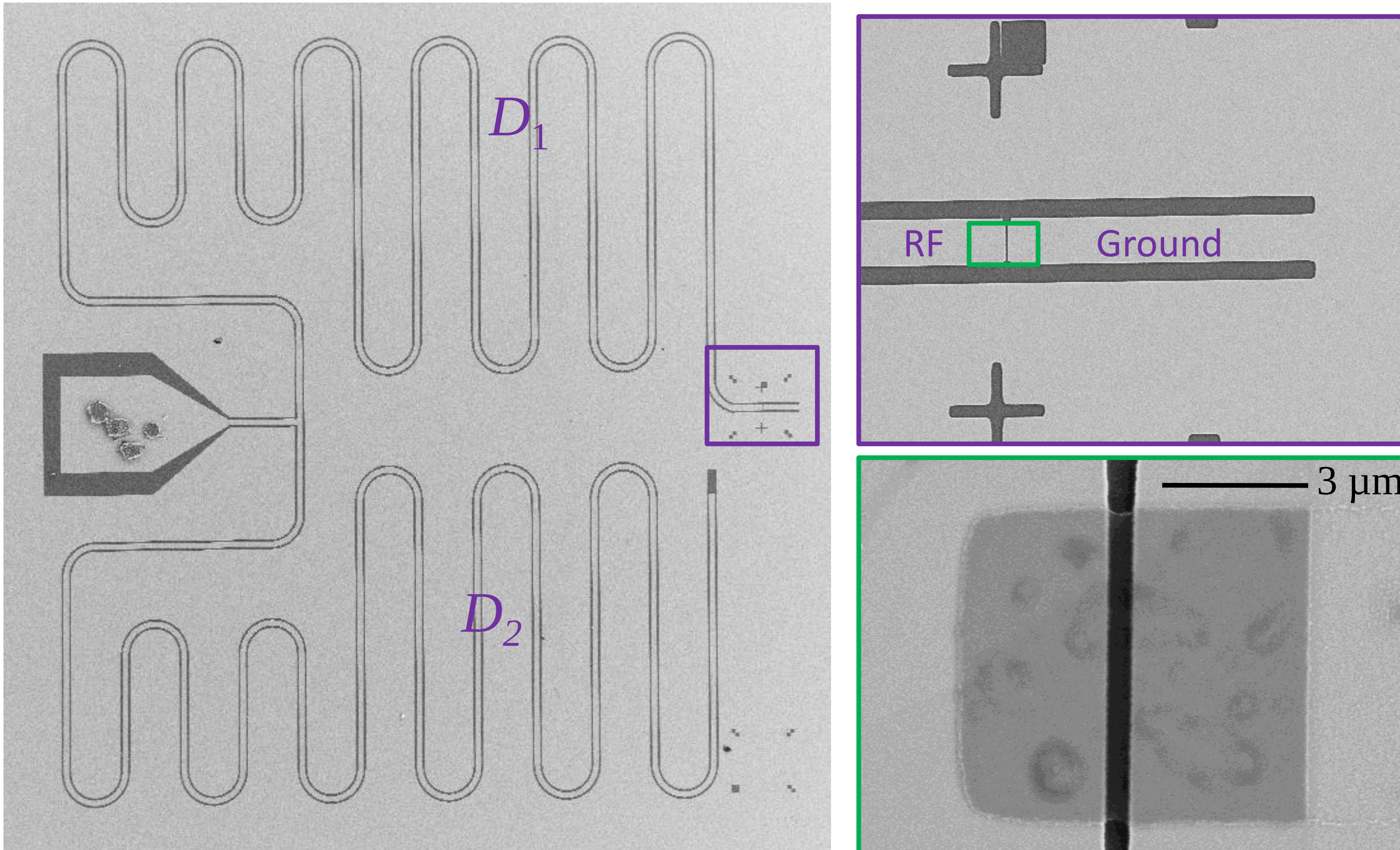
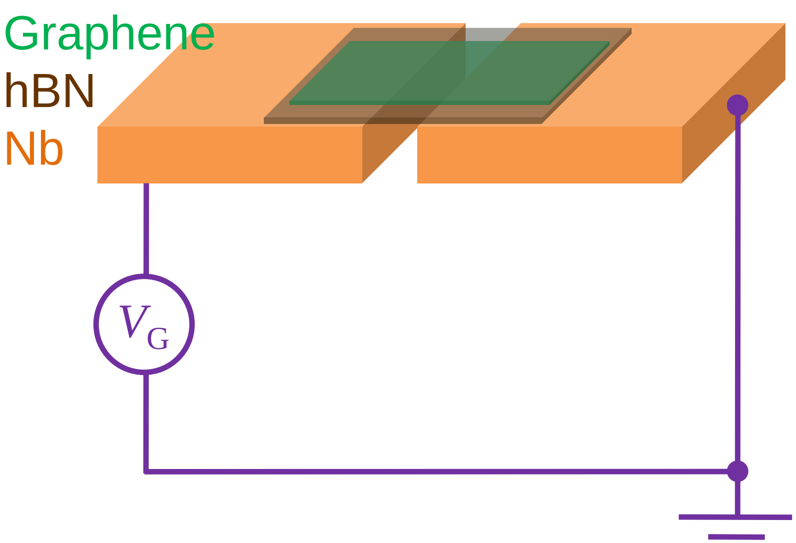
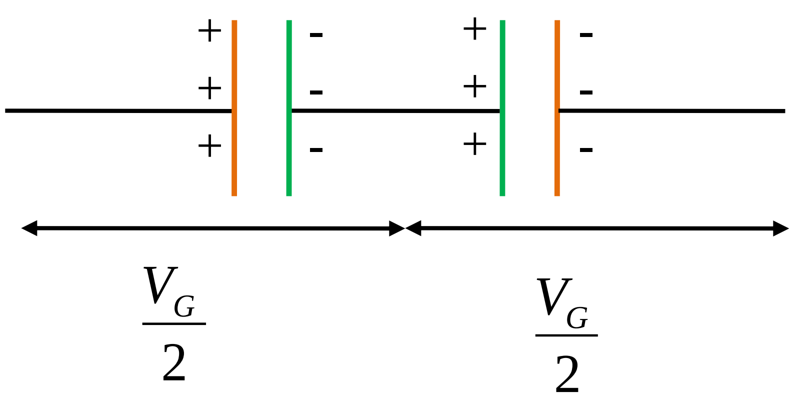


Pn measurement

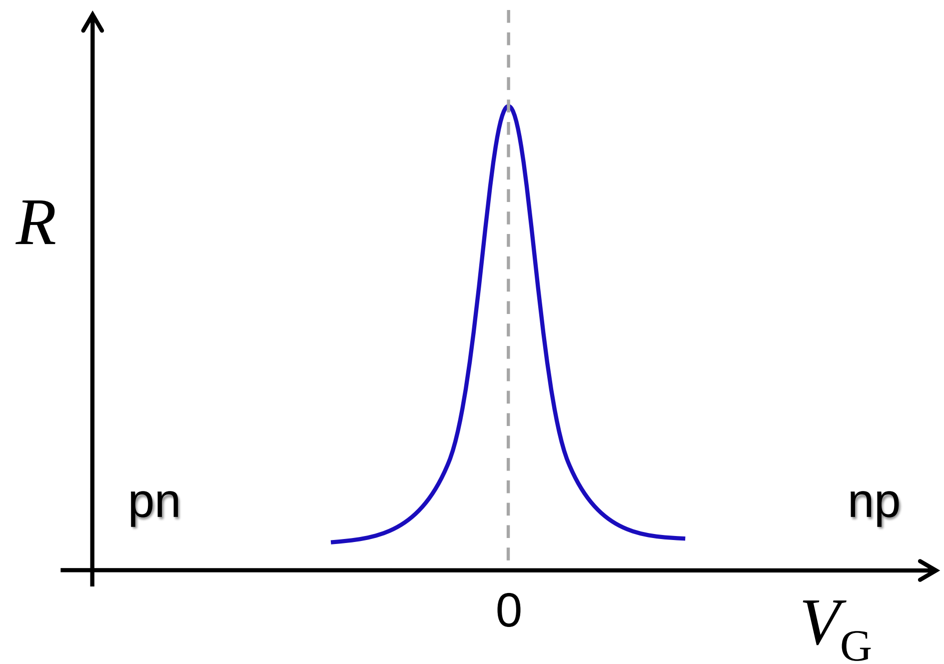


G. Hellmann at al., APL (2012)



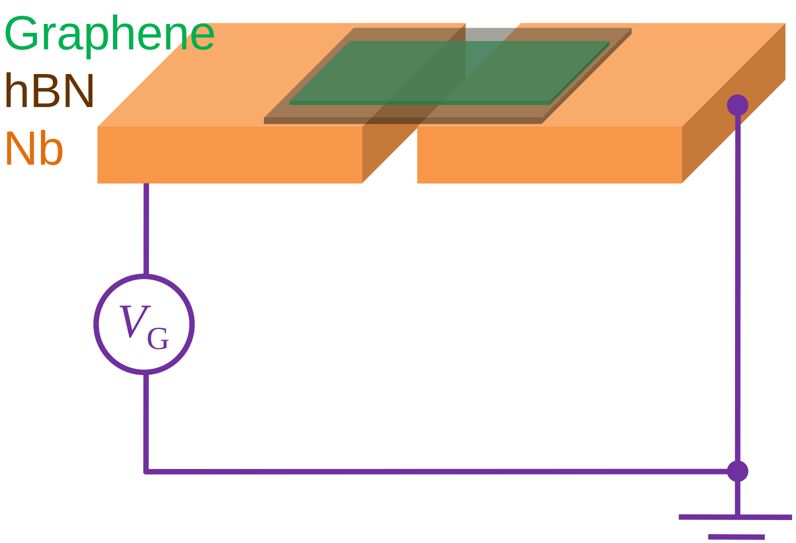


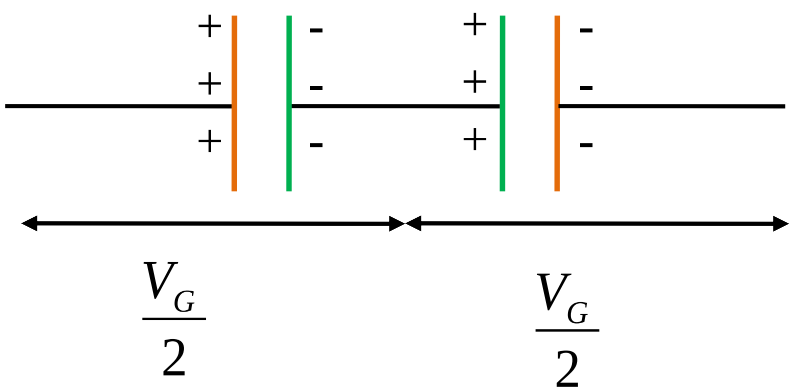
Residual doping = 0

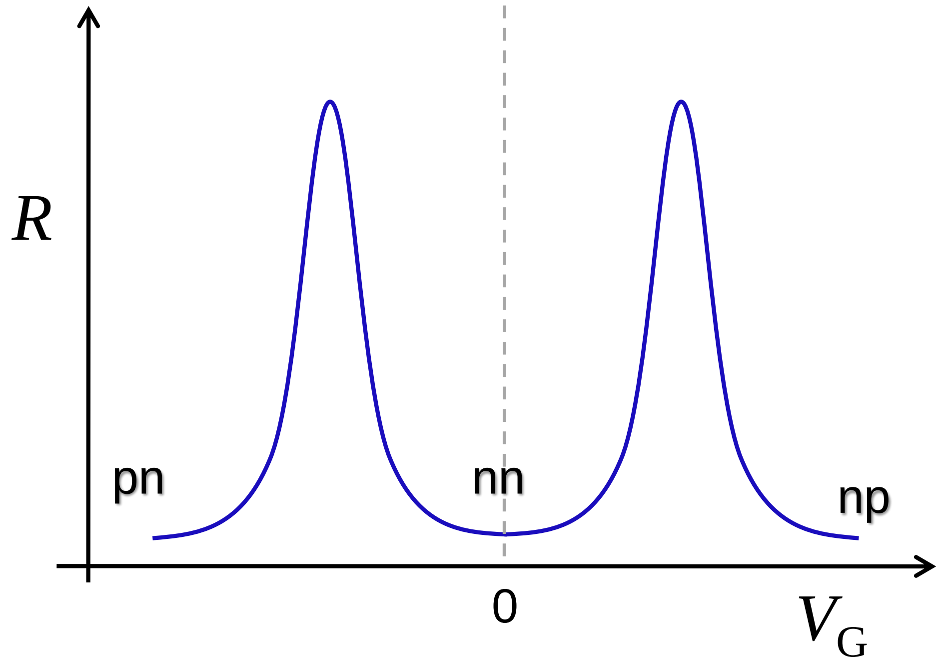


Symmetric graphene

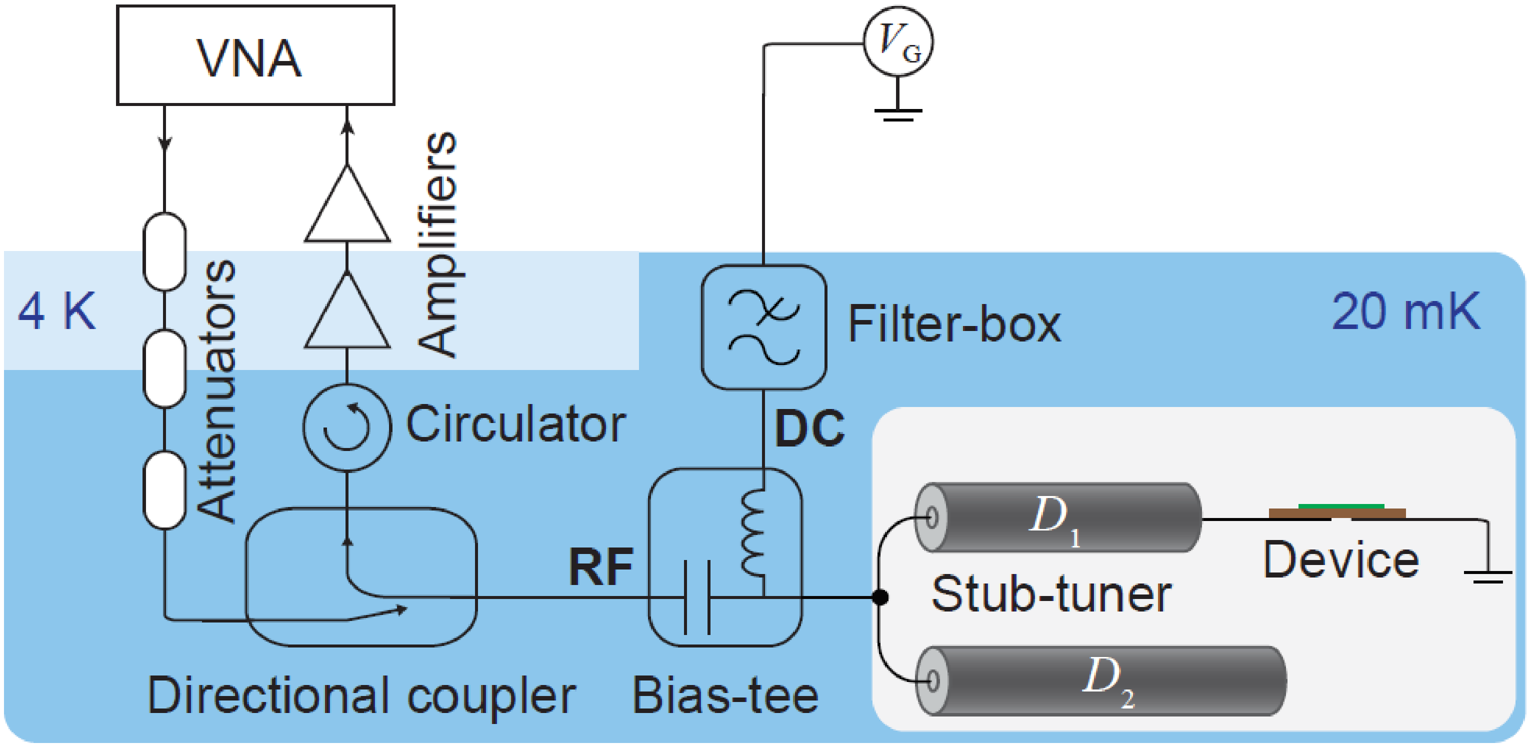
Residual doping ≠ 0

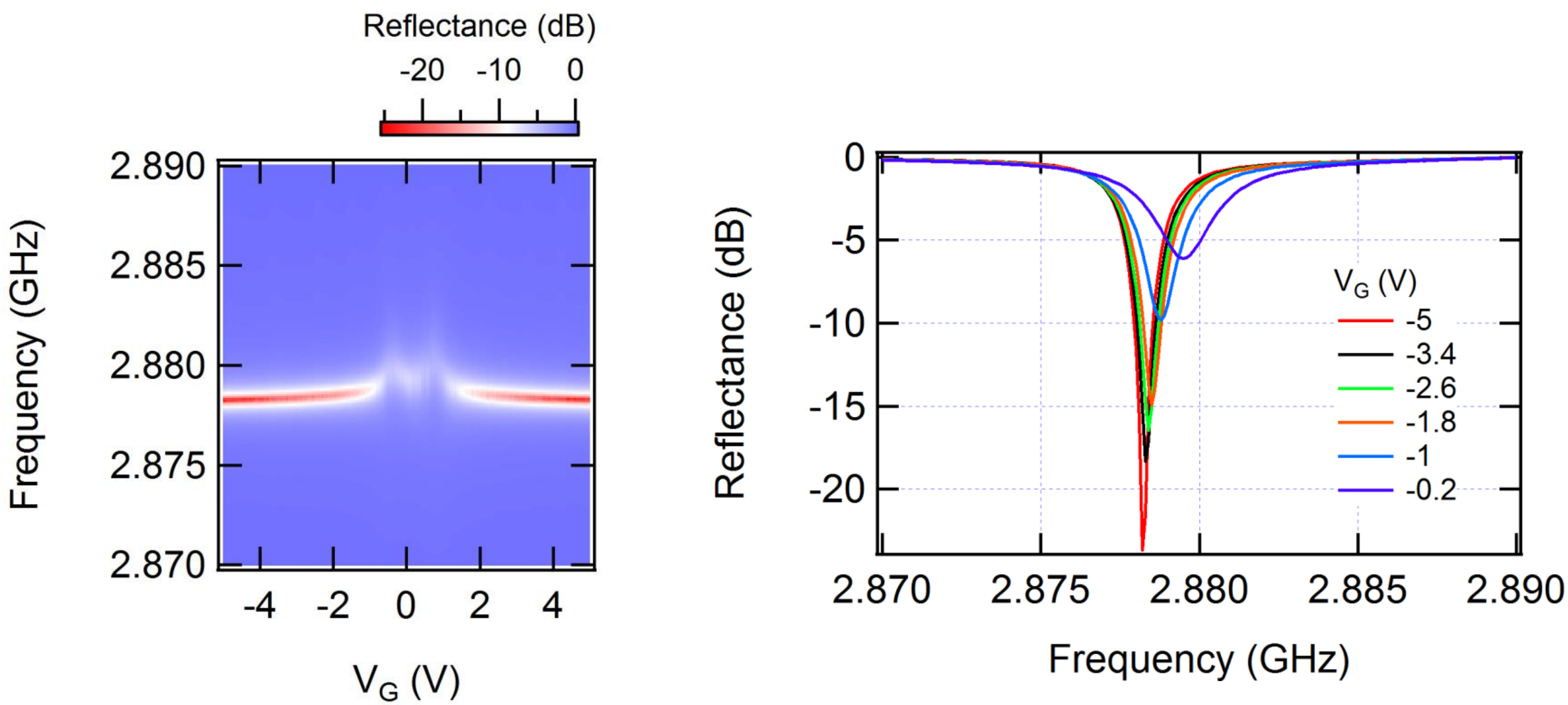




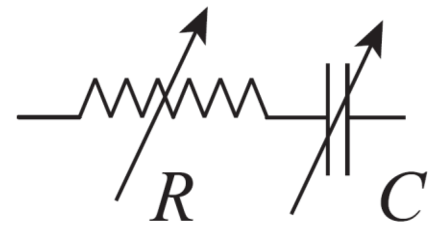


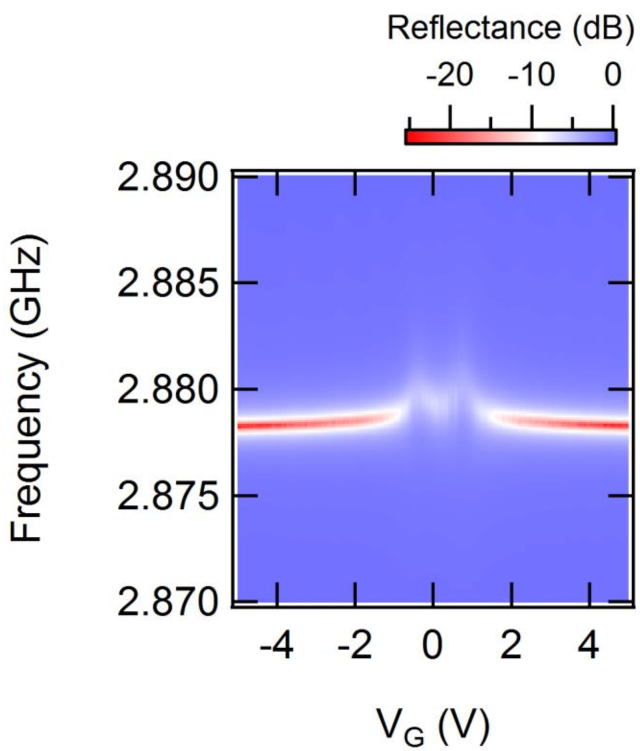
Symmetric graphene

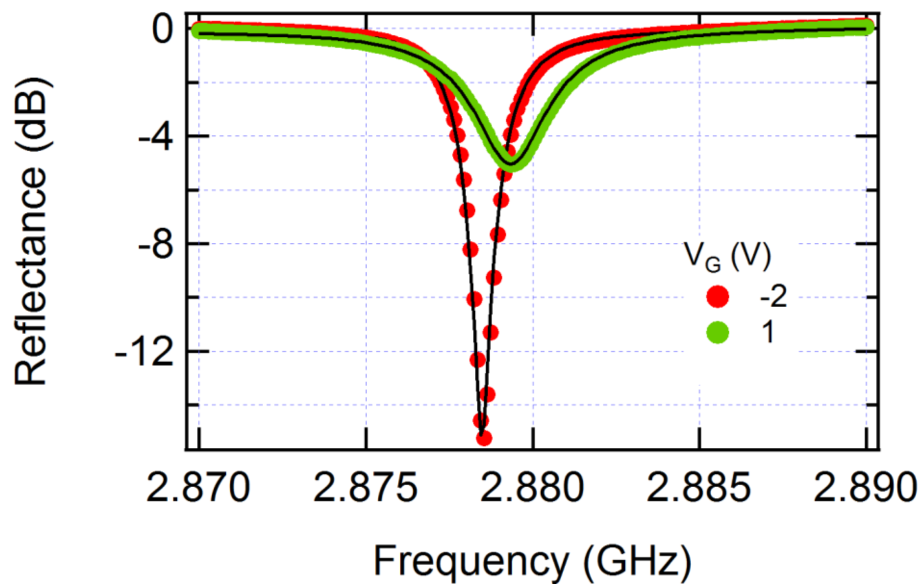

VNA
V_G
Amplifiers
4 K
Attenuators
Filter-box
20 mK
Circulator
DC
D_1
RF
Device
Stub-tuner
D_2
Directional coupler
Bias-tee



Frequency shifts and linewidth changes due to varying capacitance and resistance





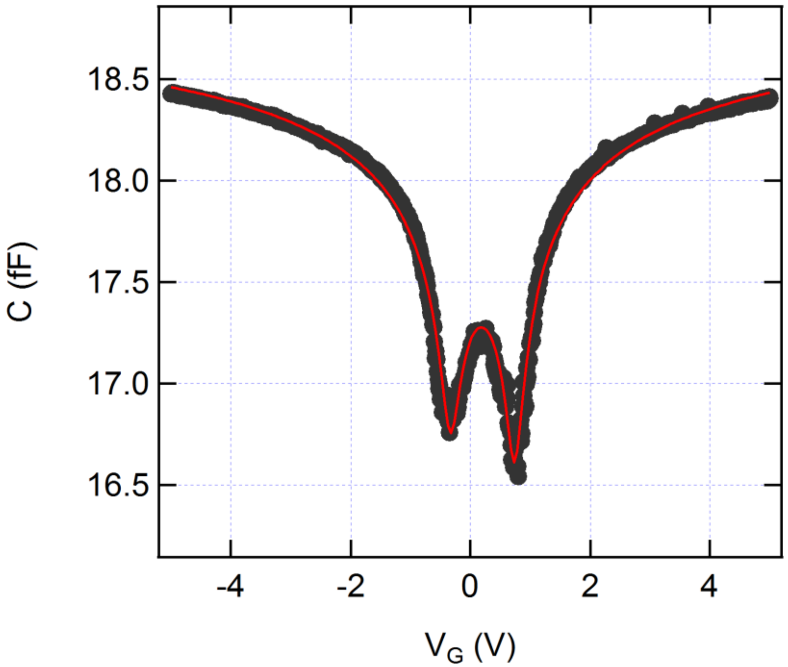


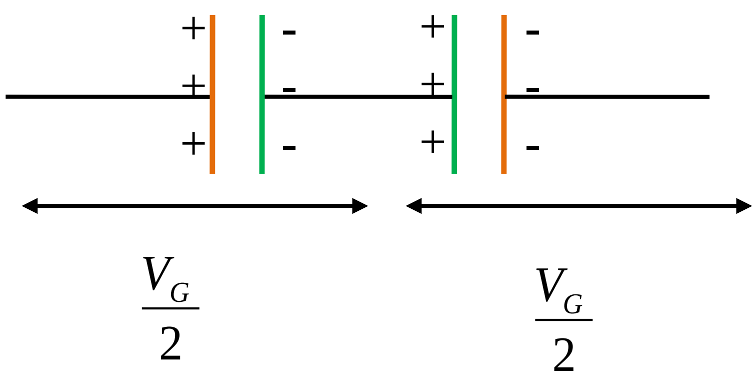
$$Z_{in} = Z_0 \left[\tanh(\gamma D_2) + \frac{Z_0 + Z_L \tanh(\gamma D_1)}{Z_L + Z_0 \tanh(\gamma D_2)} \right]$$

$$Z_L = R + \frac{1}{j\omega C}$$

R = 94.9 Ω
C = 19.8 fF

R = 263.1 Ω
C = 18.8 fF





$$C_q = \frac{2e^2 A}{\hbar v_F \sqrt{\pi}} \sqrt{n_{tot}}$$

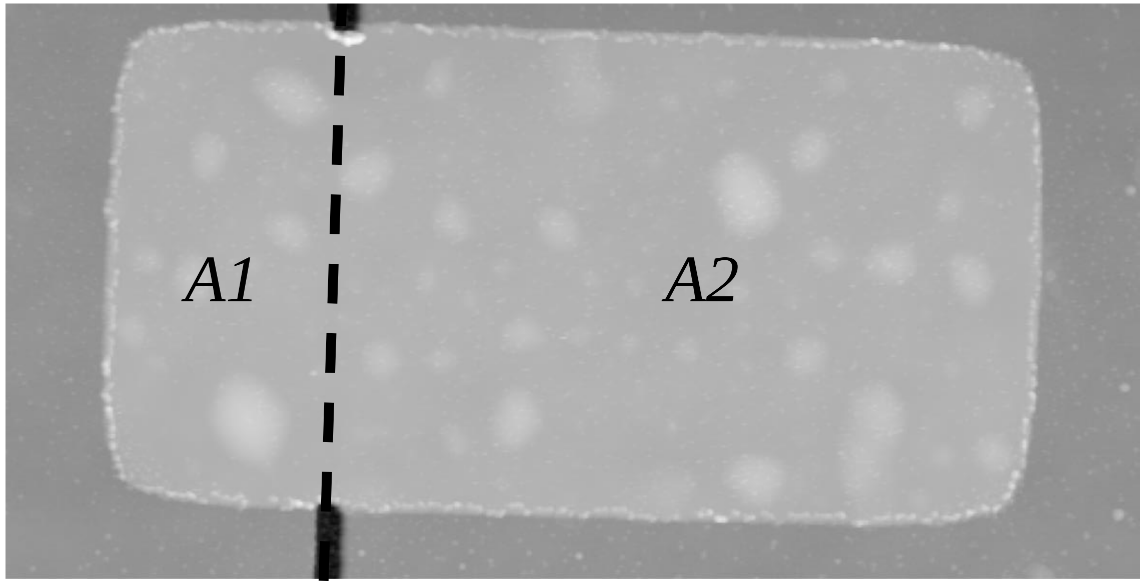
$$n_{tot} = \sqrt{n^2 + n_{imp}^2}$$

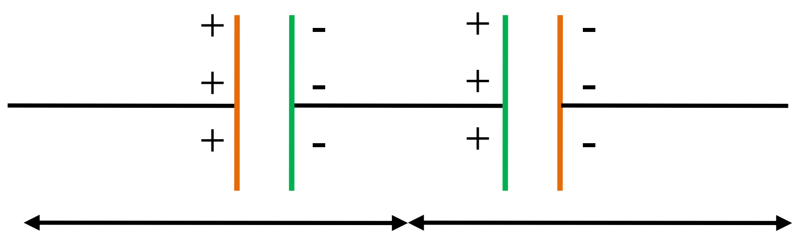
$$n = \frac{C_g (V_G - V_D)}{Ae}$$

$$n_{imp} \approx 6 \times 10^{10} cm^{-2}$$

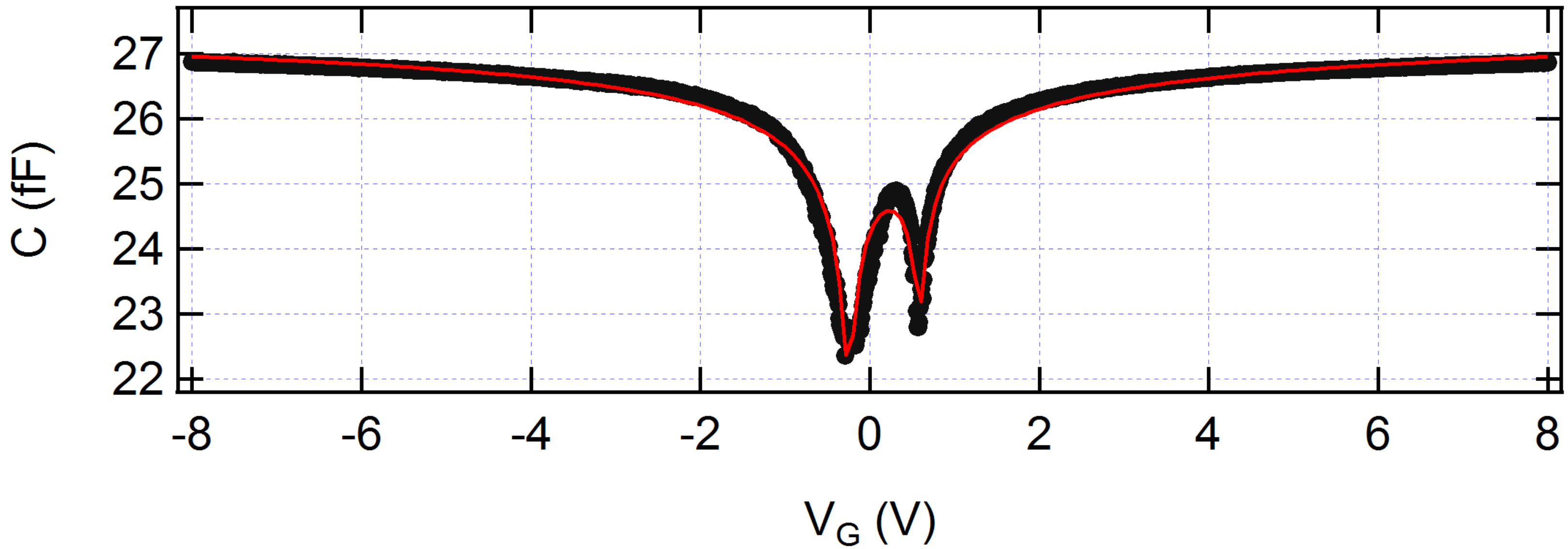
$$v_F \approx 0.95 \times 10^6 m/s$$

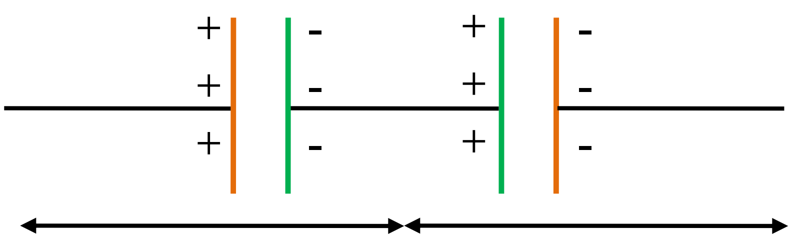
$$\varepsilon_{BN} \approx 4$$





$$V_G\left(\frac{A2}{A1+A2}\right) \qquad V_G\left(\frac{A1}{A1+A2}\right)$$



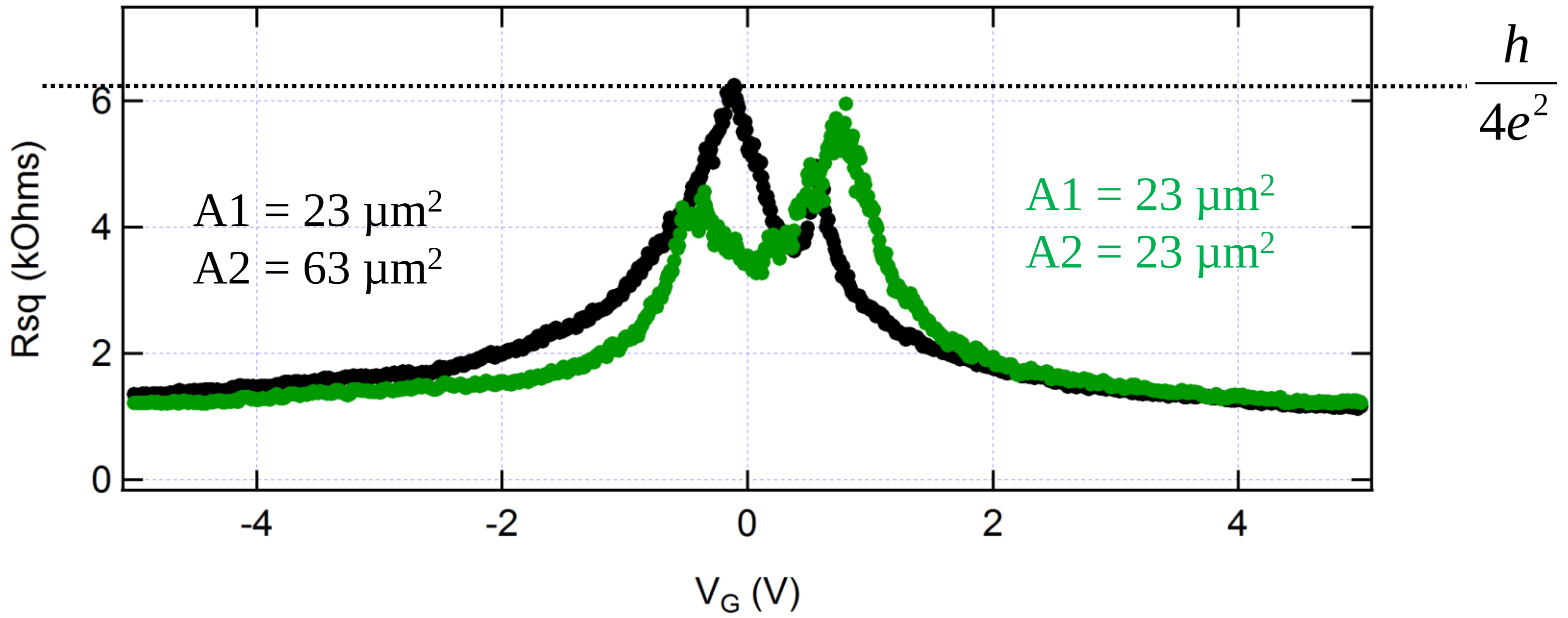


$$V_G\left(\frac{A2}{A1+A2}\right) \qquad V_G\left(\frac{A1}{A1+A2}\right)$$

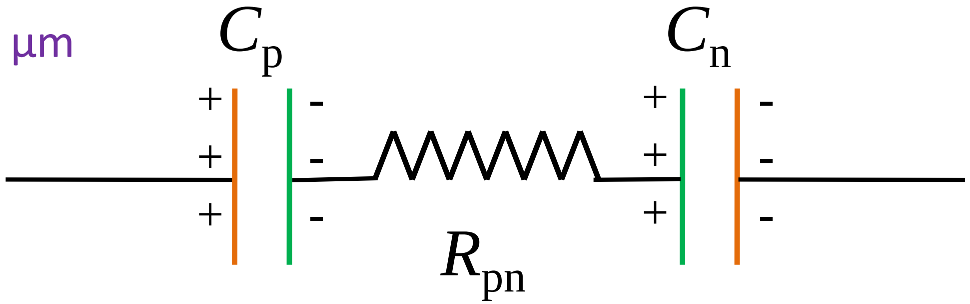
$$n_{imp} \approx 3\times10^{10}\,cm^{-2}$$

$$v_F \approx 1.03\times10^{6}\,m/s$$

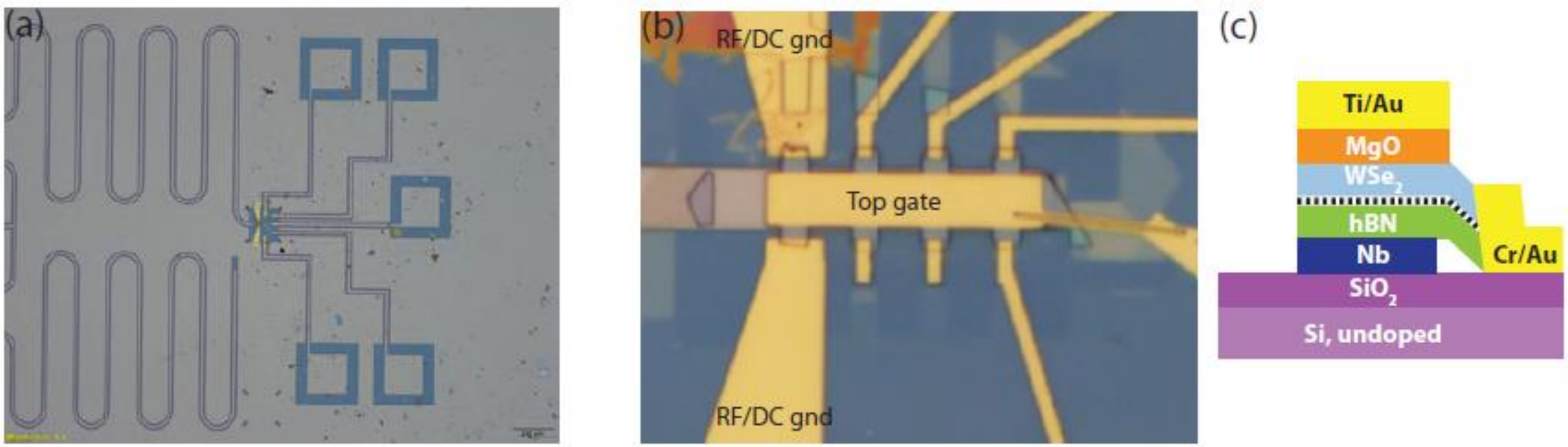
$$\varepsilon_{BN} \approx 4$$

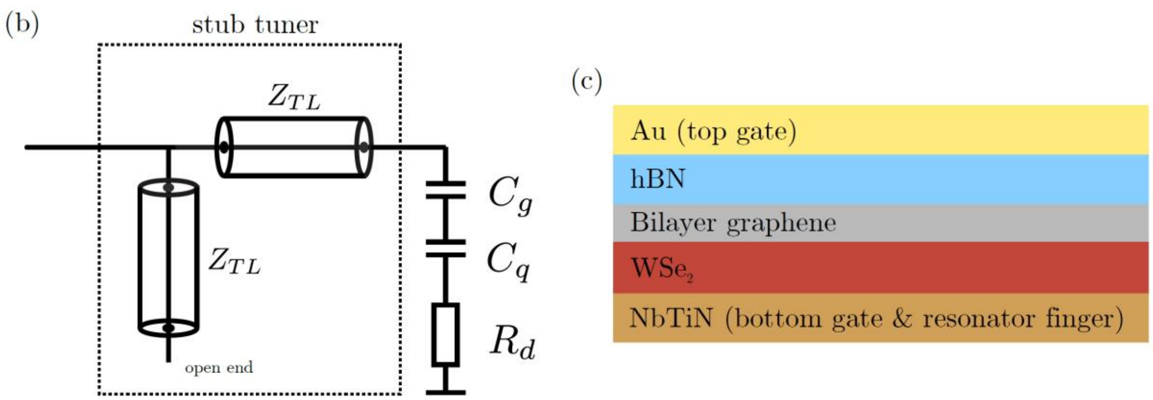


Slit width = 400 nm
Graphene width = 6.5 μm



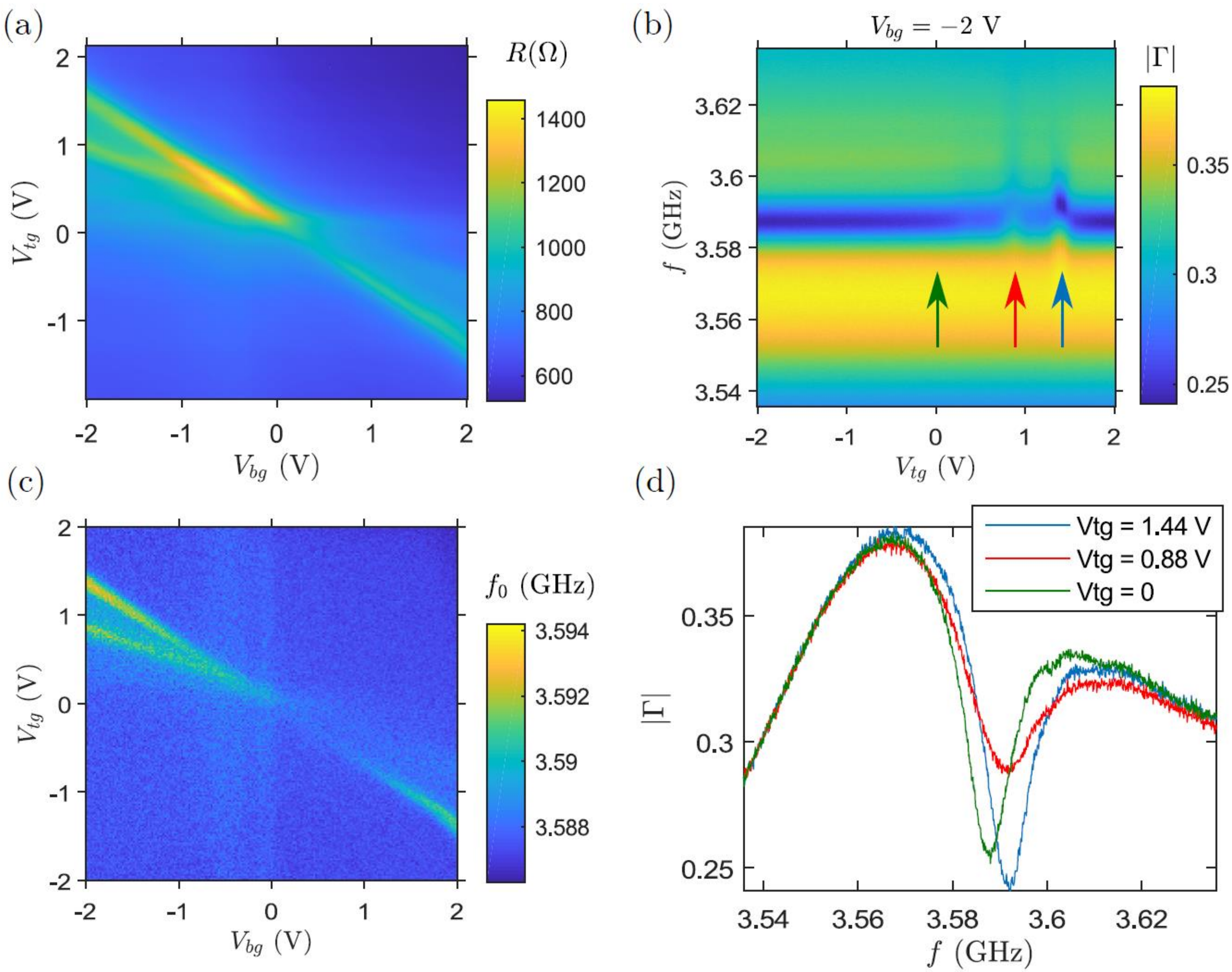
Capacitance measurements





- Couple it to SC resonator to measure DOS
- Quantum capacitance shifts the resonance frequency of the resonator

Capacitance measurements



- See E-field induced gap

DOTS?

$$\alpha_G = \frac{\Delta\mu}{|e|\Delta V_G} = \frac{C_G}{C_\Sigma}.$$

$$\alpha_G = \frac{C_G}{C_\Sigma} = \frac{\beta^+ |\beta^-|}{\beta^+ + |\beta^-|},$$

$$\beta^- = -\frac{C_G}{C_S} \quad \text{and} \quad \beta^+ = \frac{C_G}{C_\Sigma - C_S},$$