Quantum Noise of a Carbon Nanotube Quantum Dot in the Kondo Regime

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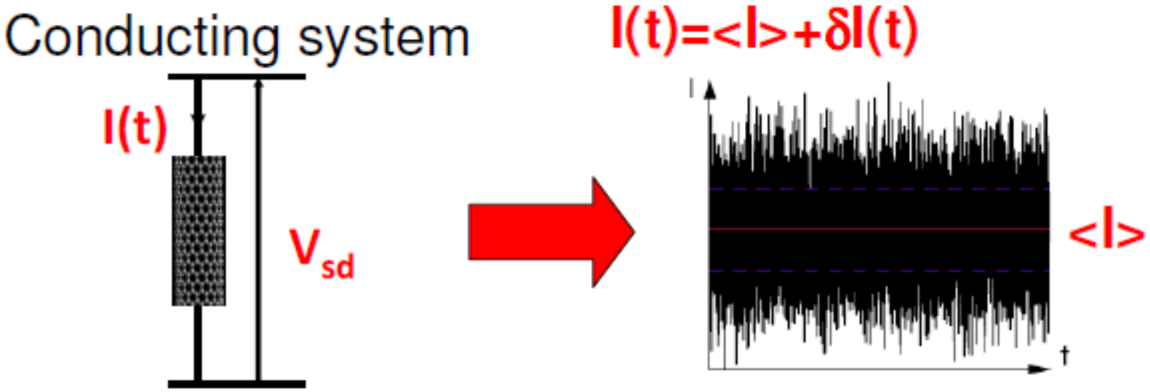
Motivation, introduction to noise

- What is electronic noise?



$$S_I(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} \langle \delta I(t+\tau)\delta I(t) \rangle$$

- Why measure noise?

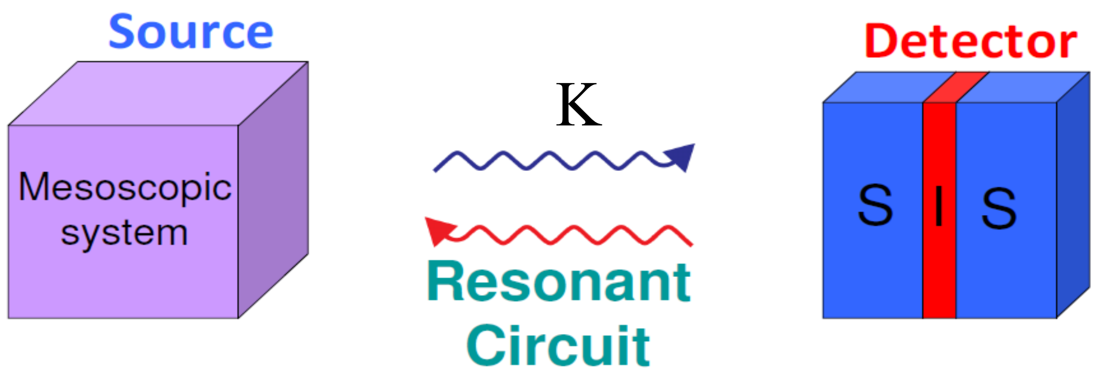
Electronic correlations, effective charge, characteristic energy scale, ...

How to measure (quantum) noise at 70 GHz ???

The experimental setup

$h\nu << k_B T$ Cross correlation technique

$h\nu >> k_B T$ Resonant circuits



Carbon nanotube in the Kondo regime

SIS junction

Noise detection with SIS junction :
Kouwenhoven's group, Science (2003)
P.M. Billangeon *et al.*,PRL (2006)

$$S_{measured}=K\ S(-\nu_0)$$

Quantum noise detection with SIS junction

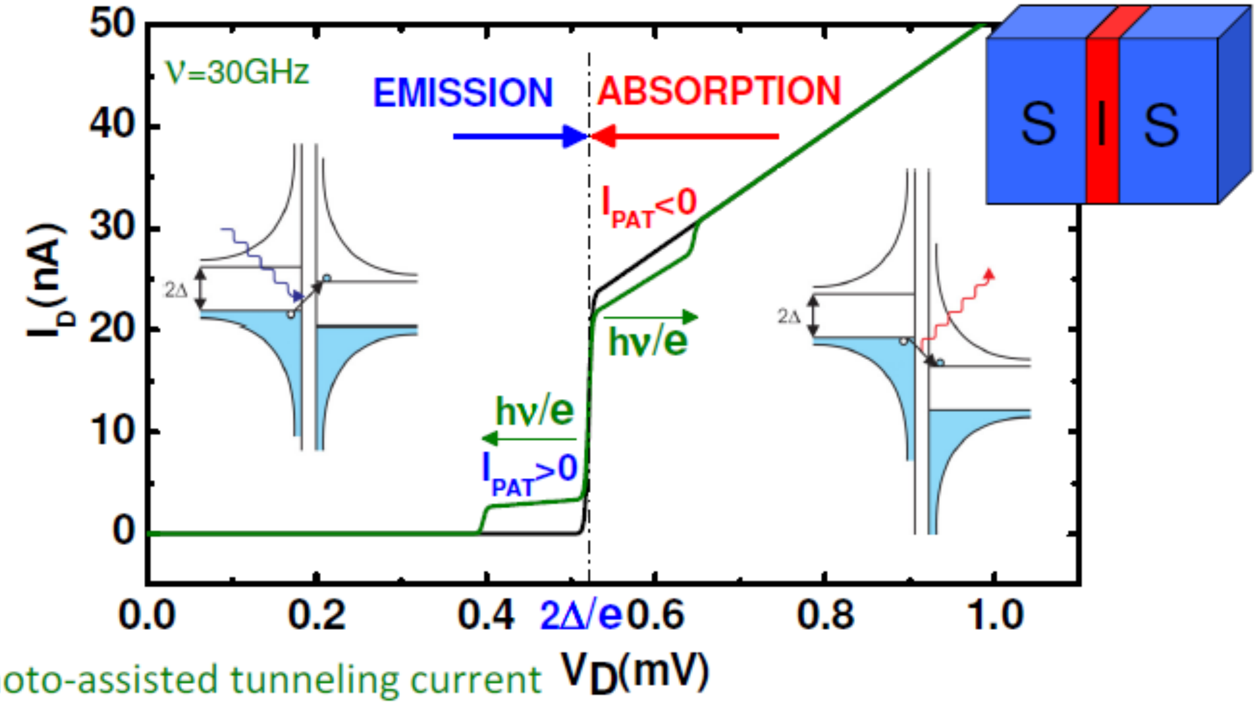


Photo-assisted tunneling current

$$k_B T \ll eV_D$$

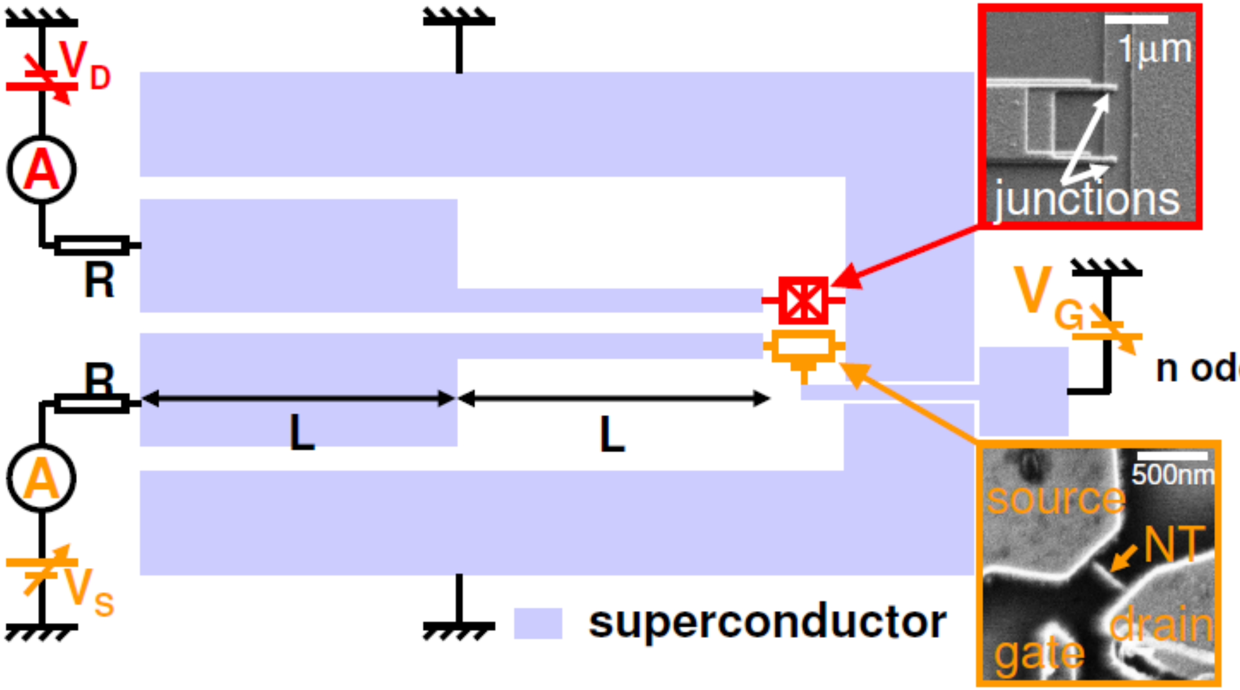
$$I_{\rm PAT}(V_D) = \int_0^{\infty} d\nu \left(\frac{e}{h\nu}\right)^2 S_V(-\nu) I_{\rm QP,0}\left(V_D + \frac{h\nu}{e}\right)$$ ← Related to the emission noise

$$+ \int_0^{eV_D/h} d\nu \left(\frac{e}{h\nu}\right)^2 S_V(\nu) I_{\rm QP,0}\left(V_D - \frac{h\nu}{e}\right)$$ ← Related to the absorption noise

$$- \int_{-\infty}^{+\infty} d\nu \left(\frac{e}{h\nu}\right)^2 S_V(\nu) I_{\rm QP,0}(V_D),$$ ← Renormalizes the elastic current

Ingold and Nazarov (1992)

Resonant coupling between a Carbon nanotube and a SIS detector



$$\nu_n = \frac{nv}{4L}$$

¼ wavelength resonant circuit

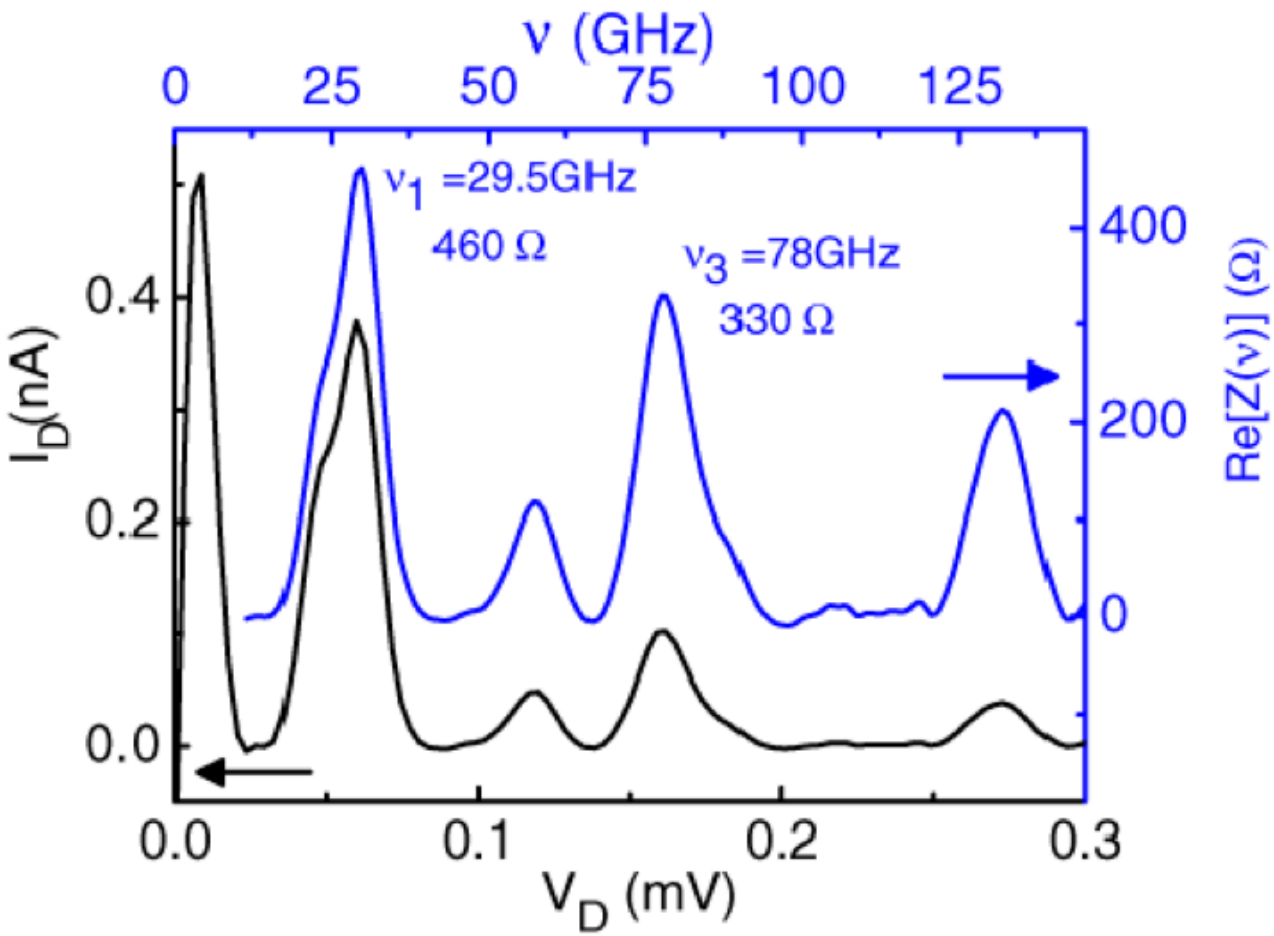
Independent DC polarizations of the source and the detector

Coupling at eigenfrequencies of the resonator (30 GHz and harmonics)

Coupling proportional to the quality factor (J. Basset *et al.* *PRL 2010*)

T = 20 mK – measurement temperature

I(V) for the detector and Resonator impedance

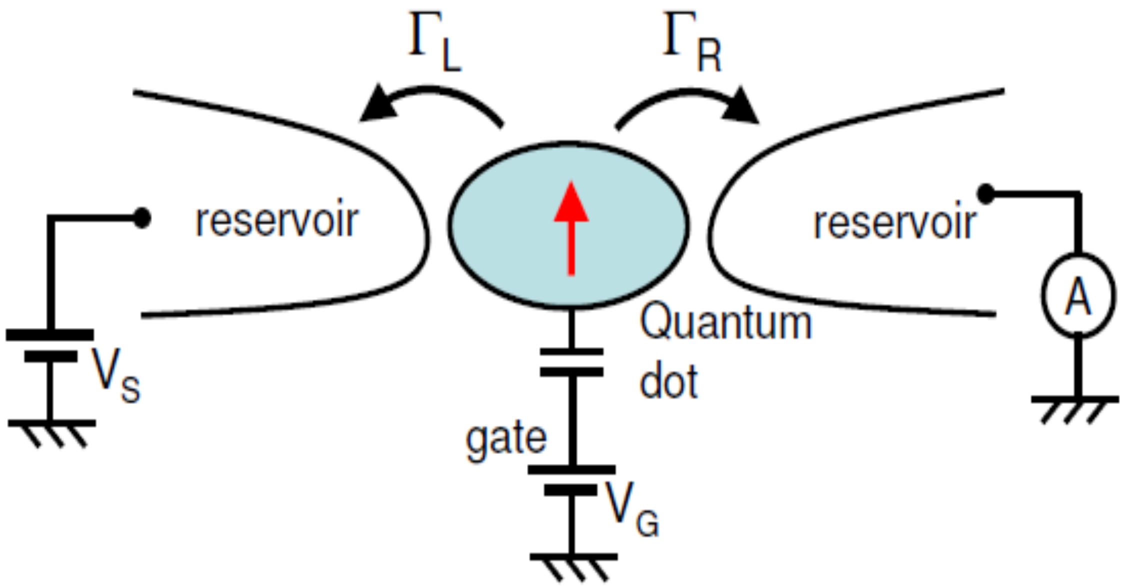


$$I(V) = \mathrm{Re}[Z(2eV/h)]I_C^2/2V$$

$Z(\omega)$ – impedance of the resonator

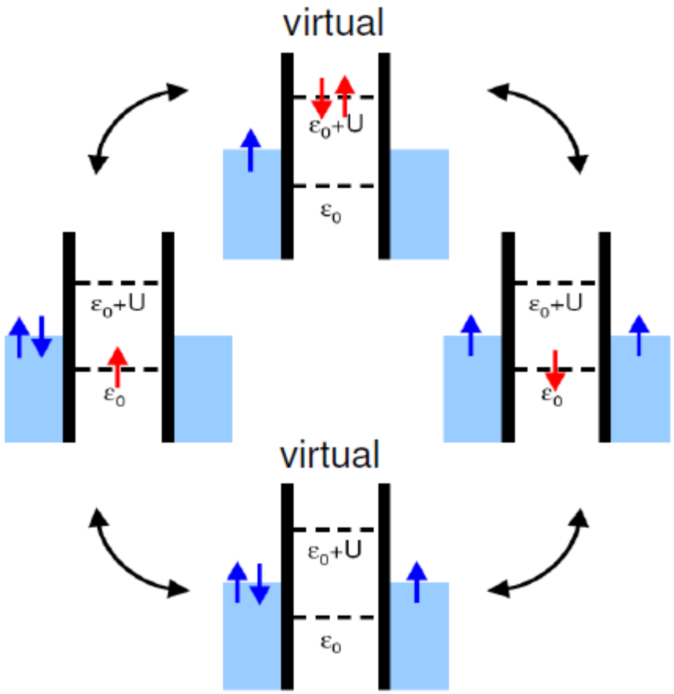
I_C – critical current across the junction

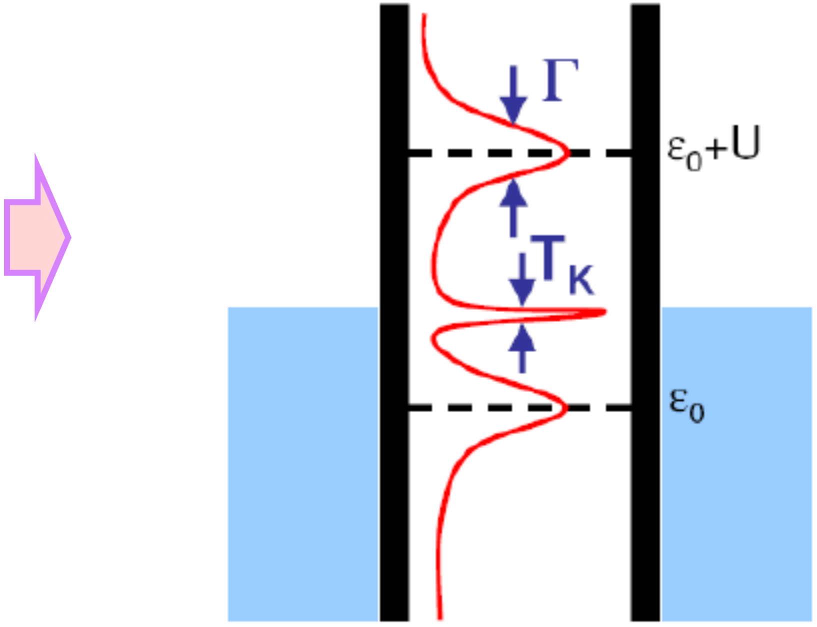
Source: Nanotube Quantum Dot with one electron



Nanotube in Kondo regime

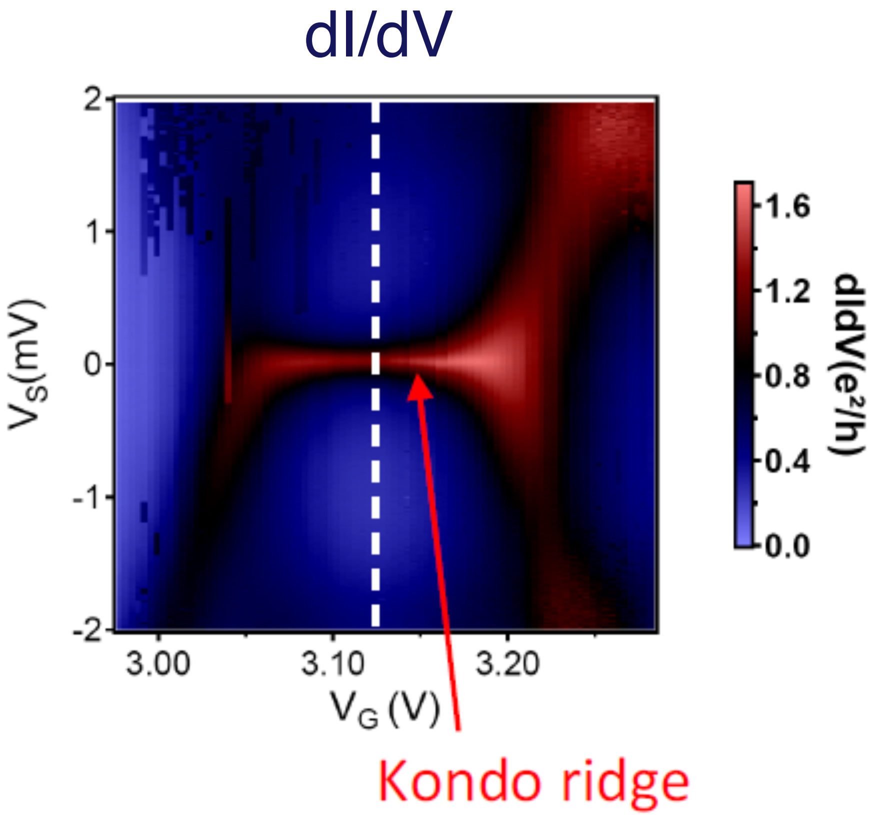
Quantum fluctuations
(odd # of electrons)

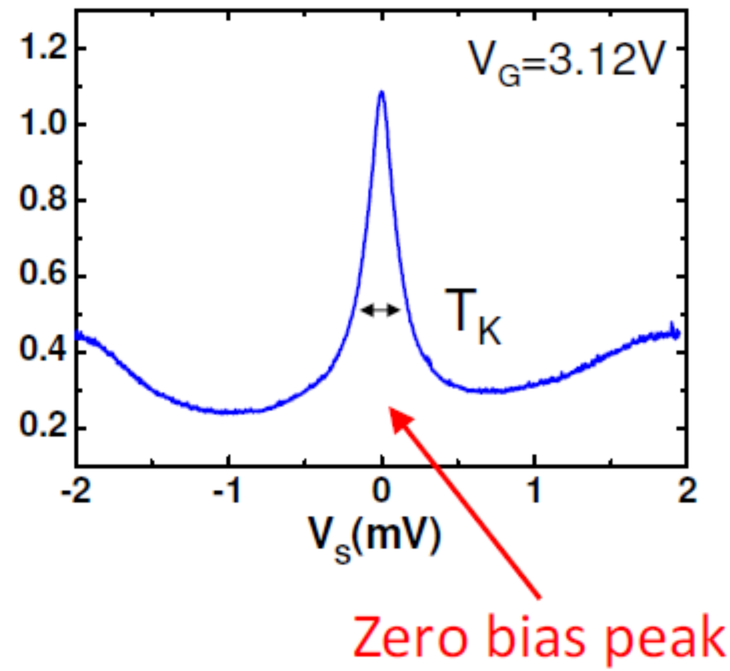




Kondo effect screens the spin

Kondo effect in nanotube



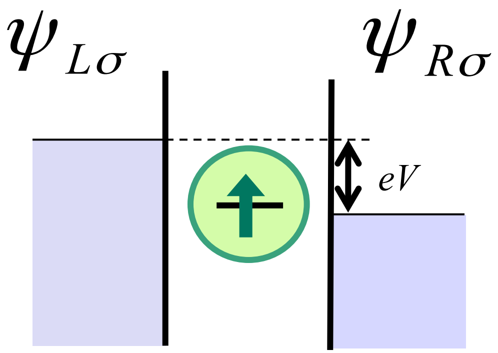


Center of the ridge ➔ T_K=1.4K ⬄ ν=30GHz

Functional RG theory

Simple model

Kondo Hamiltonian :



Dot spin

$$H_{\text{int}} = \frac{1}{2} \sum_{\alpha,\beta=L,R} \sum_{\sigma,\sigma'} j_{\alpha,\beta}\, \mathbf{S}\, \psi^{\dagger}_{\alpha\sigma} \sigma_{\sigma\sigma'} \psi_{\beta\sigma'} \ .$$

$$\alpha \in \{L, R\}$$

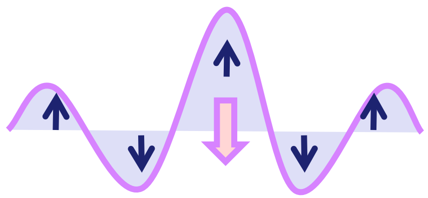
Use pseudofermions (Abrikosov)

$$\hat{S}^i \rightarrow \sum_{s,s'} \tfrac{1}{2}\ f^{\dagger}_s \sigma^i_{s,s'} f_{s'} \qquad\qquad \sum_s f^{\dagger}_s f_s = 1$$

Do perturbative RG for $S(\omega, V)$!?

Difficulties

Kondo effect



(solved originally by Anderson)

Logarithmic singularities with „fine structure”

$$\ln(\omega / D), \qquad \ln(|\omega - eV| / D)$$

Standard RG is not accurate enough

Paaske, Rosch, Kroha, Wölfle: **Current conservation???**

Pertubation theory for the coupling vertex: **Retarded interactions !**

General FRG theory for: $\max(\omega, eV, T, \mu B) \geq T_K$

Real time path integral on the Keldysh contour

Action: $\mathcal{S} = \mathcal{S}_{\text{lead}} + \mathcal{S}_{\text{spin}} + \mathcal{S}_{\text{int}}$

quadratic

Interaction part:

Keldysh label

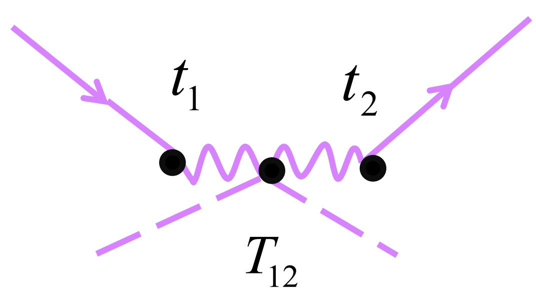
$$\mathcal{S}_{\text{int}} = \sum_{\kappa} \sum_{\alpha\beta} s_{\kappa} \, \frac{1}{4} \int \mathrm{d}t_1 \, \mathrm{d}t_2 \, g_{\alpha\beta}(t_1 - t_2) \;\; \bar{f}^{\kappa}(T_{12}) \vec{\sigma} f^{\kappa}(T_{12}) \cdot \bar{\psi}^{\kappa}_{\alpha}(t_1) \vec{\sigma} \psi^{\kappa}_{\beta}(t_2)$$

$$T_{12} = (t_1 + t_2)/2$$

Keldysh sign: $s_{\kappa} = \pm 1$

Interaction initially local:

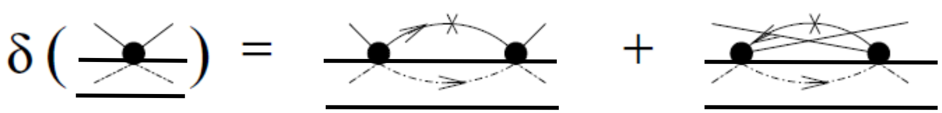
$$g^{(0)}_{\alpha\beta}(t) = j_{\alpha\beta} \; \delta(t)$$



Generating RG equations

1. Expand $\langle U(-\infty,-\infty)\rangle$ in S_{int}

2. Rescale $a \to a'$, $g_{\alpha\beta}(t,a) \to g'_{\alpha\beta}(t,a')$

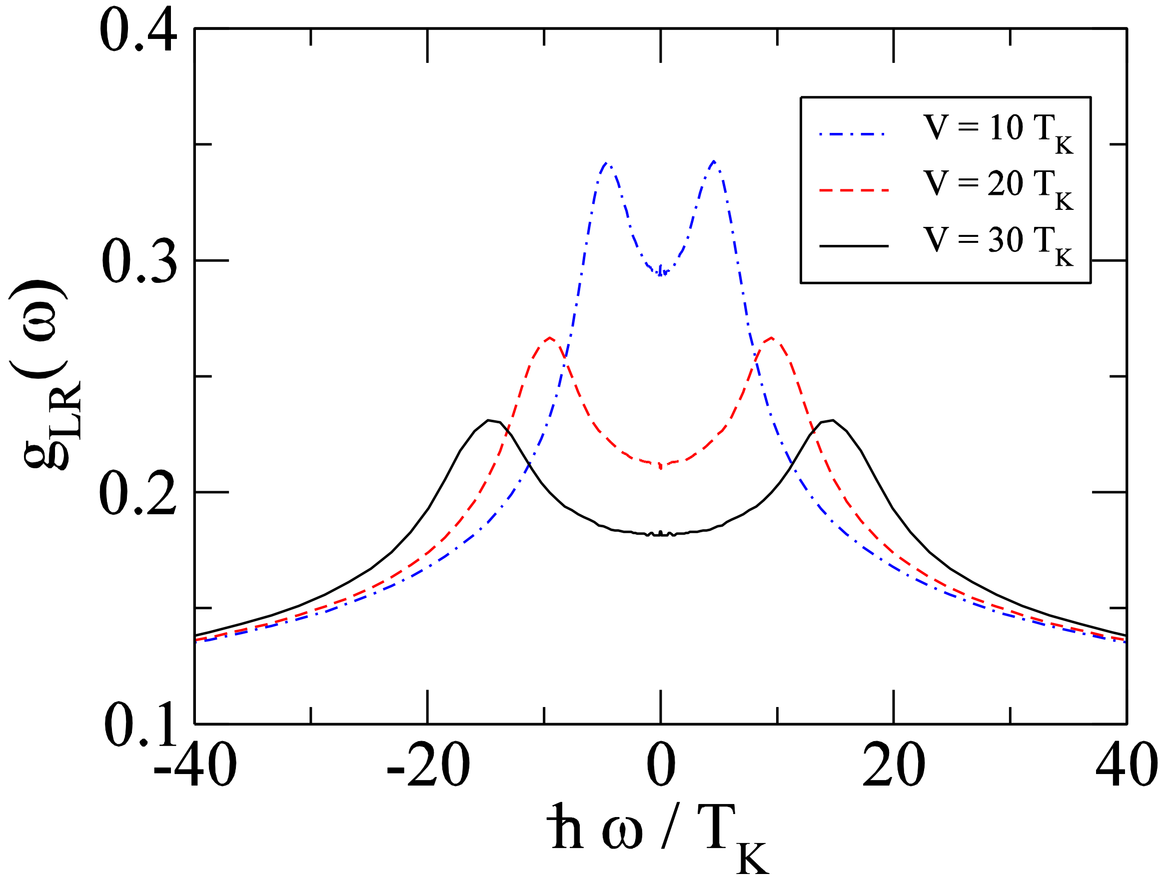


$$\frac{\mathrm{d}\mathbf{g}(\omega)}{\mathrm{d}l} = \mathbf{g}(\omega)\ \mathbf{q}(\omega,a)\ \mathbf{g}(\omega)$$

$l = \ln(a/a_0)$ $g_{\alpha\beta} \to \mathbf{g}$ Cut-off function

„Standard" RG: $g_{\alpha\beta}(t) \to \delta(t)\ \int \mathrm{d}t\ g_{\alpha\beta}(t)$

Renormalized vertex



Current vertex

Current operator from equation of motion:

$$\hat{I}_L(t) = -\hat{I}_R(t) = \sum_{\alpha\beta} \frac{1}{2} v^L_{\alpha\beta} \hat{\mathbf{S}}(t) \cdot \hat{\psi}^\dagger_\alpha(t) \sigma \hat{\psi}_\beta(t)$$

Current conservation

$$\mathbf{v}^L = -\mathbf{v}^R = \begin{pmatrix} 0 & -i\, j_{LR} \\ i\, j_{LR} & 0 \end{pmatrix}$$

Generating functional: $Z[h^\kappa_\alpha(t)] \equiv \langle e^{-i \sum_{\kappa,\alpha} \int \mathrm{d}t\; h^\kappa_\alpha(t) I^\kappa_\alpha(t)} \rangle_S$

RG:

1. Expand $Z[h^\kappa_\alpha(t)]$ in $h^\kappa_\alpha(t)$ and in S_{int}
2. Rescale $a \rightarrow a'$, $g_{\alpha\beta}(t,a) \rightarrow g'_{\alpha\beta}(t,a')$

   and $I^\kappa_\alpha(t)$

Current vertex renormalization

Current necessarily becomes non-local

time of the measurement!

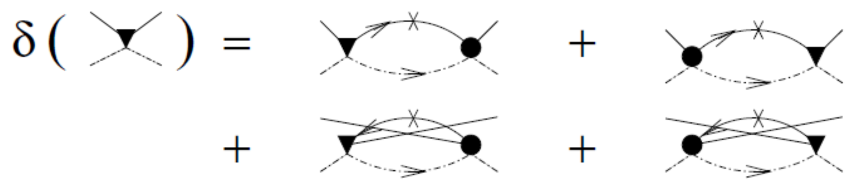
$$I_L^\kappa(t) = \frac{e^2}{4}\sum_\kappa \sum_{\alpha\beta} \int \mathrm{d}t_1 \mathrm{d}\, t_2\, V_{\alpha\beta}(t_1 - t, t - t_2, a)$$
$$\bar{f}^\kappa(t)\vec{\sigma} f^\kappa(t) \cdot \bar{\psi}_\alpha^\kappa(t_1)\vec{\sigma}\psi_\beta^\kappa(t_2)\ ,$$

Reason:

It is not enought to know the times the electrons enters or leave the dot but also the time of the current measurement must be track of

RG equations

Current vertex scaling:



$$\frac{\mathrm{d}\mathbf{V}^L(\omega_1,\omega_2)}{\mathrm{d}l} = \mathbf{V}^L(\omega_1,\omega_2)\,\mathbf{q}(\omega_2,a)\,\mathbf{g}(\omega_2) + \mathbf{g}(\omega_1)\,\mathbf{q}(\omega_1,a)\,\mathbf{V}^L(\omega_1,\omega_2)$$

Initial condition: $\mathbf{V}^{L/R}(\tau_1,\tau_2,a_0) = \delta(\tau_1)\,\delta(\tau_2)\,\mathbf{v}^{L/R}$

Current conservation:

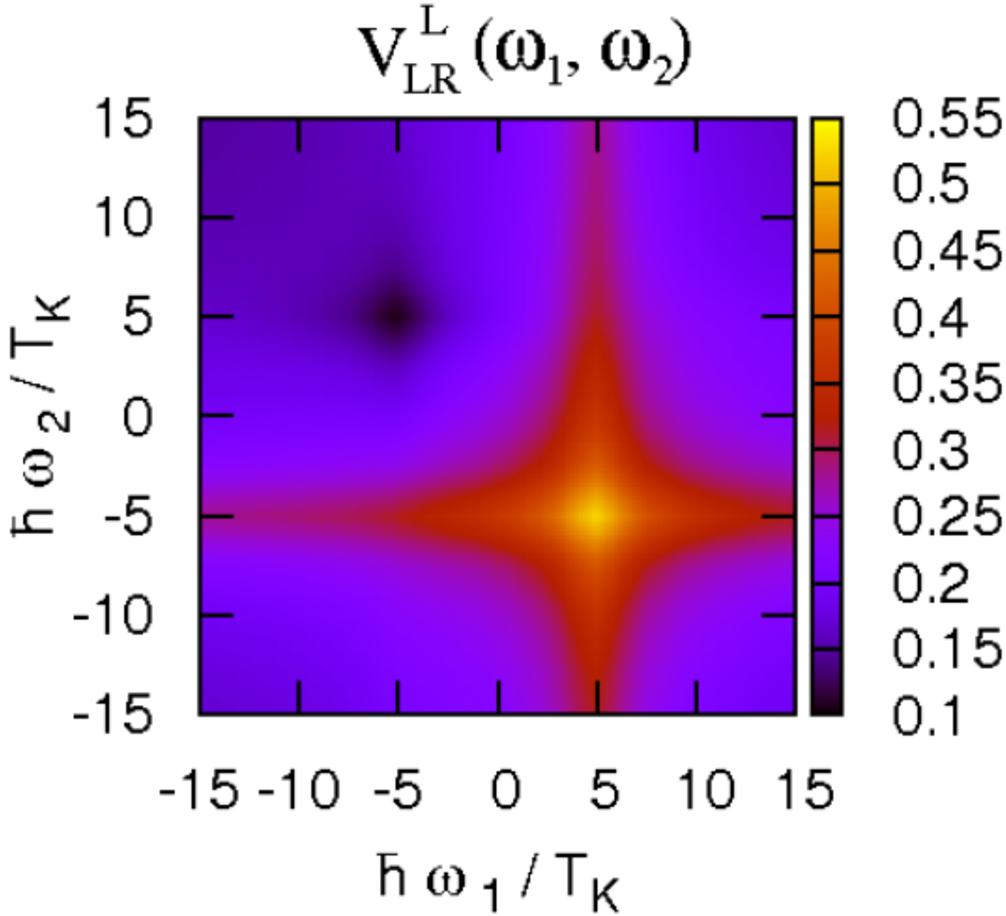
$$\mathbf{v}^L = -\mathbf{v}^R \quad \Rightarrow \quad V^L_{\alpha\beta}(\omega_1,\omega_2) = -V^R_{\alpha\beta}(\omega_1,\omega_2)$$

$$I^\kappa_L(t) + I^\kappa_R(t) \equiv 0$$

Automatically satisfied at the functional level !

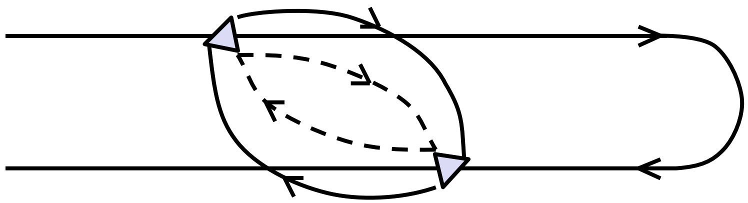
Renormalized current vertex

$eV/T_K = 10$



Computing the noise

Functional RG noise formula:



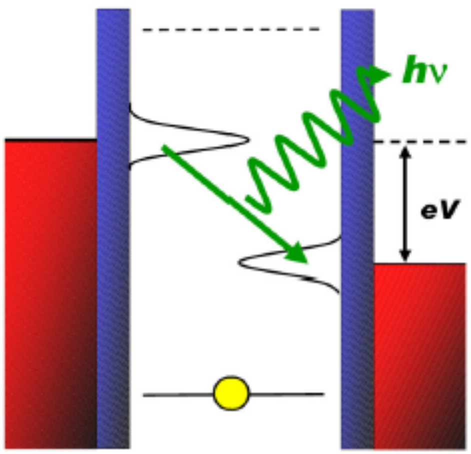
$$S^{>}_{LL}(\omega) = \frac{e^2}{2} S(S+1) \int \frac{d\tilde{\omega}}{2\pi} \mathrm{Tr}\{\mathbf{V}^L(\tilde{\omega}_-, \tilde{\omega}_+) \mathbf{G}^{>}(\tilde{\omega}_+) \mathbf{V}^L(\tilde{\omega}_+, \tilde{\omega}_-) \mathbf{G}^{<}(\tilde{\omega}_-)\} .$$

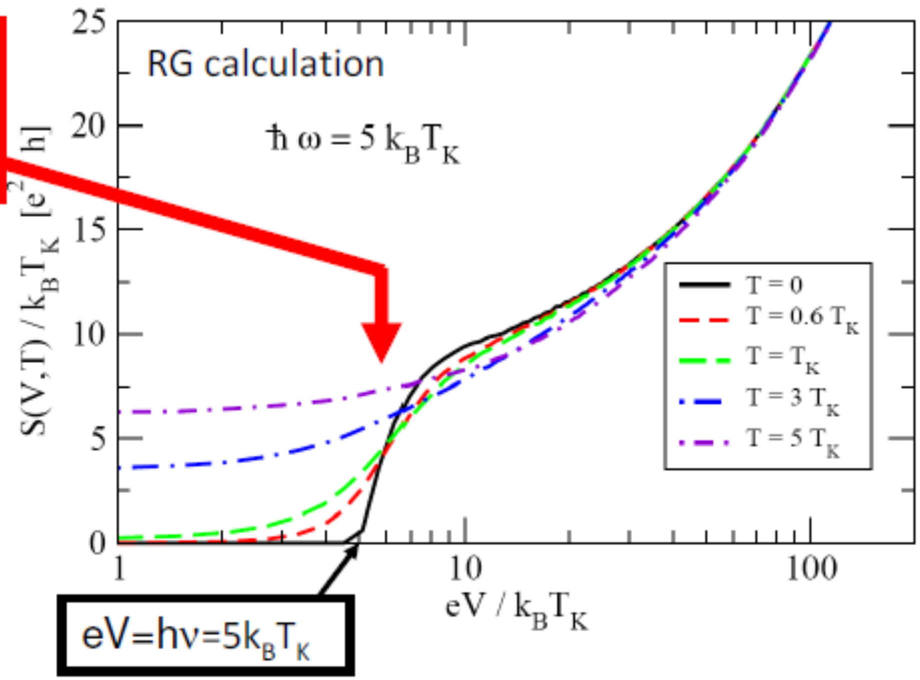
$$\tilde{\omega}_{\pm} = \tilde{\omega} \pm \frac{\omega}{2} \qquad G^{>/<}_{\alpha\beta}(\omega) = \pm i\ 2\pi\ \delta_{\alpha\beta}\ f(\pm(\omega - \mu_\alpha))$$

Theoretical Prediction

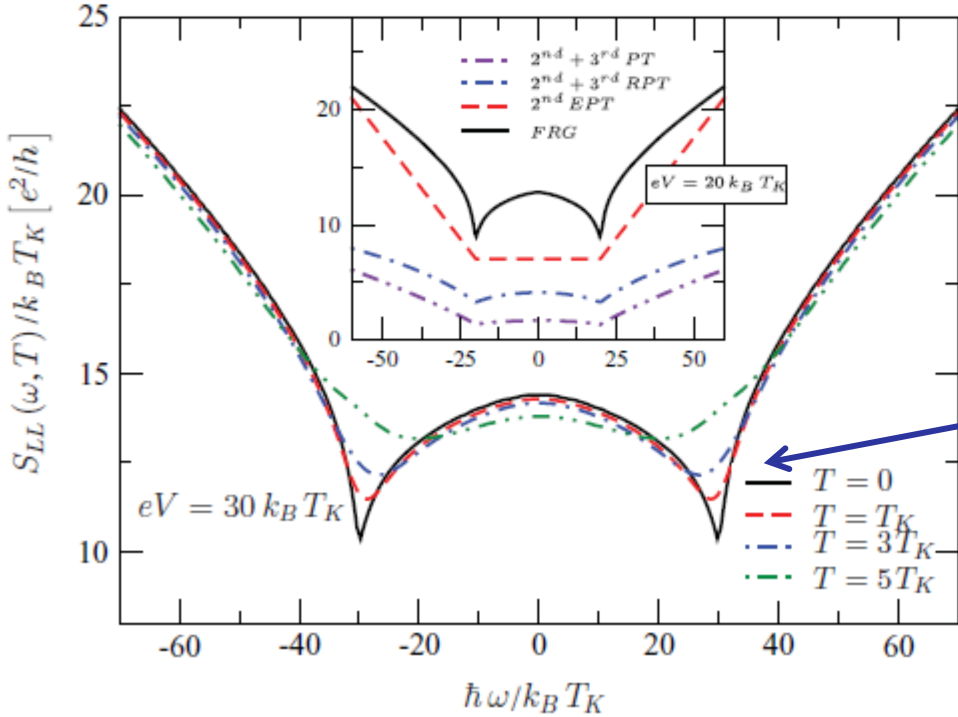
Signature of the Kondo effect on noise :

Logarithmic singularity at V=hν/e





Symmetrized noise spectra



Non-equilibrium Kondo effect

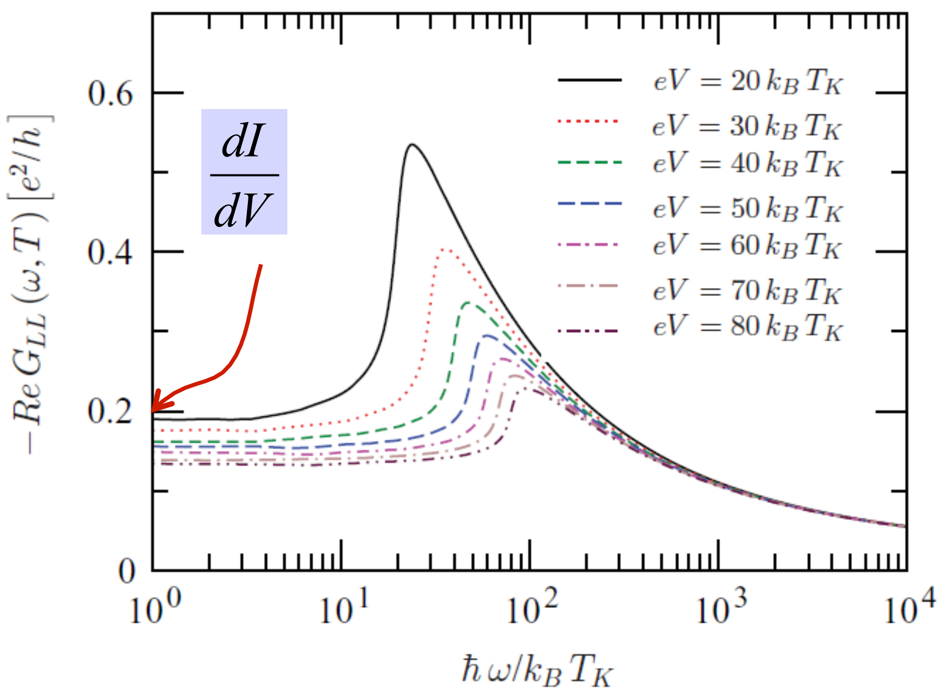
Non-equilibrium ac linear conductivity

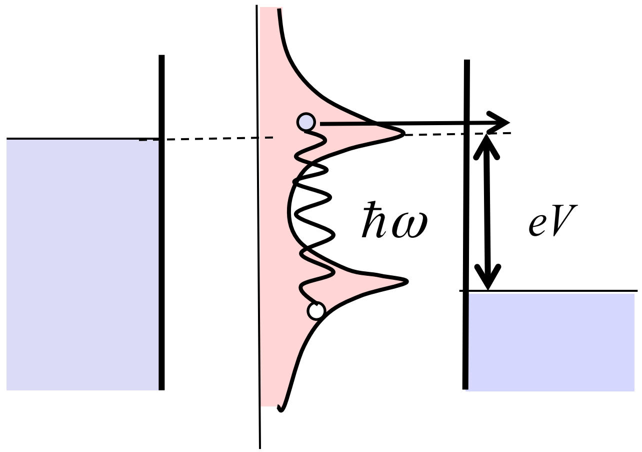
$$V_{SD} = V + \delta V_{\omega} \cos(\omega t) \quad \Rightarrow \quad I(t) = G(\omega, V)\, \delta V_{\omega} \cos(\omega t) + \ldots$$

Non-equlibrium ac-conductance

$$\mathrm{Re}\, G_{LL}(\omega, V) = -\frac{1}{\hbar\omega}[S^{>}_{LL}(\omega) - S^{<}_{LL}(\omega)]$$

Safi(2009)





Back to experiment…

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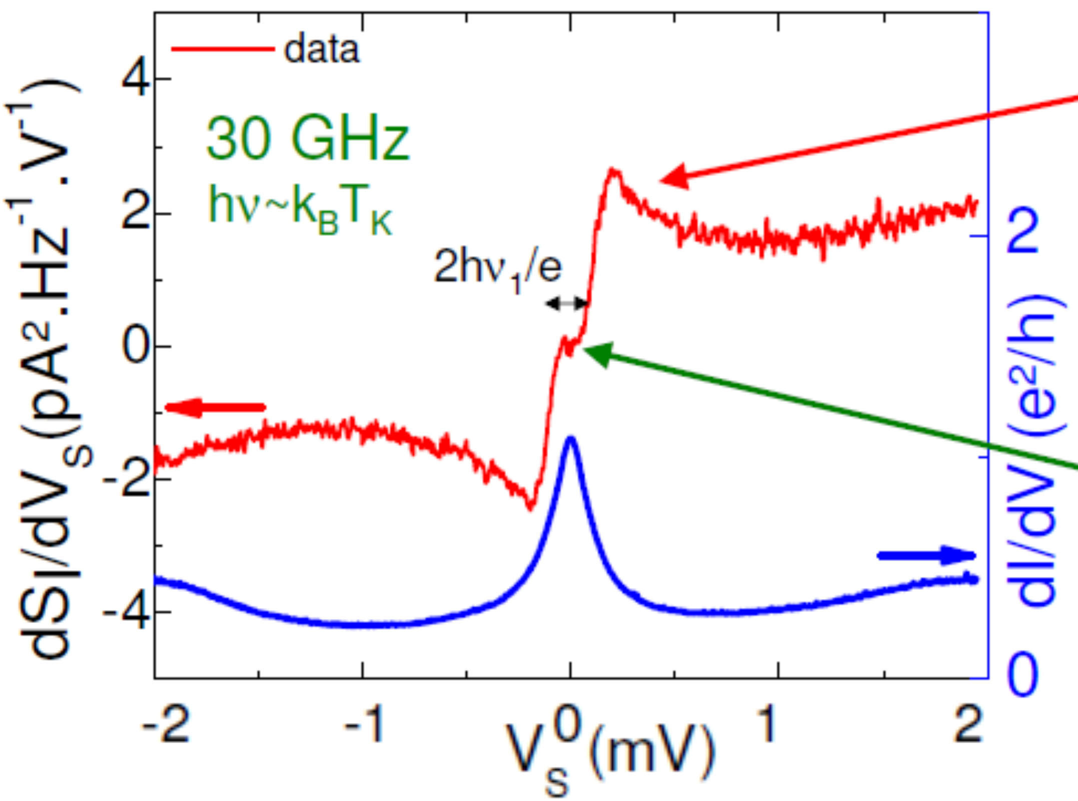
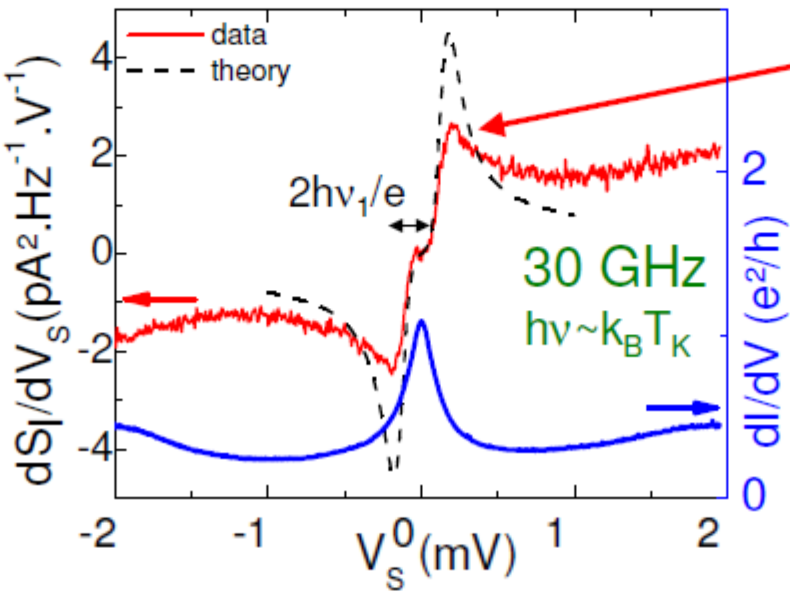


Small singularity related to the Kondo resonance at $h\nu \sim k_B T_K$

No emission noise if $|eV_S| < h\nu$

Back to experiment…



Singularity related to the Kondo resonance at $h\nu \sim k_B T_K$
➔ Qualitatively consistent but not quantitatively

Theory: no free fitting parameter!

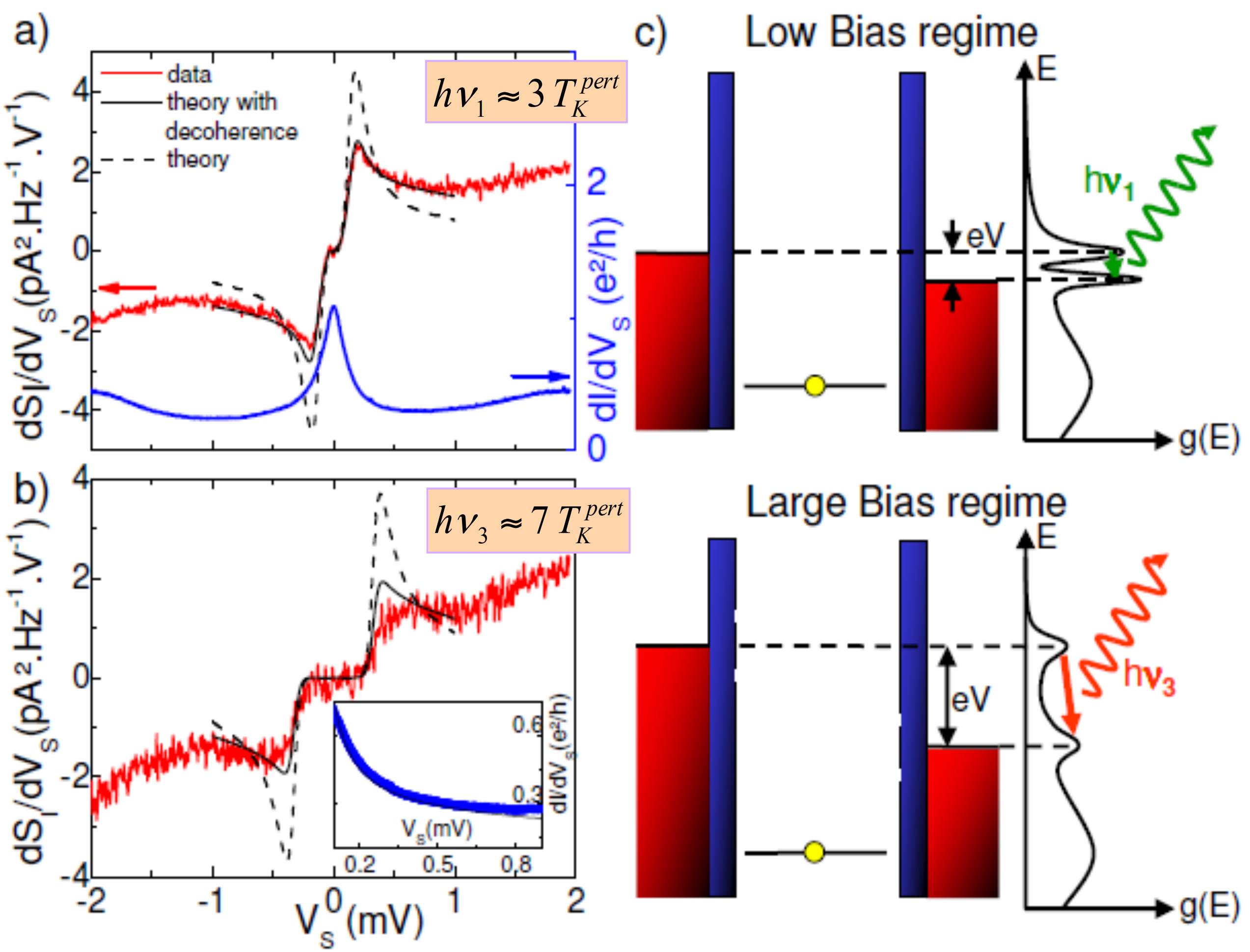
- Kondo temperature TK=1.4K
- asymmetry a=0.67

⇨ **Additional decoherence ???**

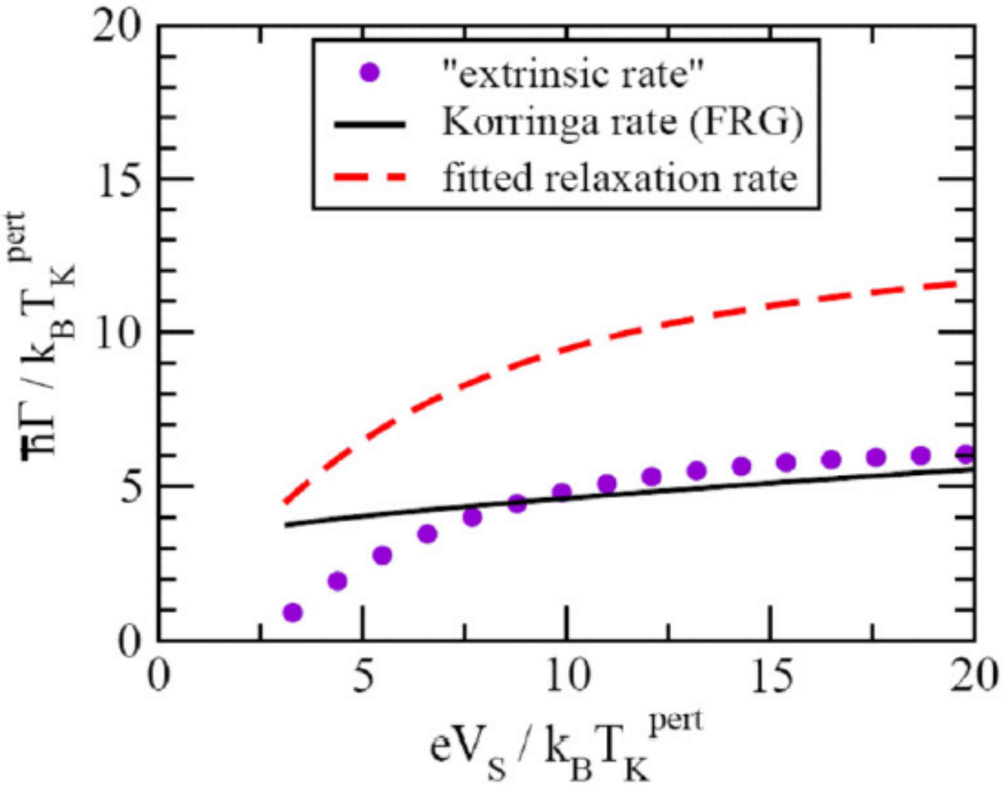
(Monreal *et al. PRB 05;* Van Roermund *et al. PRB 10;* De Franceschi *et al. PRL 02*)

Fit of the noise spectra: voltage dependent relaxation

Single choice of $\Gamma_{spin}(V)$ must fit everything…



Fitted the coherence rate…



Approximate form: $h/\tau_S \approx \alpha\, k_B T_K^{\mathrm{RG}} \mathrm{atan}(\frac{\beta e V_S}{k_B T_K^{\mathrm{RG}}})$

with α=14, β=0.15

Conclusions

Theory:

- Real time functional RG formalism

  ⇨ Current-conserving scheme
- $S(\omega,V)$ shows strong anomalies at $\omega = \pm eV$
- $G(\omega,V)$ shows split Kondo resonance

Experiment:

- Non-equilibrium quantum noise measurment
- Logarithmic anomalies observed
- Additional spin relaxation must be assumed to agree with theory

J. Basset et al, PRL **108**, 046802 (2012)