On-chip superconducting resonators

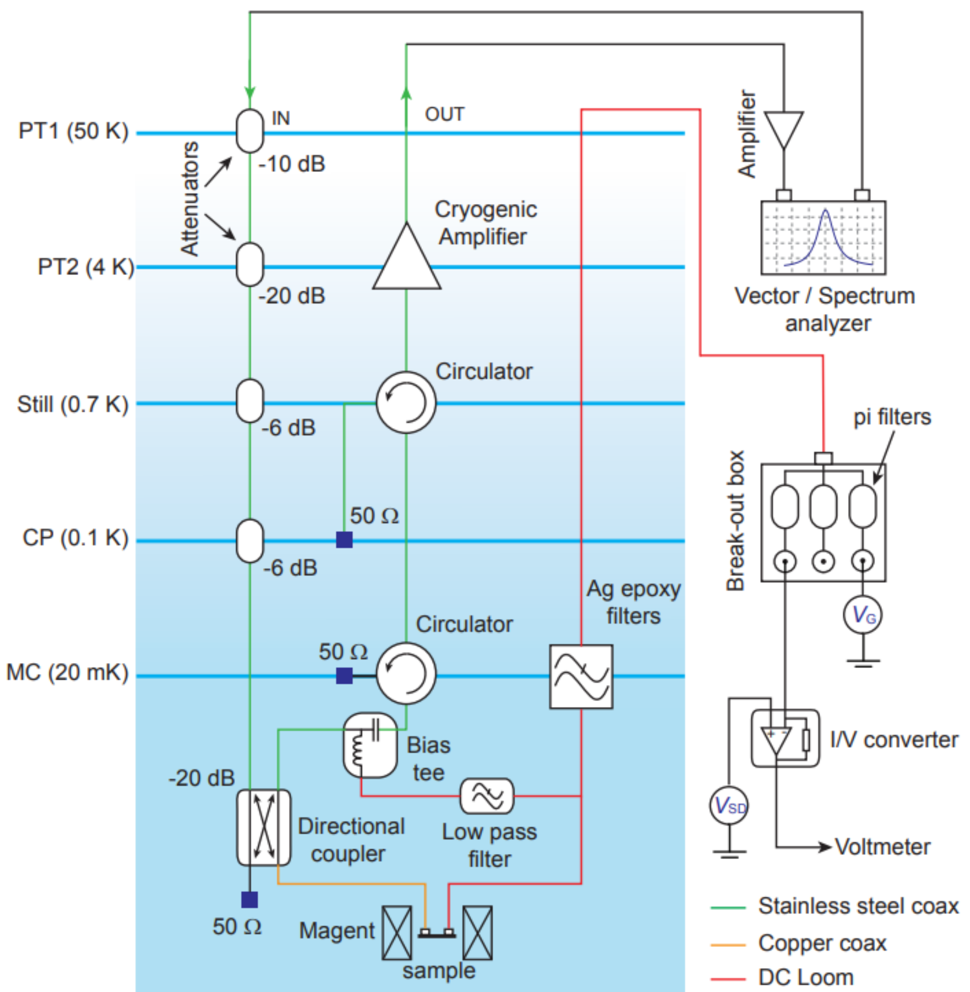
Gergő Fülöp
RF seminar
2019-03-11

From the previous episode …

- Coaxial cables
- Attenuators
- Directional coupler
- Bias tee
- Isolator/circulator
- Filters
- Amplifiers
- DC block

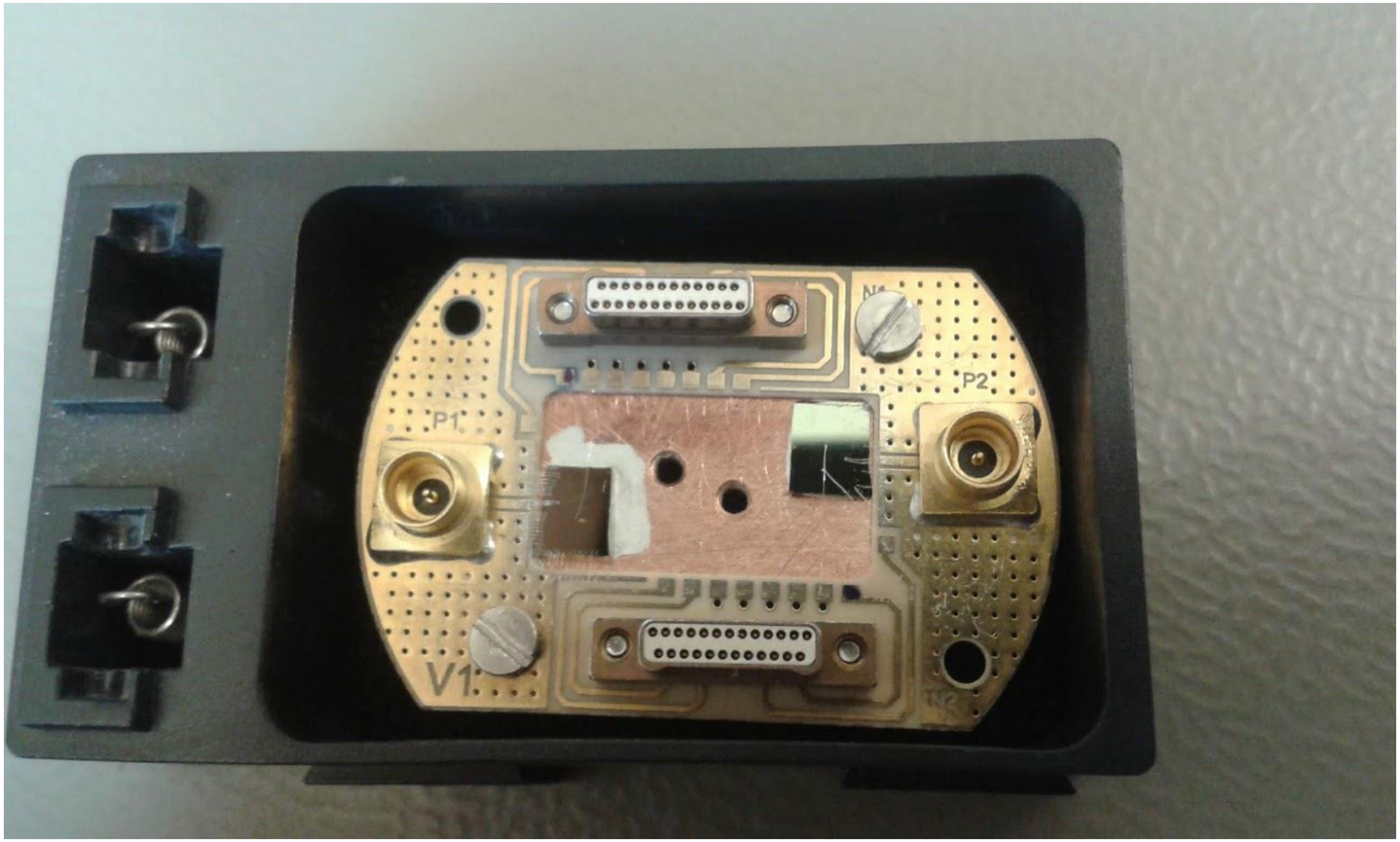
- Mixer
- (Power detector)
- Multiplexer
  - Diplexer

Oxford Triton dilfridge



V. Ranjan, PhD thesis

What is on the chip?

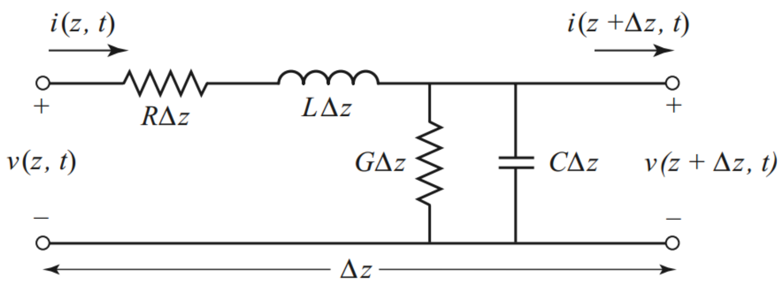


Overview

- Distributed resonators
  - Transmission lines
    - CPWs
    - striplines
  - CPW resonators
    - Quarter-wave resonator
    - Half-wave resonator
    - Stub tuner
  - Loss
  - Ground plane
- Lumped element resonators
- Coupling to devices
- Kinetic inductance
  - Stripline resonators
  - High-impedance resonators
- Remarks

Transmission lines

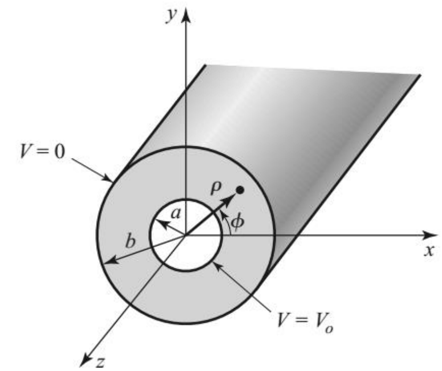
- Distributed model



Propagation constant:

$$\gamma = \alpha + j\beta$$

$$\frac{V_o^+}{I_o^+} = Z_0$$

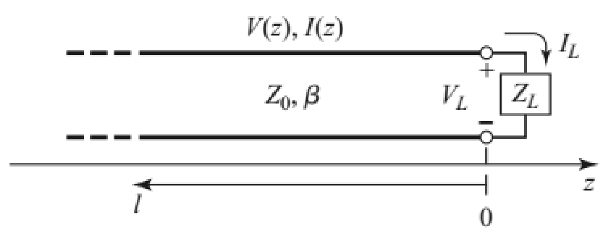


$$Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

- Lossless TL

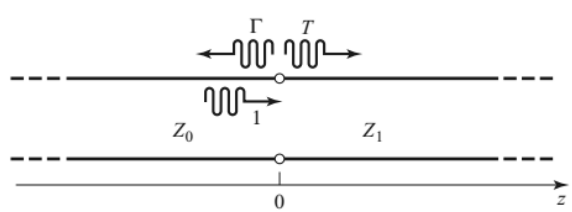
$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}, \qquad Z_0 = \sqrt{\frac{L}{C}},$$

- Reflection
  - From the end of the TL



$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

  - Interfaces

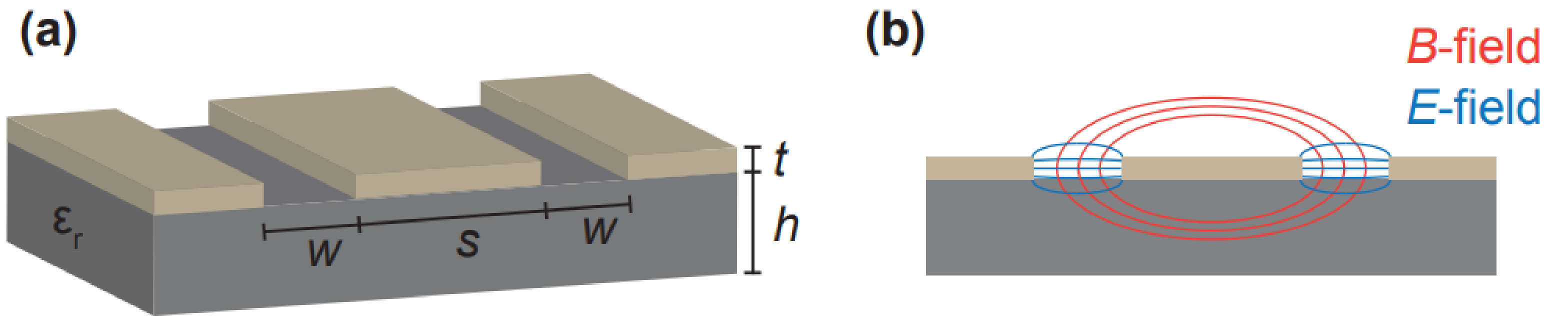


$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

D. Pozar

Transmission lines

- Coplanar waveguide



- Characteristic impedance

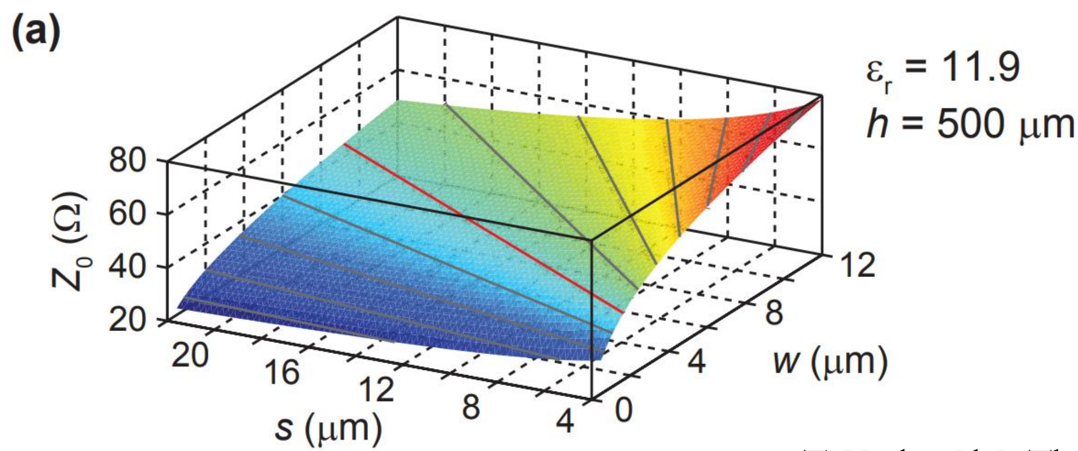
$$\epsilon_{\text{eff}} = 1 + \frac{\epsilon_r - 1}{2} \cdot \frac{K(k_1)K(k_0')}{K(k_1')K(k_0)}$$

$$k_0 = \frac{s}{s+2w}$$

$$k_0' = \sqrt{1-k_0^2}$$

$$k_1 = \frac{\sinh(\pi s/4h)}{\sinh(\pi(s+2w)/4h)}$$

$$k_1' = \sqrt{1-k_1^2}.$$

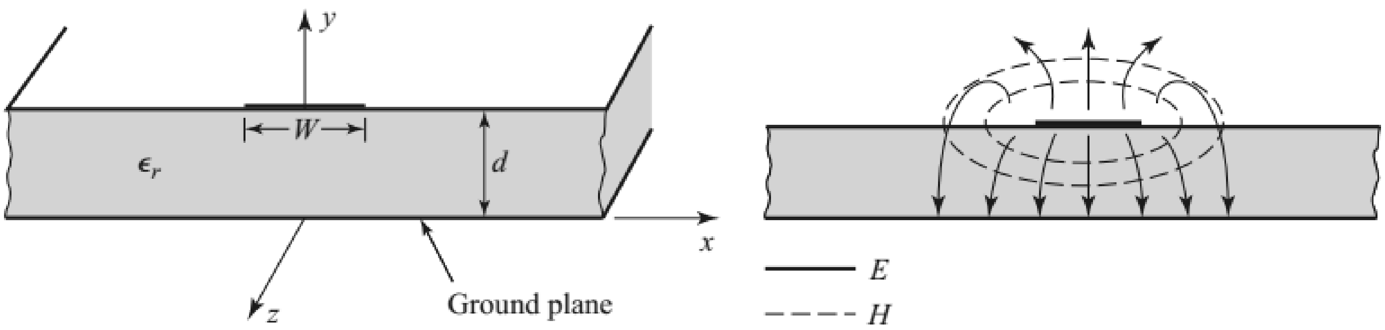


T. Hasler, PhD Thesis

Transmission lines

- Stripline / microstrip

$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12d/W}}$$



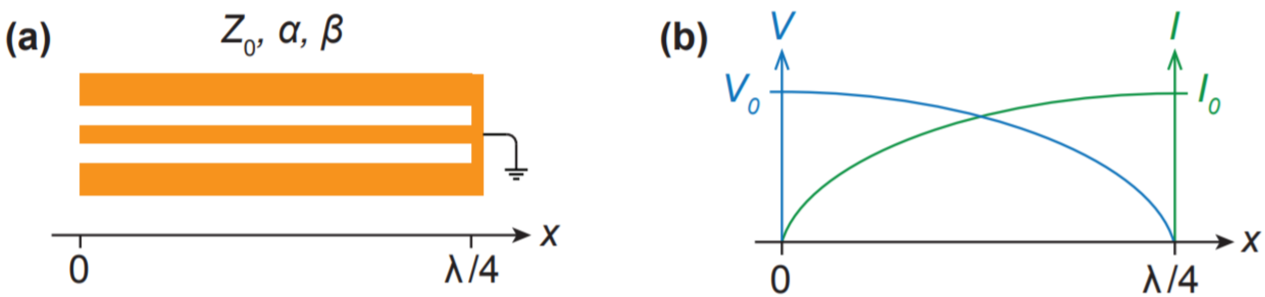
(a) (b)

- Characteristic impedance

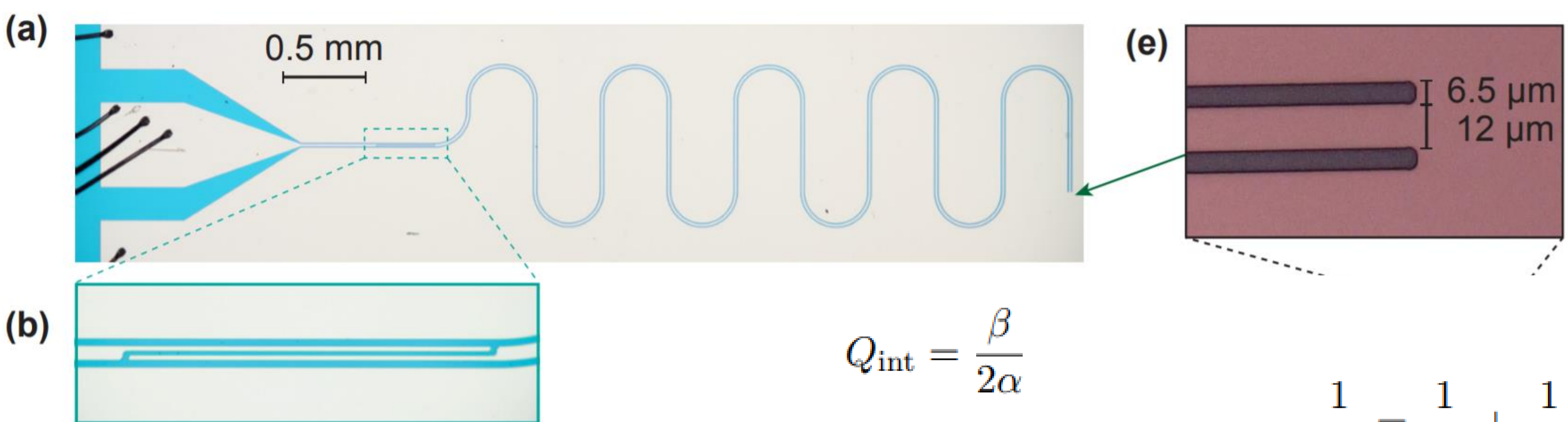
$$Z_0 = \begin{cases} \dfrac{60}{\sqrt{\epsilon_e}} \ln\left(\dfrac{8d}{W} + \dfrac{W}{4d}\right) & \text{for } W/d \leq 1 \\ \dfrac{120\pi}{\sqrt{\epsilon_e}\,[W/d + 1.393 + 0.667 \ln(W/d + 1.444)]} & \text{for } W/d \geq 1. \end{cases}$$

Quarter-wave CPW resonator

- Model
  - CPW with open end + shorted end



- Realization $l \approx \lambda/4$



$$Q = \omega \cdot \frac{\text{energy stored in resonator}}{\text{power dissipated}}$$

$$Q_{\text{int}} = \frac{\beta}{2\alpha}$$

$$Q_{\text{c}} = \frac{1}{8\pi \left(Z_0 f C_{\text{c}}\right)^2}.$$

$$\frac{1}{Q_{\text{l}}} = \frac{1}{Q_{\text{i}}} + \frac{1}{Q_{\text{c}}}$$

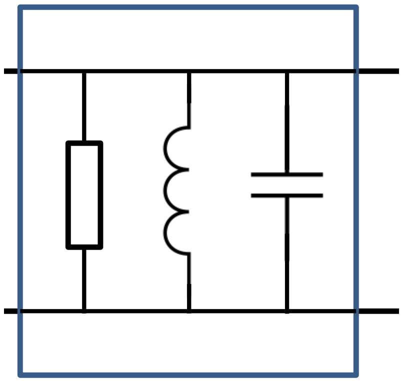
T. Hasler, PhD Thesis

Quality factor

- Definition

$$Q = \omega \frac{\text{average energy stored}}{\text{energy loss/second}}$$

- Internal, intrinsic, external, loaded, total […] Q



$$Q_0 = \omega_0 RC = \frac{R}{\omega_0 L}$$

$$Q_e = \frac{R_L}{\omega_0 L}$$

$$\frac{1}{Q_l} = \frac{1}{Q_0} + \frac{1}{Q_e}$$

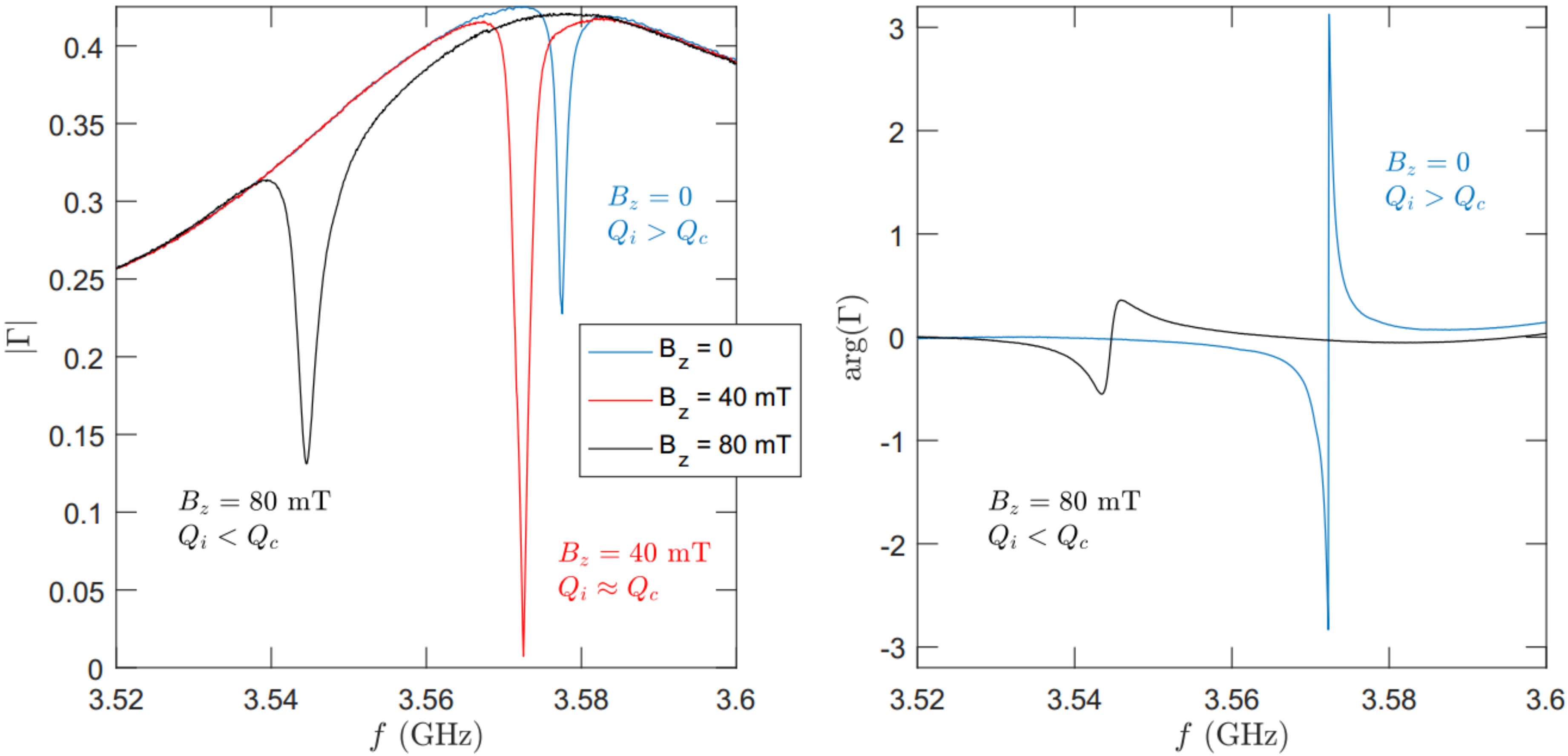
$$\kappa_L = \kappa_i + \kappa_c$$

- Determination

$$Q_l = \frac{f_0}{\text{FWHM}}$$

Quarter-wave CPW resonator

- 14 nm NbTiN film, out-of-plane B field

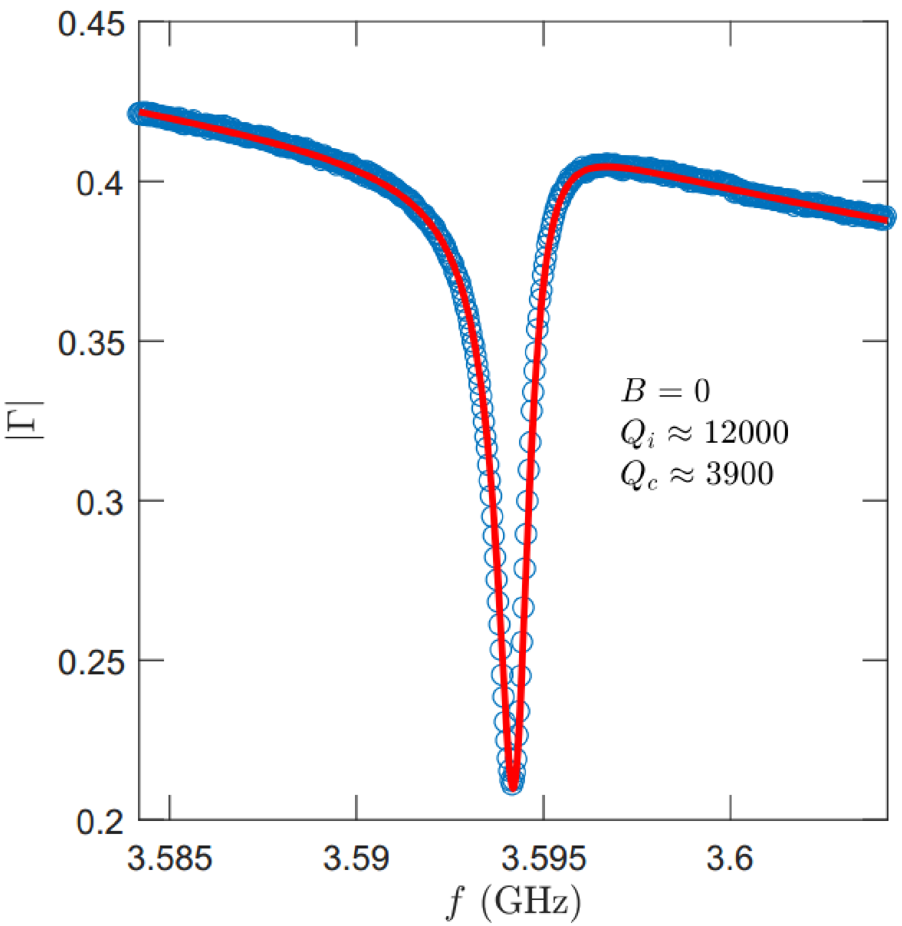


(a) B_z-dependence, magnitude traces

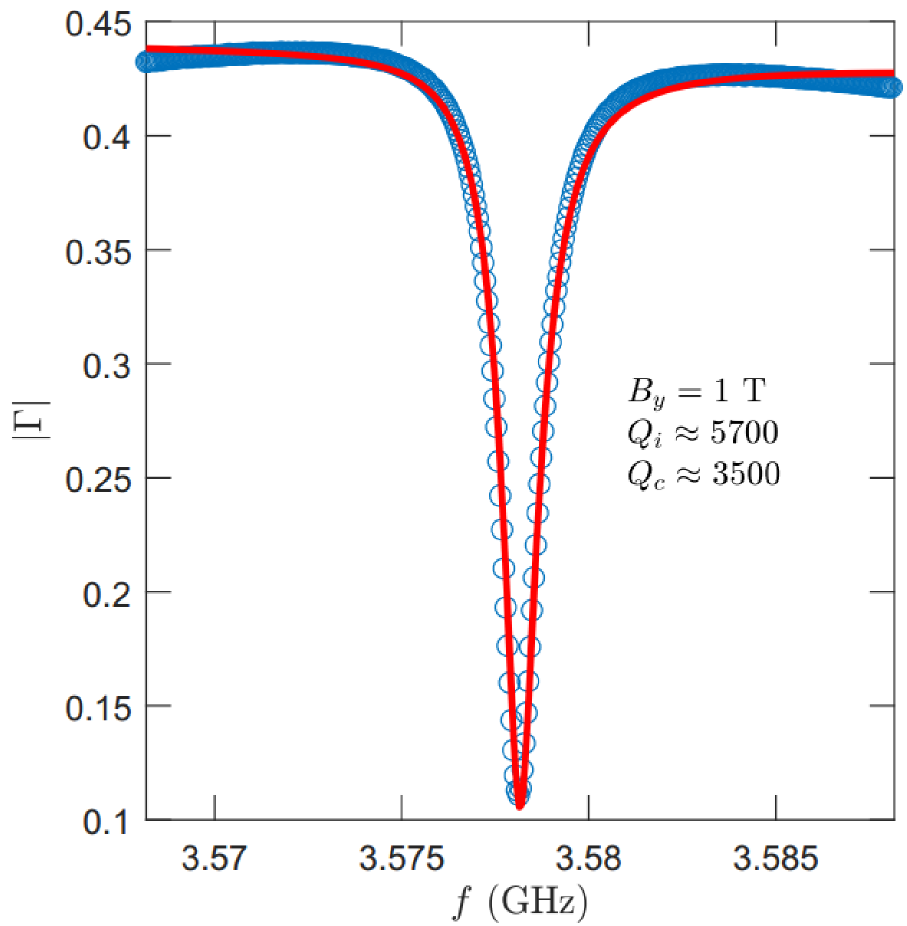
(b) B_z-dependence, phase traces

Quarter-wave CPW resonator

- 14 nm NbTiN film, in-plane B field



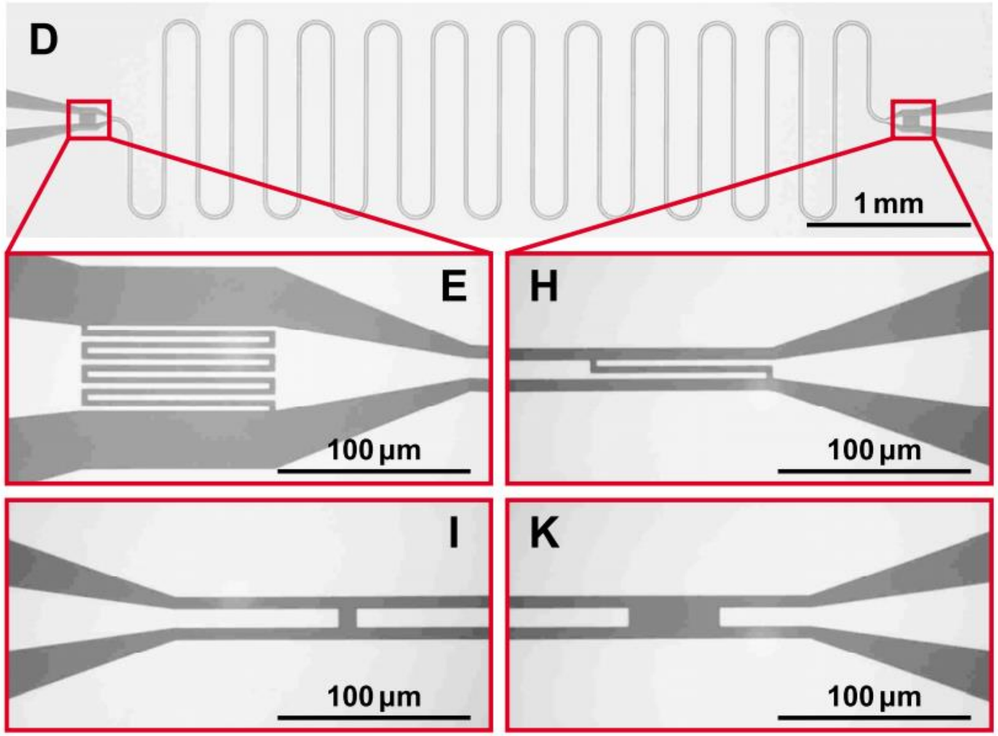
(a) Fitted resonance curve, $B = 0$

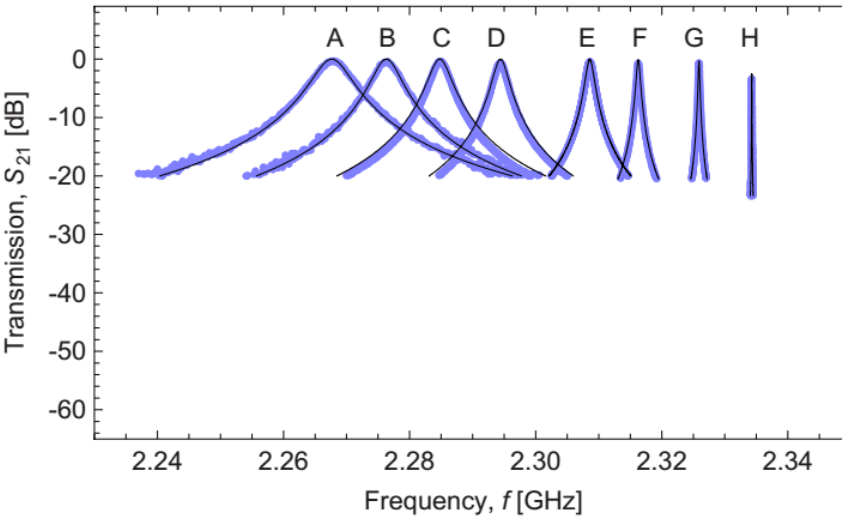


(b) Fitted resonance curve, $B_y = 1$ T

Half-wave CPW resonator

- Capacitor – CPW – Capacitor
  - Fabry-Perot cavity



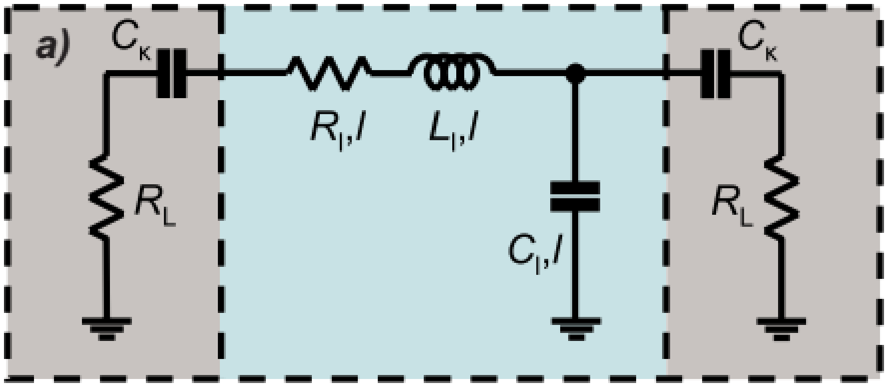


J. Appl. Phys. 104, 113904 (2008);

Half-wave CPW resonator

- Capacitor – CPW – Capacitor

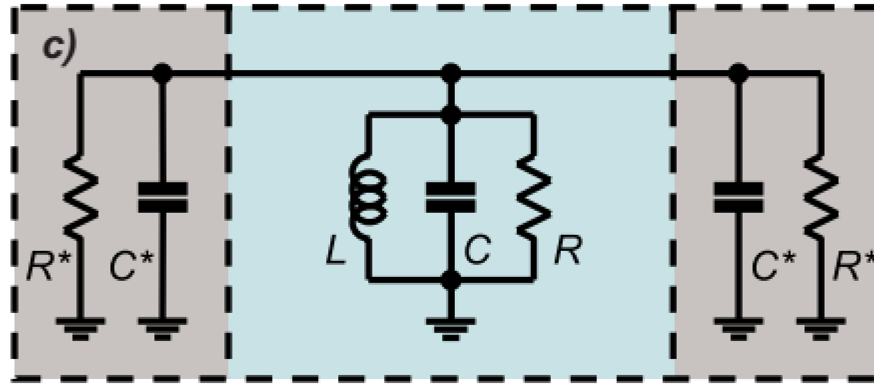
Distributed model



$$Z_{TL} = Z_0 \frac{1 + i \tan \beta l \tanh \alpha l}{\tanh \alpha l + i \tan \beta l},$$

$$\approx \frac{Z_0}{\alpha l + i \frac{\pi}{\omega_0}(\omega - \omega_n)}.$$

Lumped element model



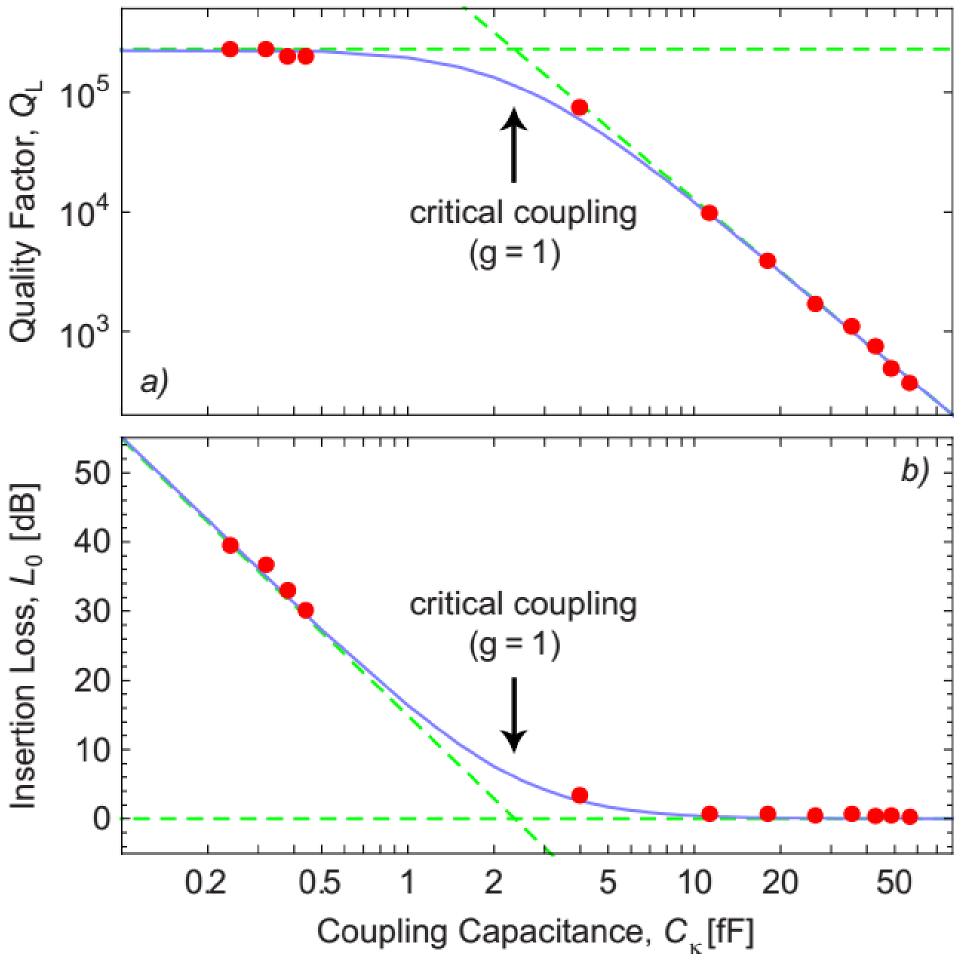
$$Z_{LCR} = \left(\frac{1}{i\omega L_n} + i\omega C + \frac{1}{R} \right)^{-1},$$

$$\approx \frac{R}{1 + 2iRC(\omega - \omega_n)},$$

J. Appl. Phys. 104, 113904 (2008);

Half-wave CPW resonator

- Capacitor – CPW – Capacitor



$$Q_{\text{int}} = \omega_n RC = \frac{n\pi}{2\alpha l},$$

$$Q_{\text{ext}} = \frac{\omega_n R^* C}{2}.$$

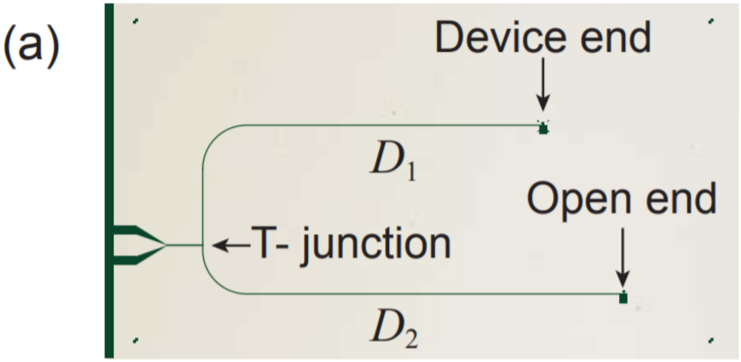
$$L_0 = -20 \log\left(\frac{g}{g+1}\right)\text{dB}$$

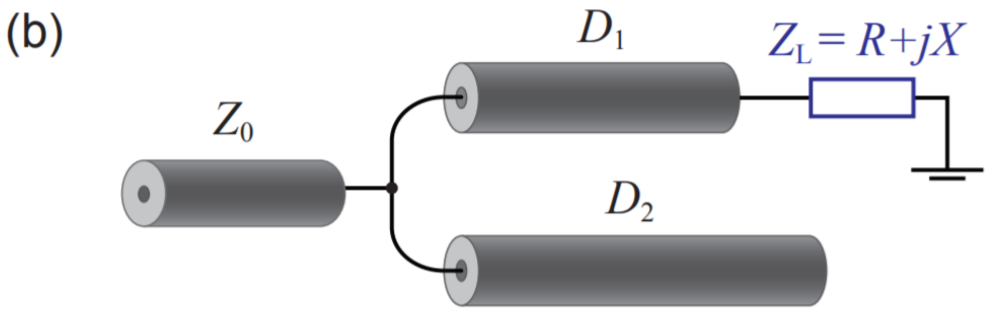
$$g = Q_{\text{int}}/Q_{\text{ext}}$$

J. Appl. Phys. 104, 113904 (2008);

CPW Stub tuner

- Stub tuner

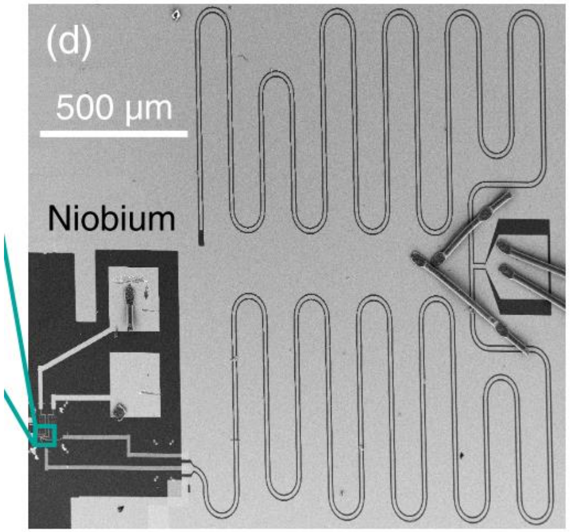




  - Measure admittance (resistance, capacitance)
  - Measure noise

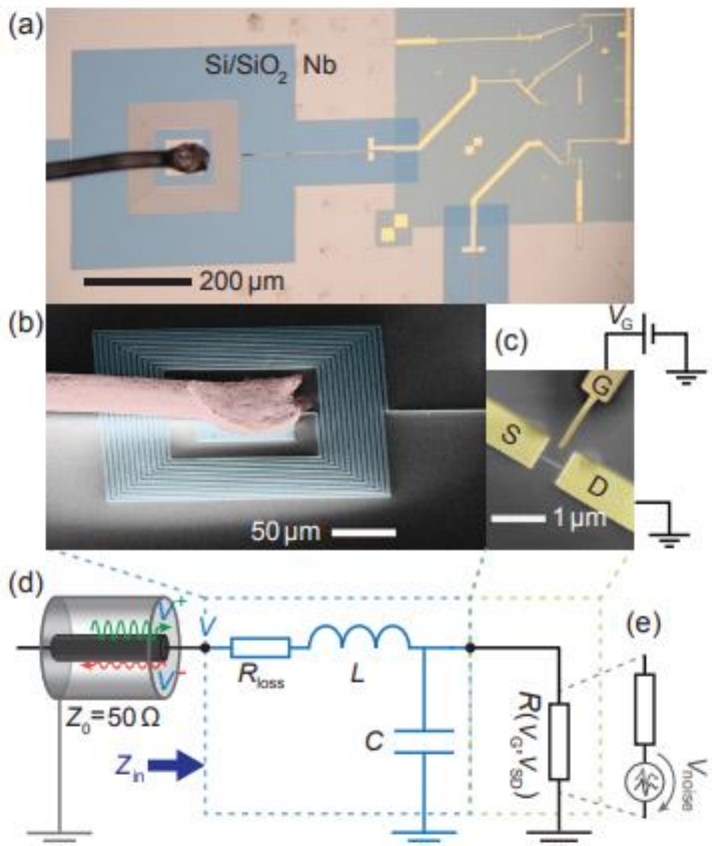
V. Ranjan, PhD Thesis

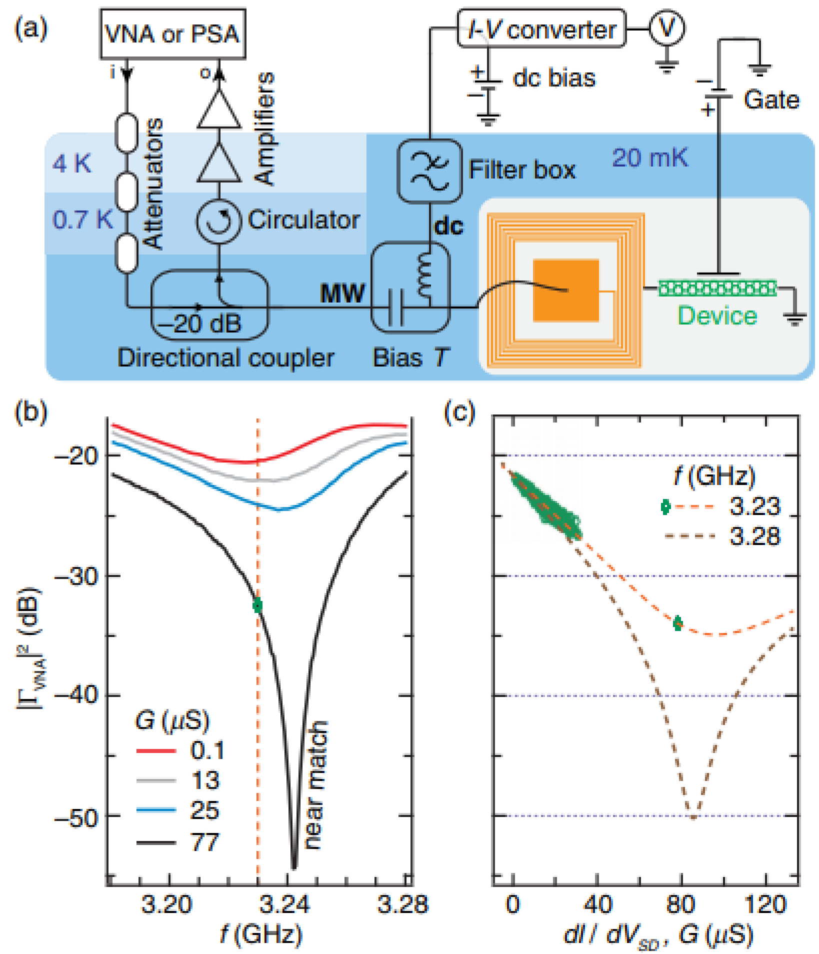
- Compact form
  - Meander
  - Bond bridges



T. Hasler, PhD Thesis

- Simple LC

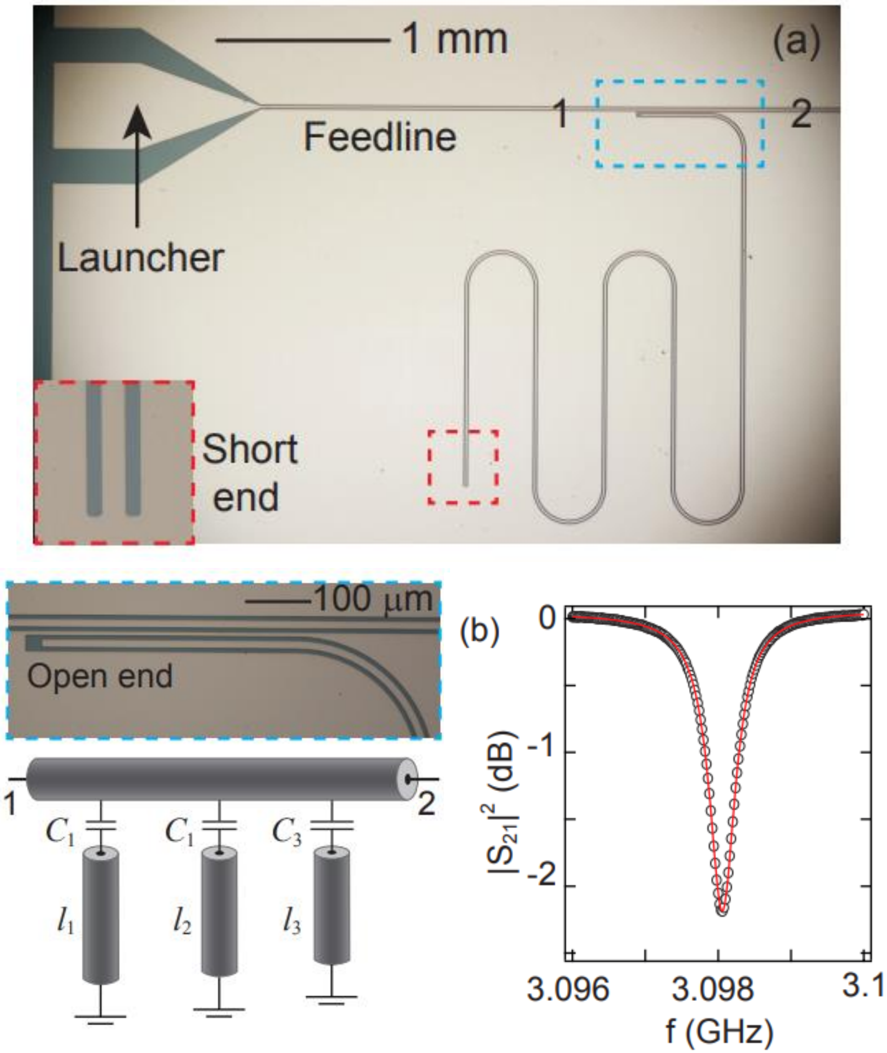




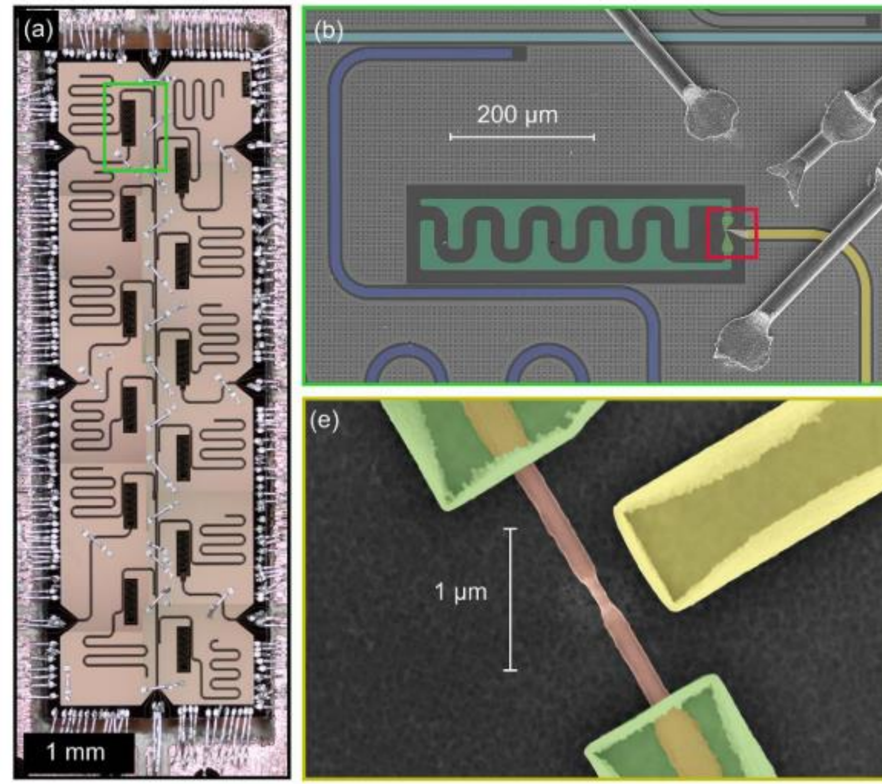
PHYSICAL REVIEW APPLIED 8, 054006 (2017)

Feedline

- Feedline with "hanging" quarter-wave resonators



V. Ranjan, PhD Thesis



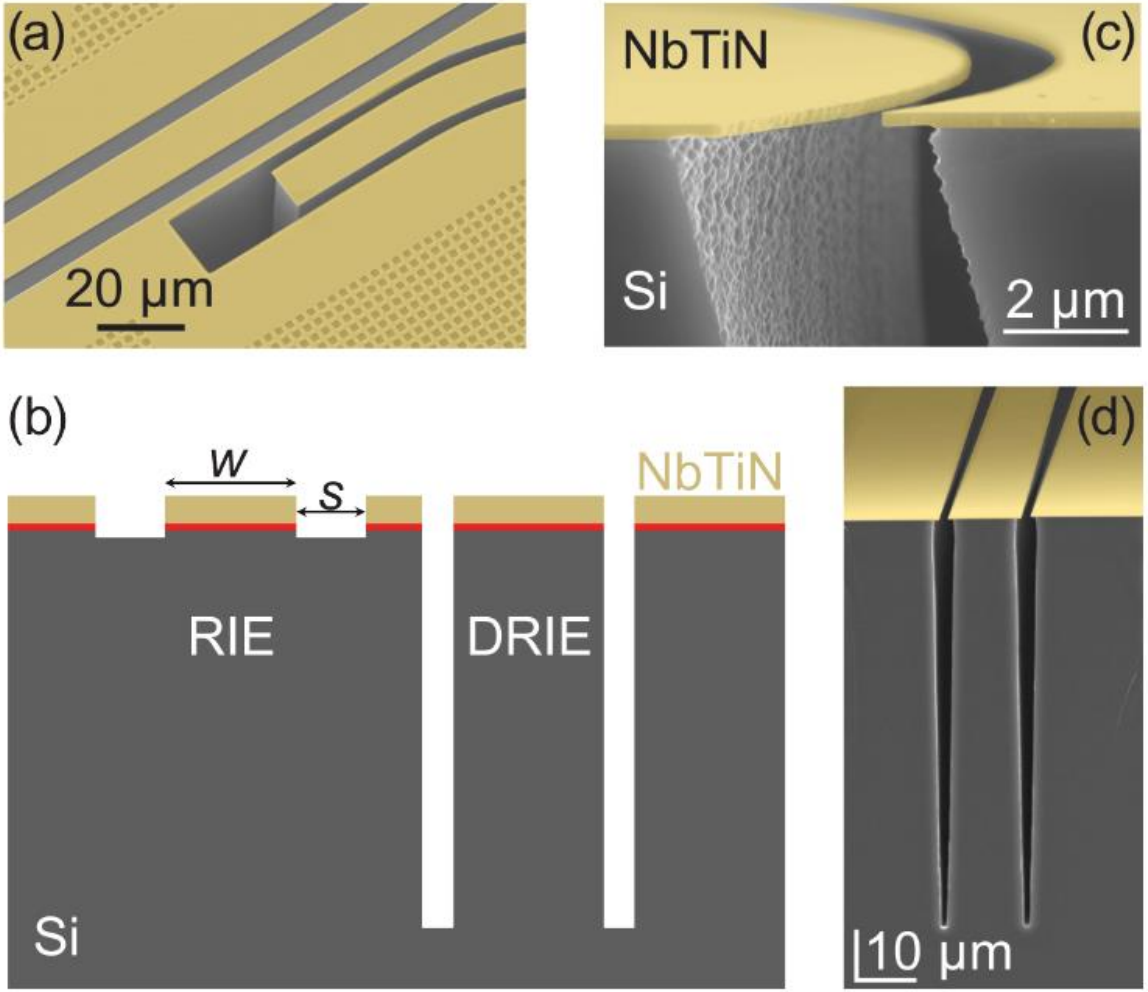
Phys. Rev. Lett. **120**, 100502 (2018)

Loss

- Conductor losses
  - Superconducting film
  - Dissipation for non-zero frequencies
- TLS
  - Mostly at interfaces
  - Nitrides are better (?)
- Common materials:
  - Oxideless Si wafer
  - Sapphire wafer (transparent!)
  - Kapton substrate (flexible, for break junctions)
    - E.g. Coherent manipulation of Andreev Bound States in an atomic contact, Camille Janvier

Loss

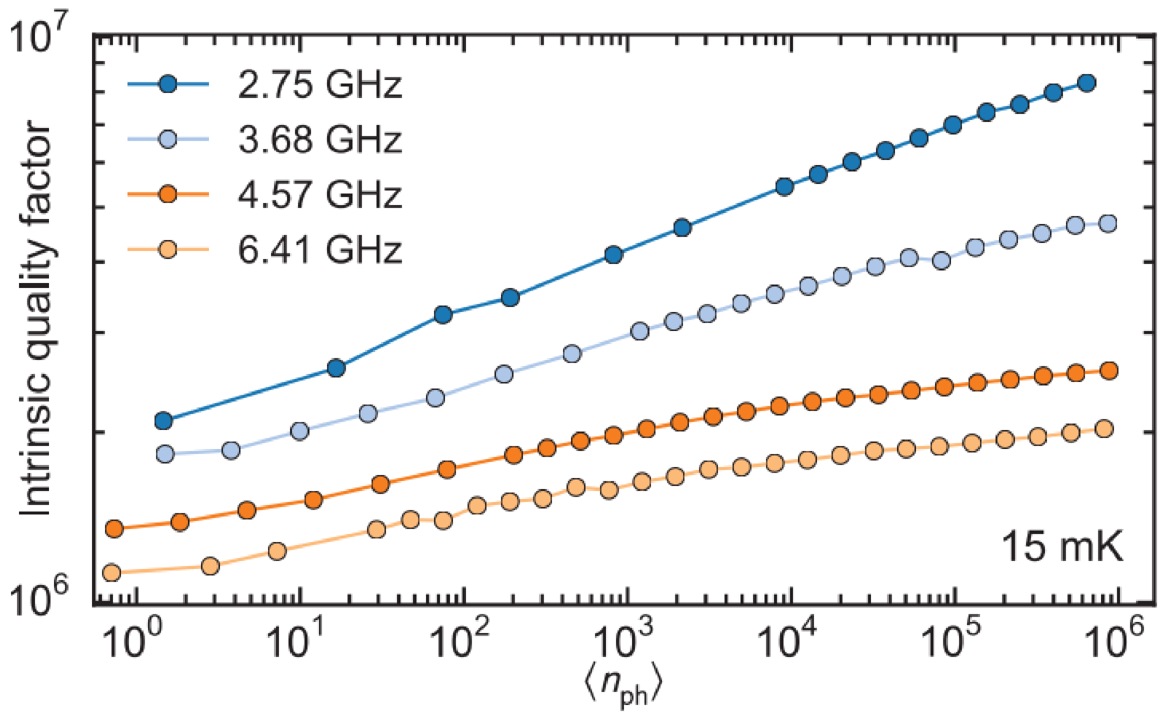
- Annealing
- Trenches
- Holes in the film
  - Vortex pinning
  - Suppress spurious modes



Appl. Phys. Lett. **106**, 182601 (2015)

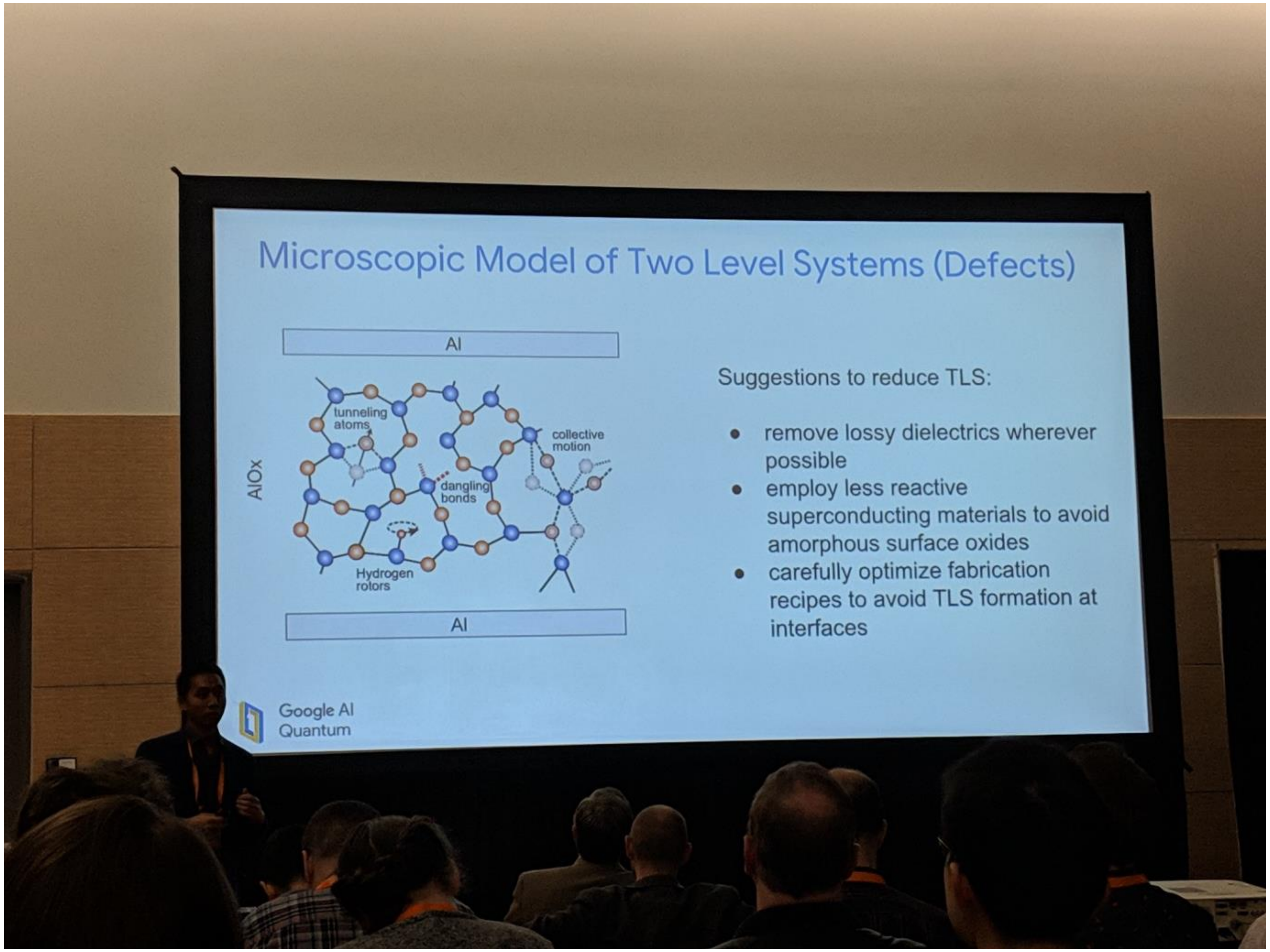
Loss

- Annealing
- Trenches
- Holes in the film
  - Vortex pinning
  - Suppress spurious modes



Appl. Phys. Lett. **106**, 182601 (2015)

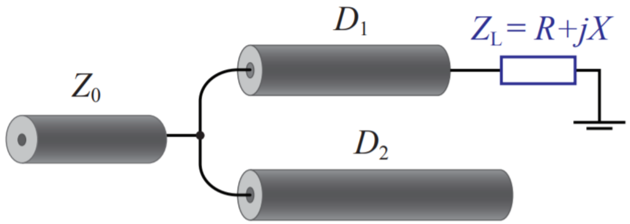
TLS



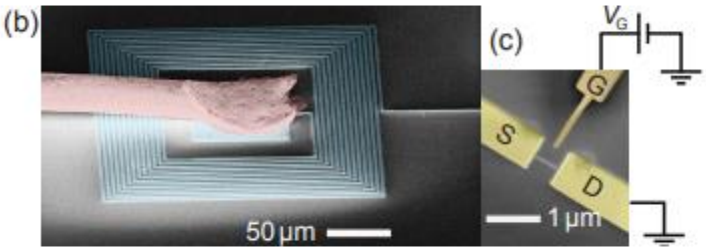
https://twitter.com/quantumVerd/status/1103749781015482369

Resonator-device coupling

- Galvanic

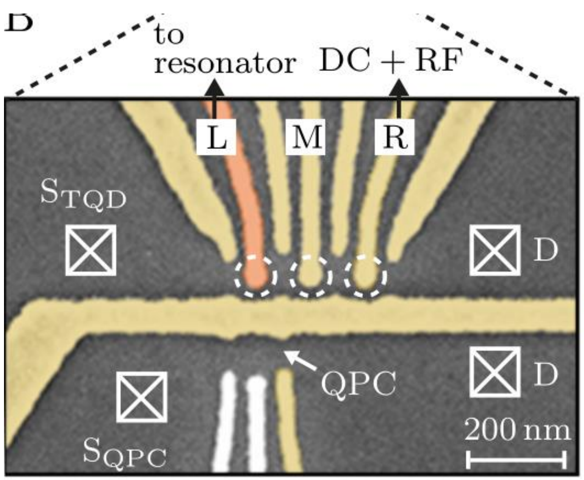


V. Ranjan, PhD Thesis

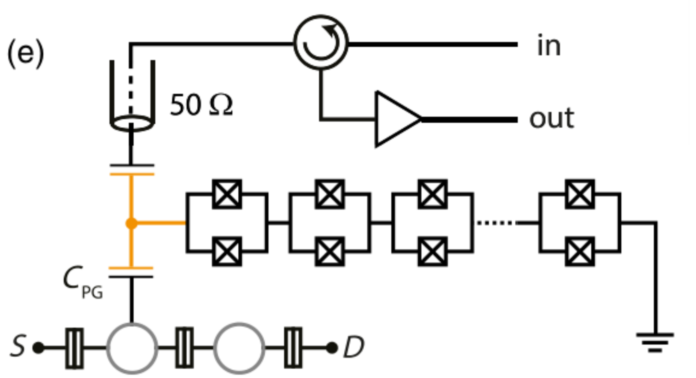


PHYSICAL REVIEW APPLIED 8, 054006 (2017)

- Capacitive



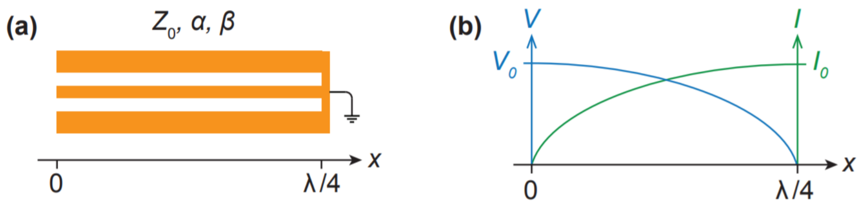
Nature **560**, 179–184 (2018)

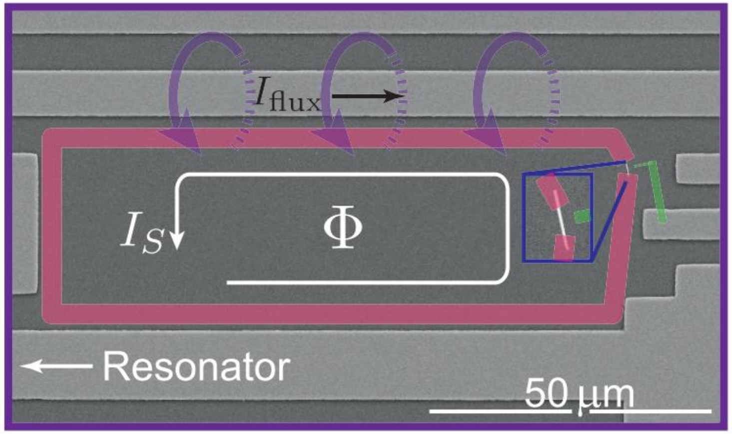


PHYS. REV. X 7, 011030 (2017)

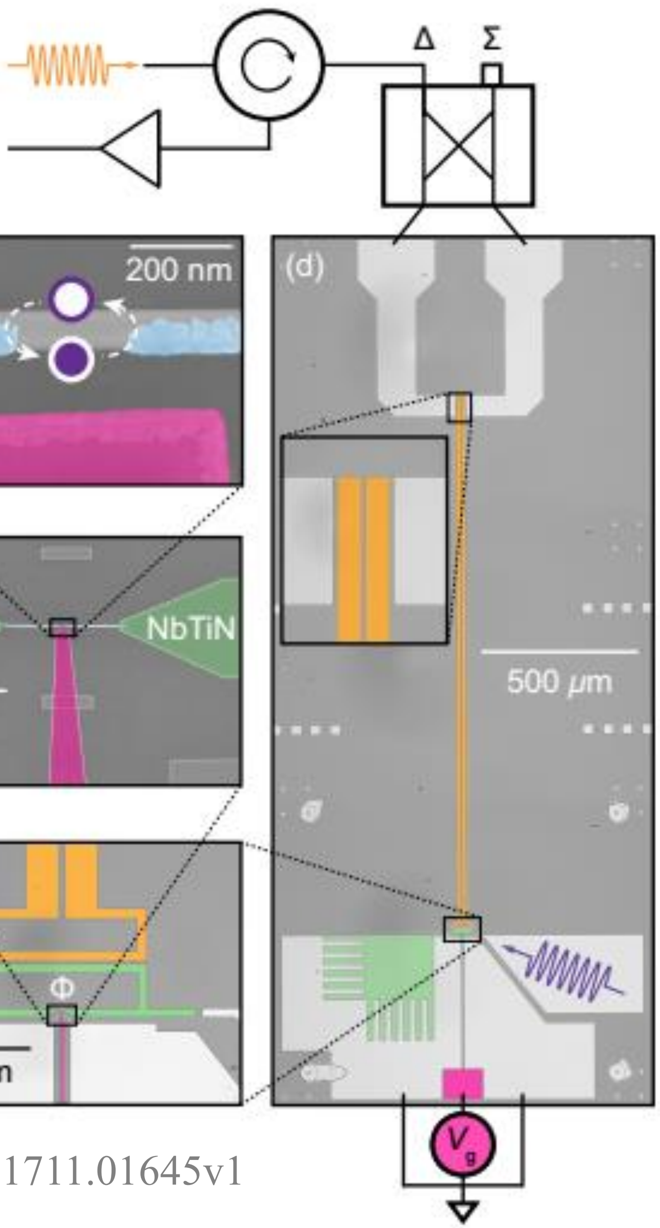
Resonator-device coupling

- Inductive





R. Haller



arXiv:1711.01645v1

Kinetic inductance

- Drude model

$$\sigma = \frac{ne^2\tau}{m(1+i\omega\tau)} = \frac{ne^2\tau}{m(1+\omega^2\tau^2)} - i\frac{ne^2\omega\tau^2}{m(1+\omega^2\tau^2)}$$

  - Low frequencies or short collision time → imaginary part is negligible
- Kinetic energy of Cooper pairs
  - Strip of length l, cross section A

$$\frac{1}{2}(2mv^2)(n_s l A) = \frac{1}{2}L_K I^2 \qquad L_K = \left(\frac{m}{2n_s e^2}\right)\left(\frac{l}{A}\right)$$

  - Scales like resistance!
- From BCS theory:

$$L_k^{\square}(0) = \frac{\hbar R_N^{\square}}{\pi\Delta_0} \qquad \Delta_0 = 2.08 k_B T_c$$

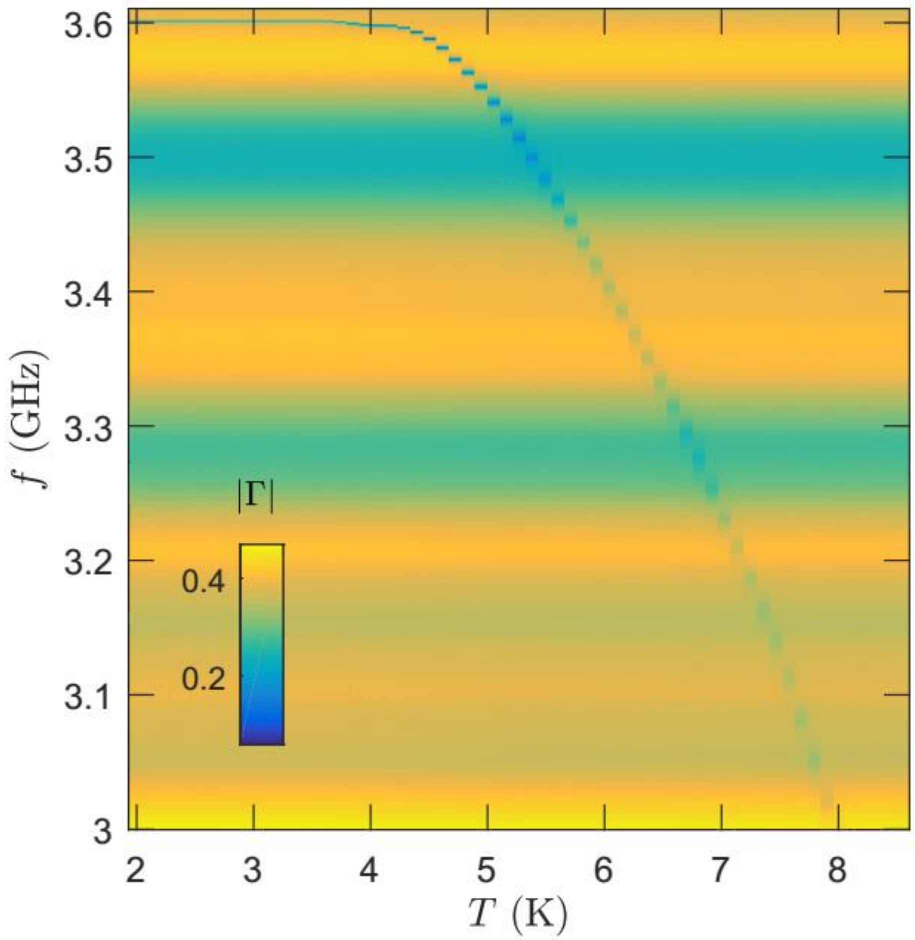
- Tunability:
  - Temperature
  - Current

arXiv:1802.01723v1

A.J. Annunziata *et al.*, "Tunable superconducting nanoinductors," *Nanotechnology* **21**, 445202 (2010)

Kinetic inductance

Quarter-wave resonator, 14 nm NbTiN film



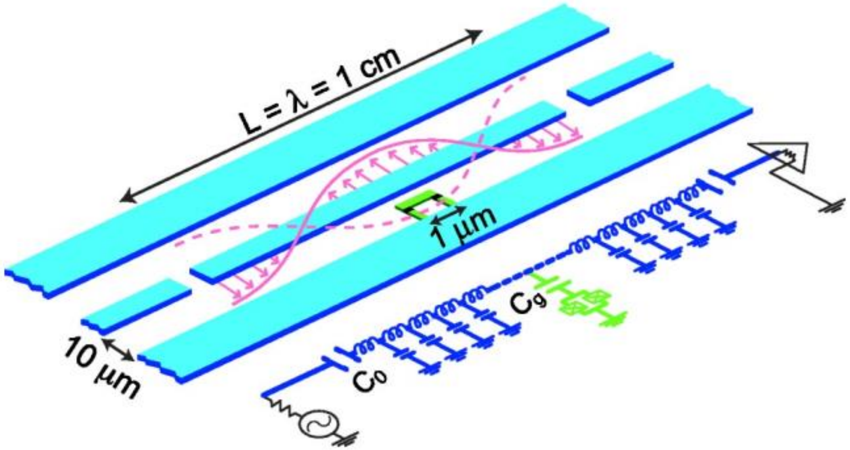
- Tunability:
  - Temperature
  - Current

arXiv:1802.01723v1

A.J. Annunziata *et al.*, "Tunable superconducting nanoinductors," *Nanotechnology* **21**, 445202 (2010)

Kinetic inductance, high-Z resonators

- Motivation
  - 1D cavity



  - Small volume / distance → large E field

$$V_{\rm rms}^0 \sim \sqrt{\hbar\omega_{\rm r}/cL}$$

$$10\ \text{GHz}\ (h\nu/k_B \sim 0.5\ \text{K})$$

$$10\ \mu\text{m gap}$$

$$V_{\rm rms} \sim 2\ \mu\text{V}$$

$$\mathcal{E}_{\rm rms} \sim 0.2\ \text{V/m}$$

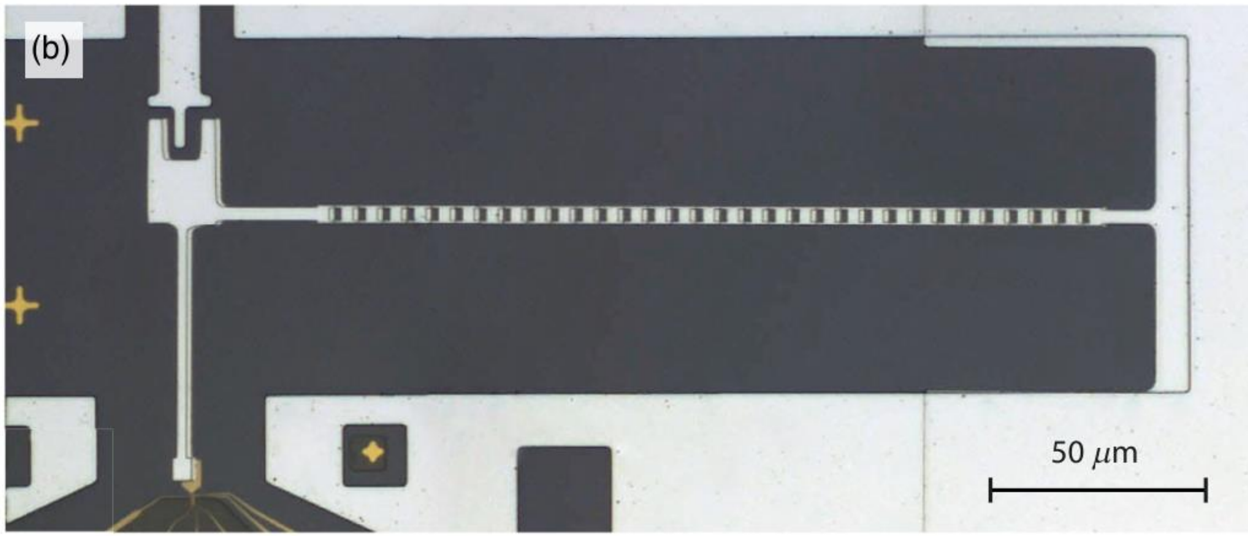
  - Coupling strength to qubits
  - Strong coupling regime

$$g \gg \kappa, \gamma$$

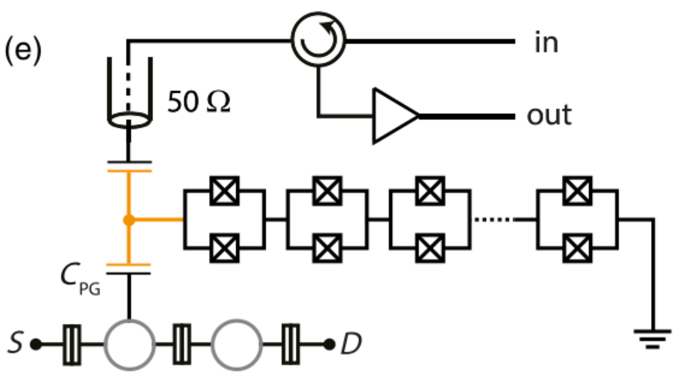
PHYSICAL REVIEW A **69**, 062320 (2004)

Kinetic inductance, high-Z resonators

- Realization in CPW geometry
  - SQUID array (32 SQUIDs)
  - Z = 1.3 – 1.8 kOhm
  - Tunable with B field



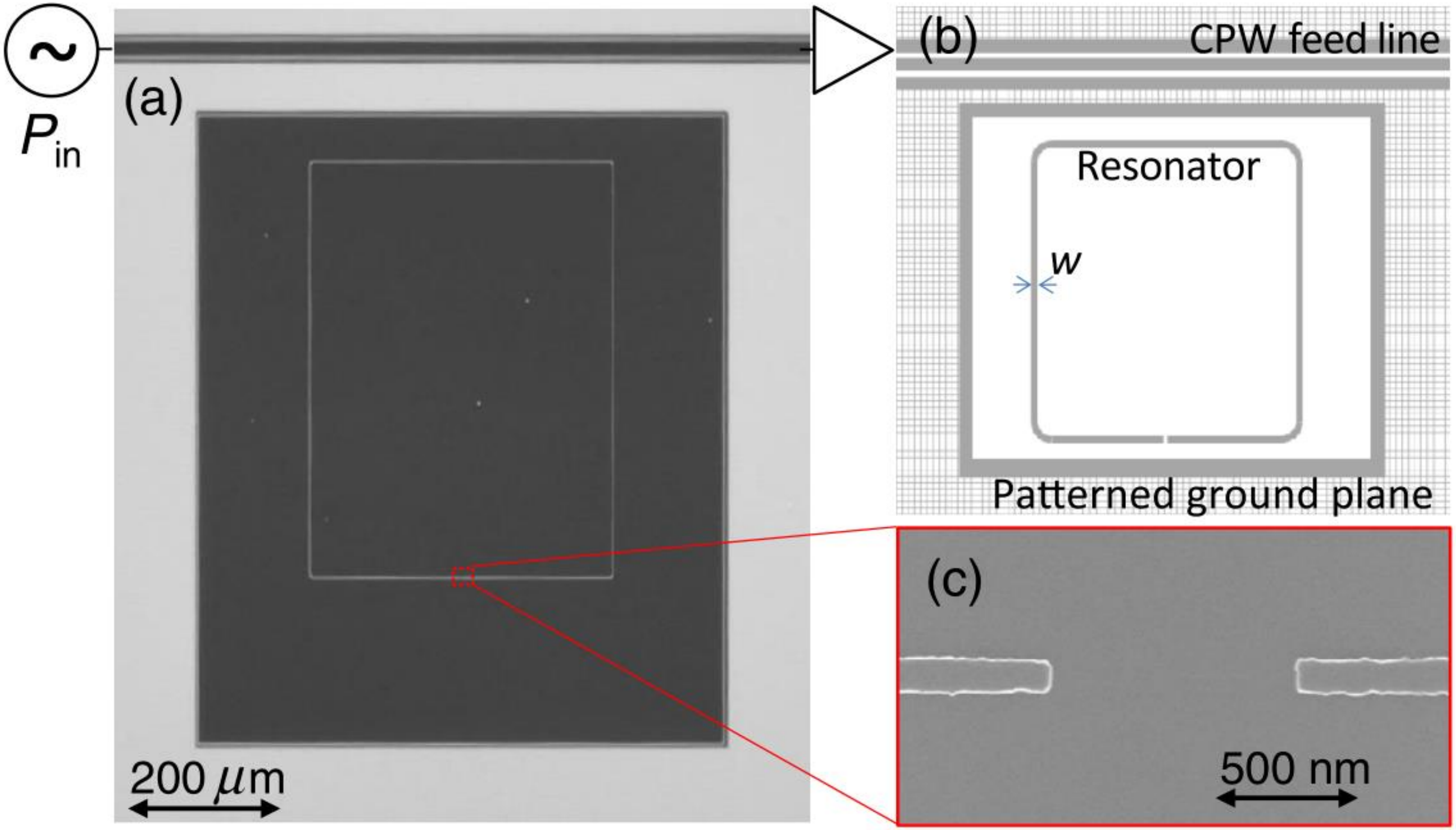
(e)



PHYS. REV. X 7, 011030 (2017)

Kinetic inductance, high-Z resonators

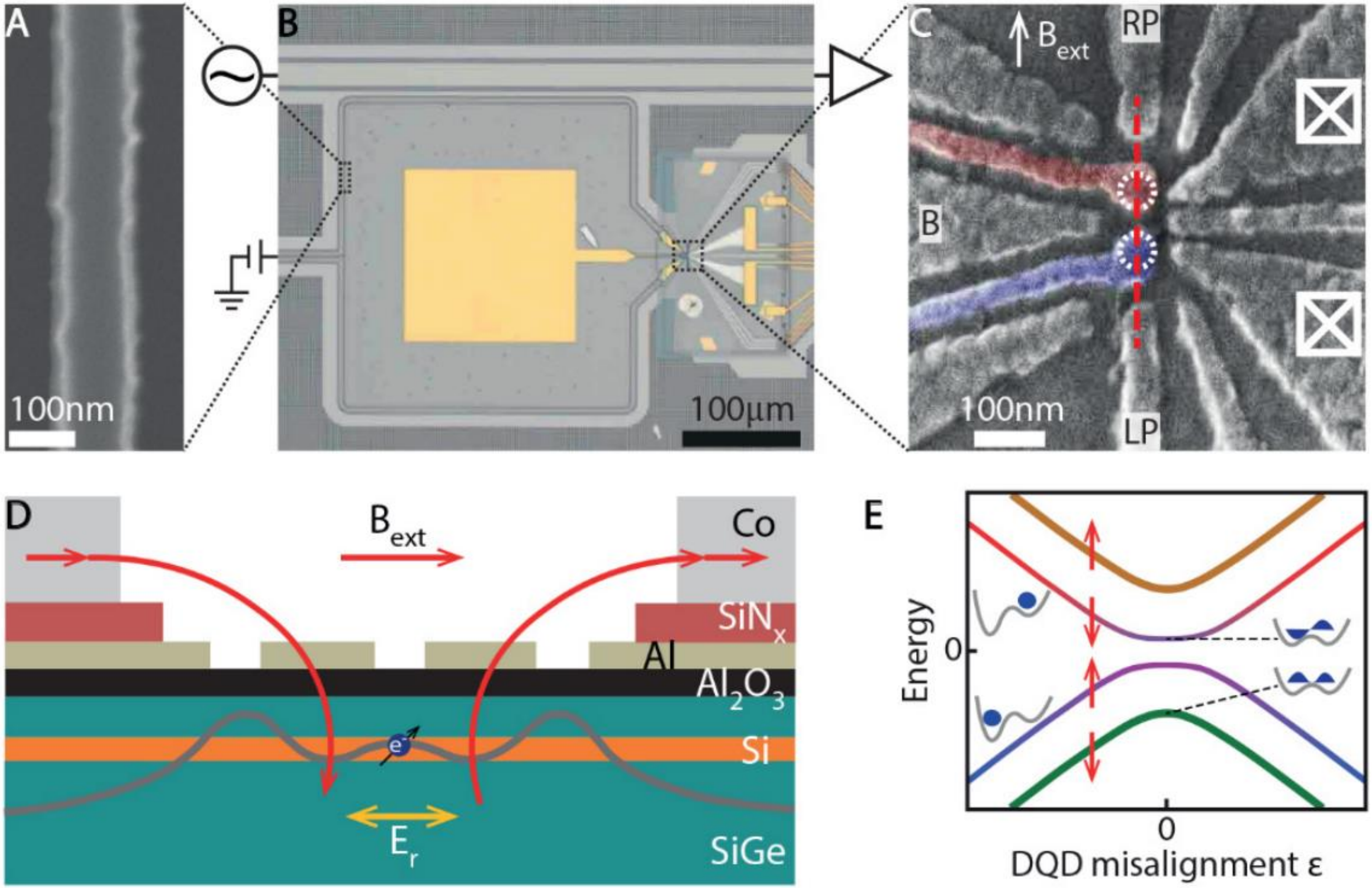
- Realization: CPW feedline + stripline
  - NbTiN, Z = 4 kOhm



PHYSICAL REVIEW APPLIED 5, 044004 (2016)

Kinetic inductance, high-Z resonators

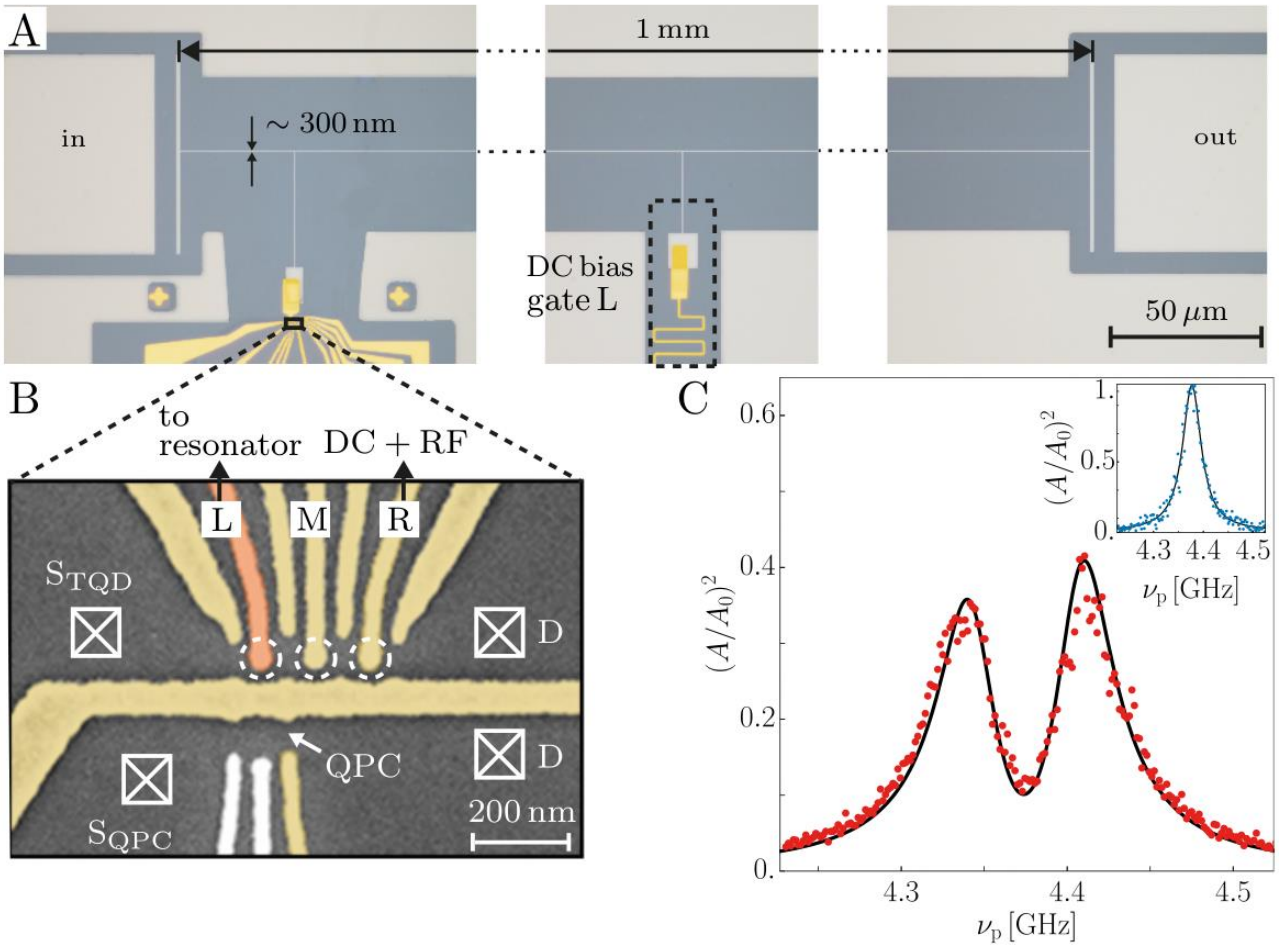
- Realization: CPW feedline + stripline



Science 09 Mar 2018:
Vol. 359, Issue 6380, pp. 1123-1127

Kinetic inductance, high-Z resonators

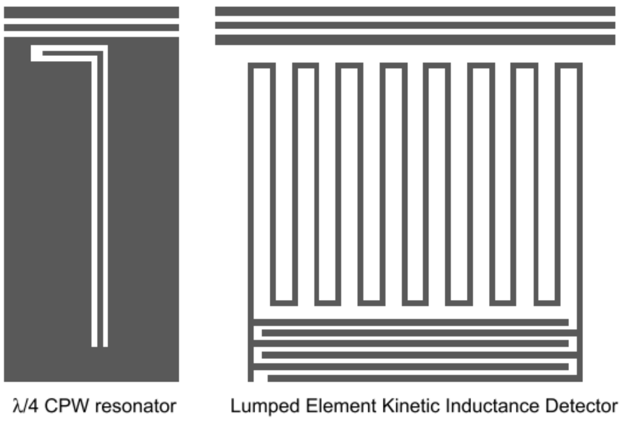
- Realization in CPW geometry
  - Thin, narrow center conductor (300 nm, NbTiN)
  - Z = 1.3 kOhm



Nature **560**, 179–184 (2018)

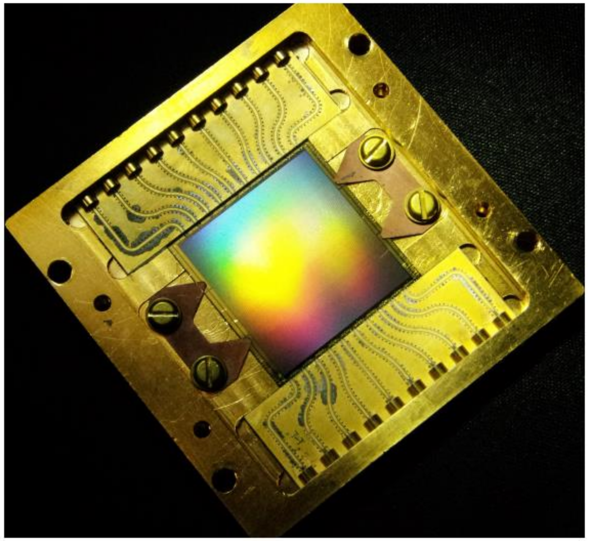
Remarks

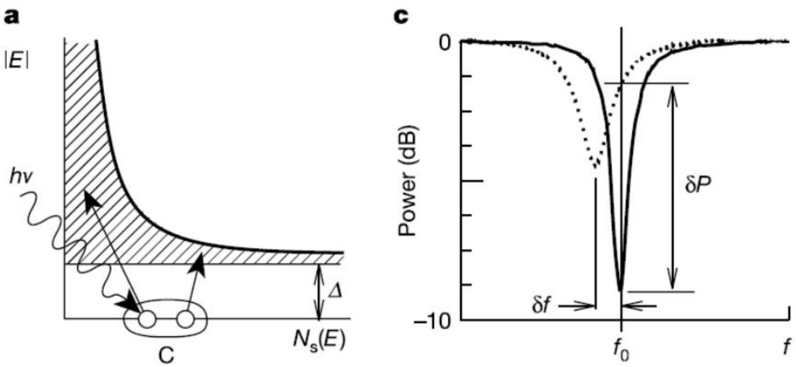
- Kinetic inductance detector
  - Distributed or lumped



Journal of Low Temperature Physics 167 (3-4), 292-304

  - 20k pixels

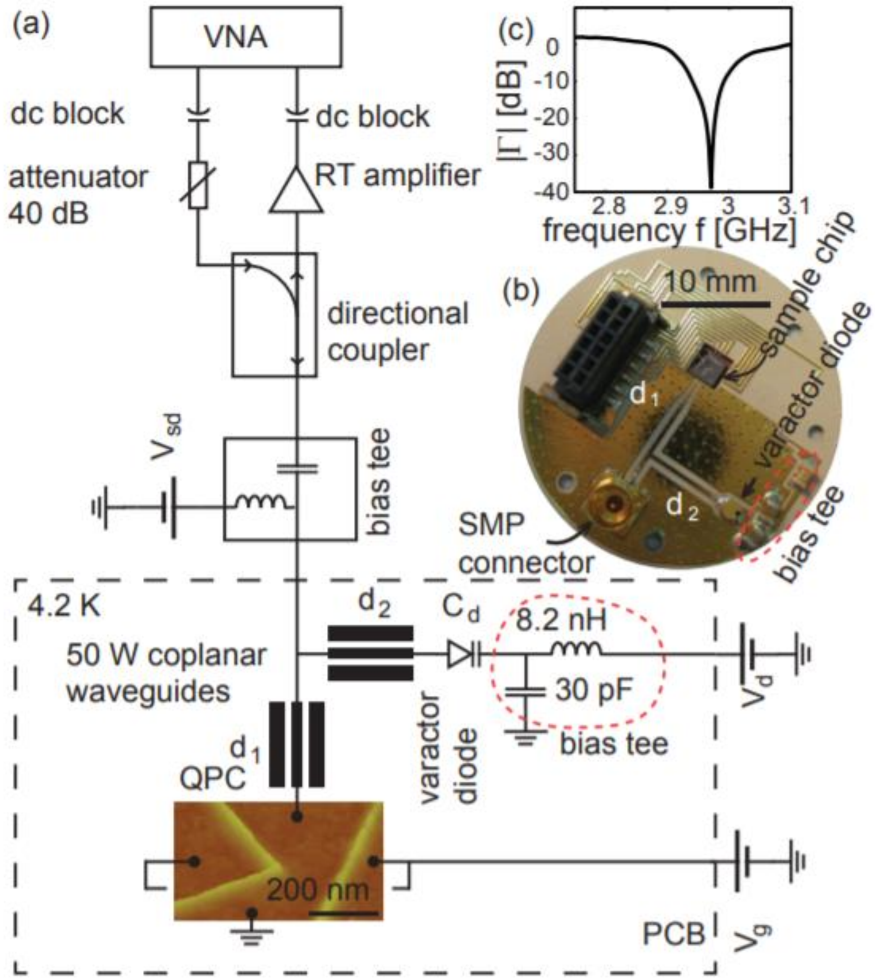




http://web.physics.ucsb.edu/~bmazin/Papers/szypryt_dissertation.pdf

Remarks

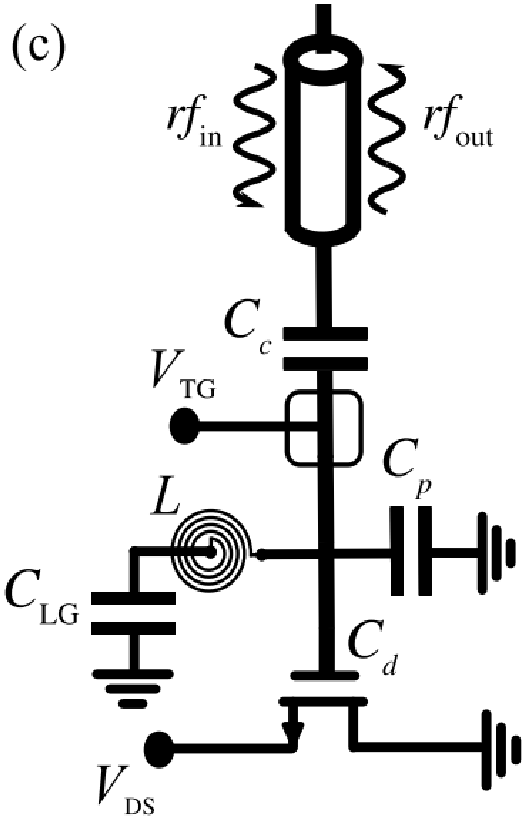
- Off-chip resonator: stub tuner

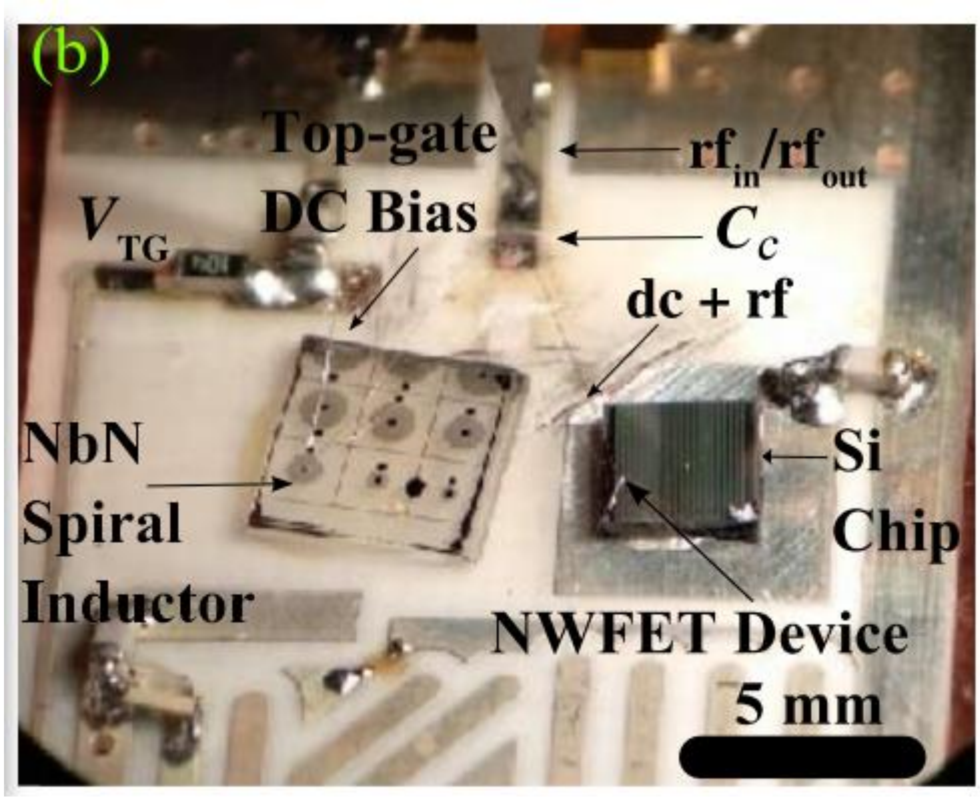


Appl. Phys. Lett. **101**, 042112 (2012);

Remarks

- Off-chip resonator: SC meander inductor

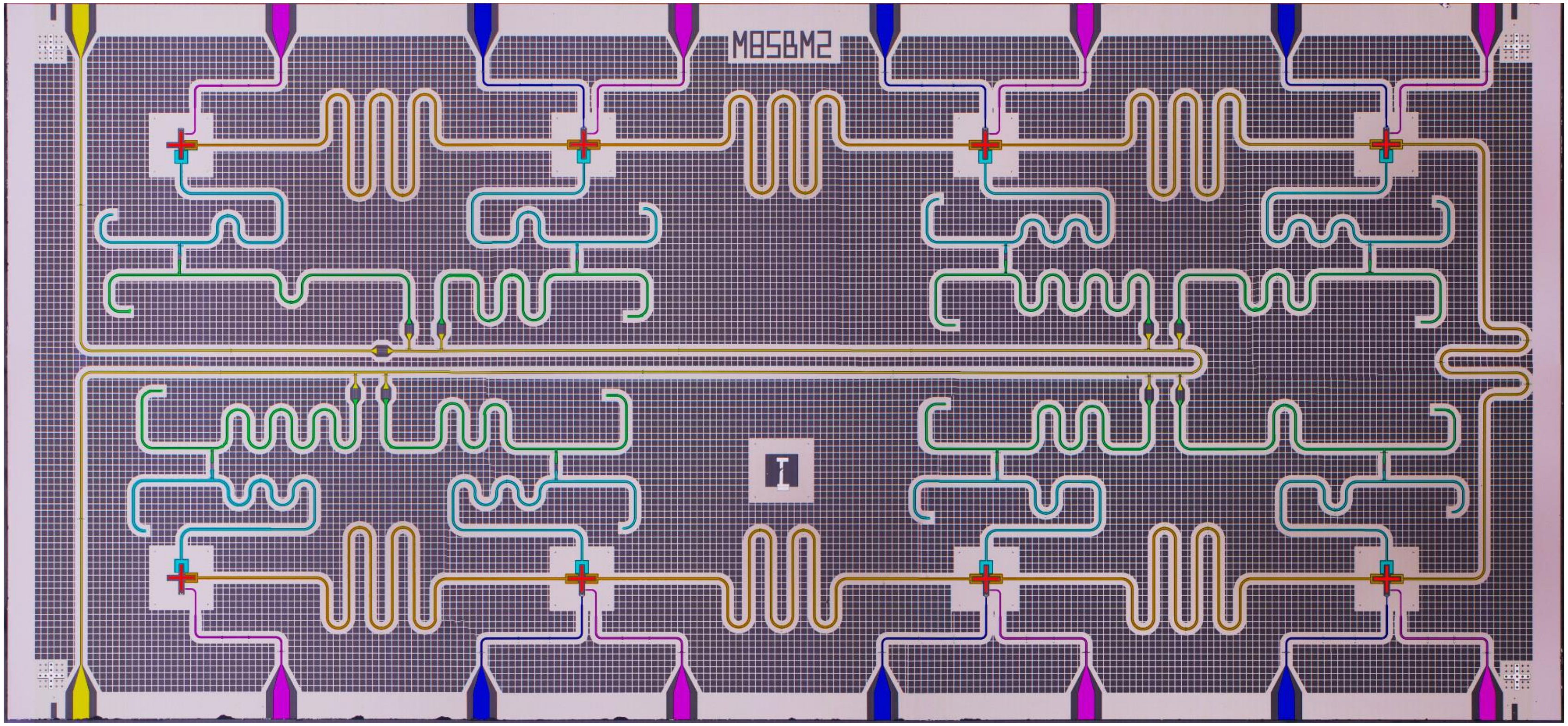




PHYSICAL REVIEW APPLIED 10, 014018 (2018)

Many-qubit architecture

- SC resonator for/as
  - Qubit read-out
  - Quantum bus
  - Purcell filter



ETH, https://qt.eu/discover/technology/