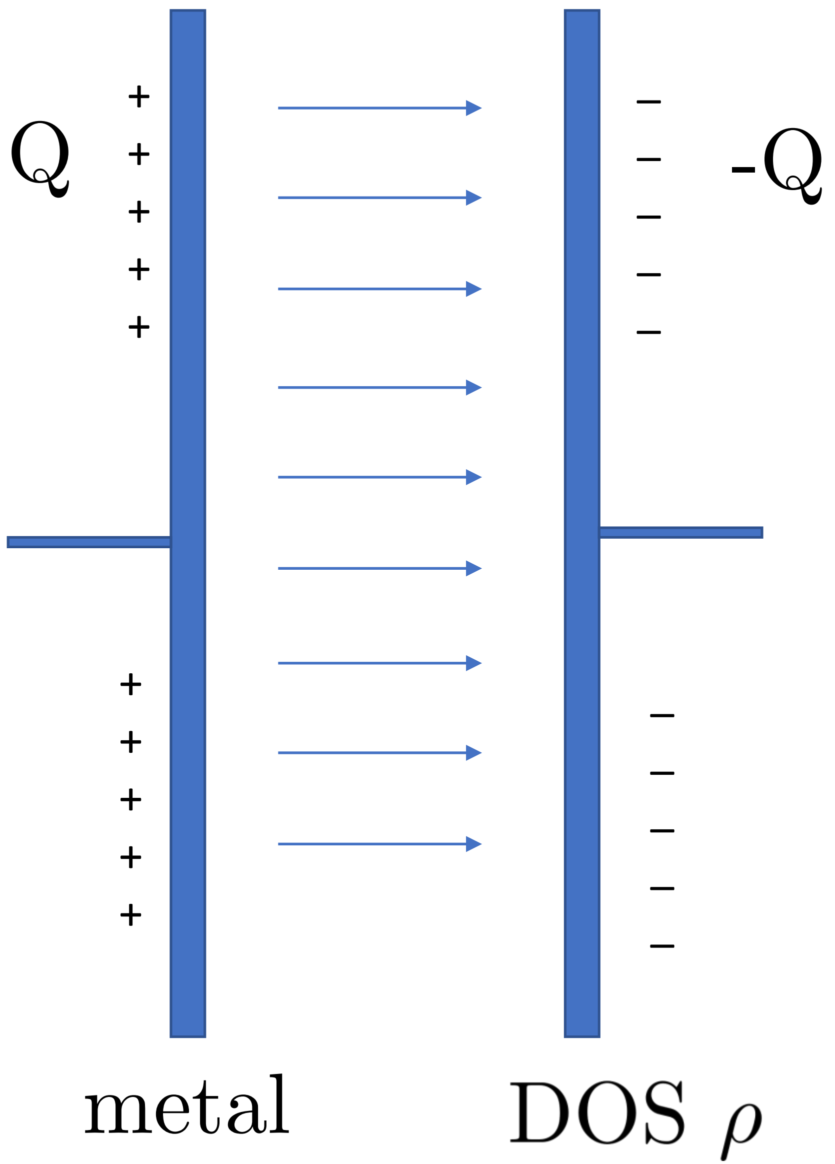
Quantum capacitance of quantum dots

RF seminar

17 June 2019

Quantum capacitance (finite DOS)



$$\Delta V_{\mathrm{Galvani}} = \frac{Q}{C_{\mathrm{geom}}}$$

$$\Delta\mu = \frac{Q}{e\rho} \longrightarrow \Delta V_{\mathrm{quantum}} = \frac{Q}{e^2\rho}$$

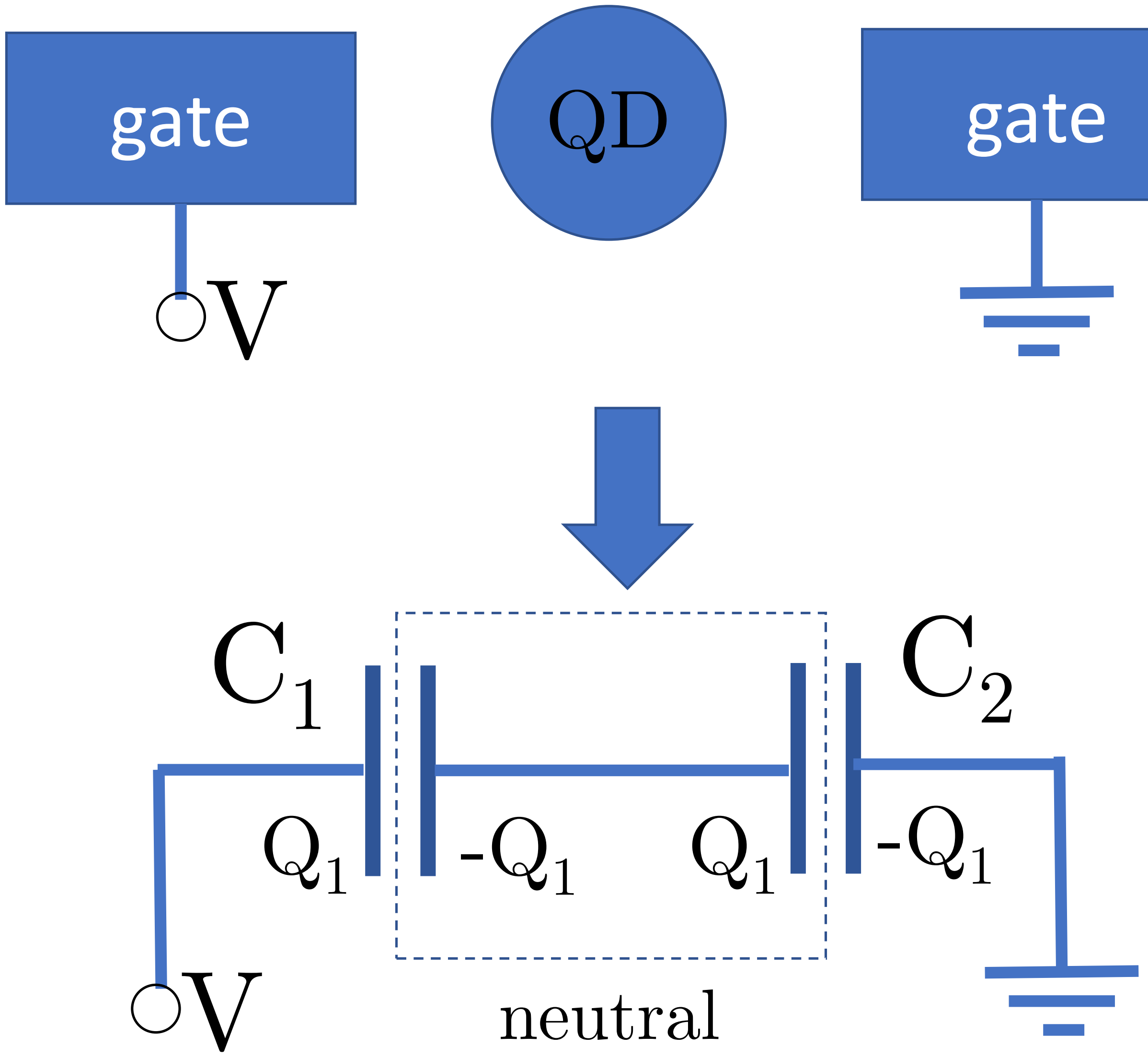
$$C_{\mathrm{quantum}} = e^2\rho$$

$$\Delta V = Q\left(\frac{1}{C_{\mathrm{geom}}} + \frac{1}{C_{\mathrm{quantum}}}\right)$$

In series

More details: previous talk, Makk Péter

Geometrical capacitance



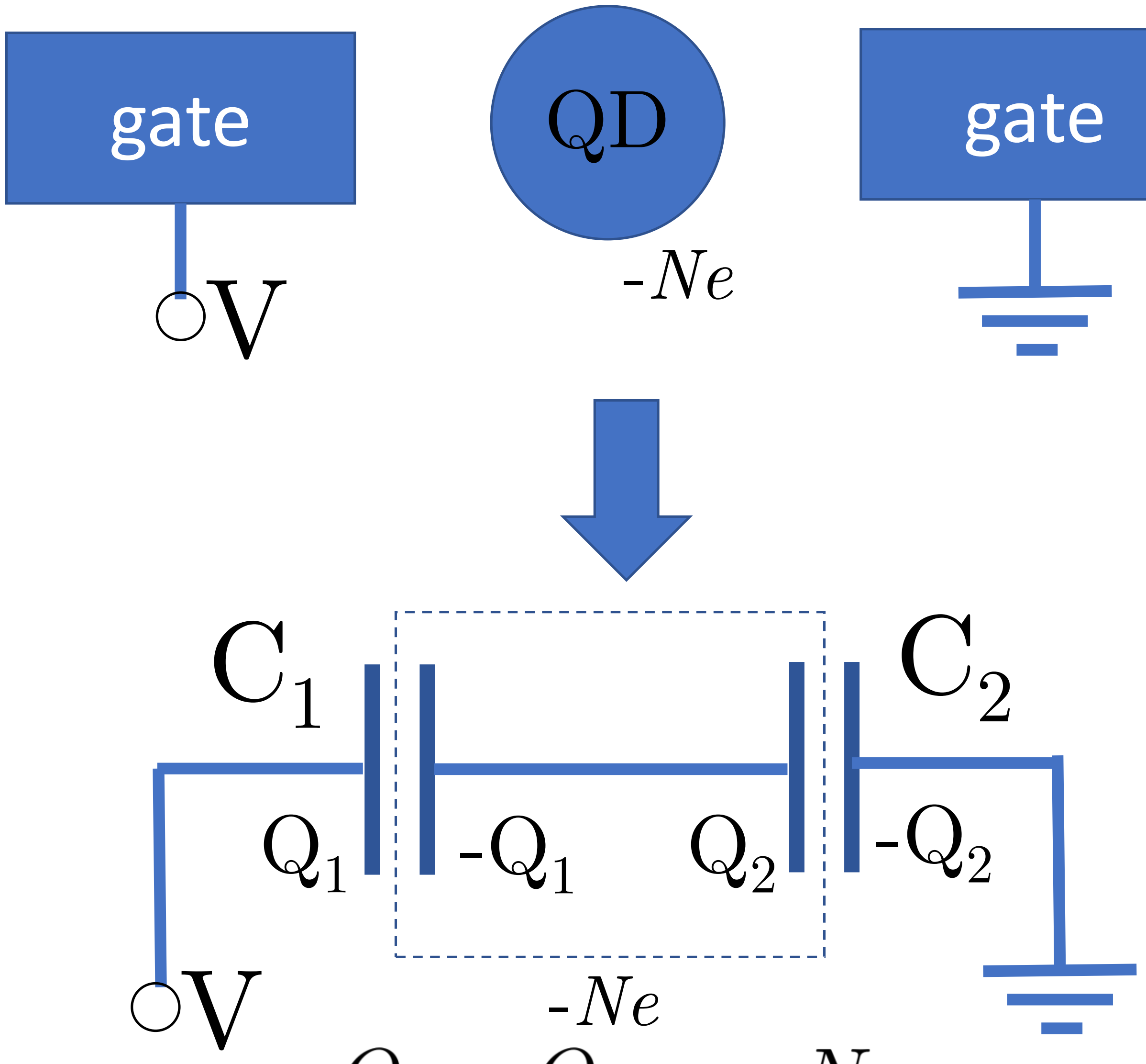
$$Q_1\left(\frac{1}{C_1}+\frac{1}{C_2}\right)=V$$

$$\frac{1}{C_\mathrm{g}}$$

capacitors in series

$$V_\mathrm{dot}=\frac{C_g}{C_2}V$$

Differential capacitance



$$Q_2 - Q_1 = -Ne$$

$$\frac{Q_1}{C_1} + \frac{Q_2}{C_2} = V$$

$$Q_1 = C_{\mathrm{g}} V + \frac{C_{\mathrm{g}}}{C_2} Ne$$

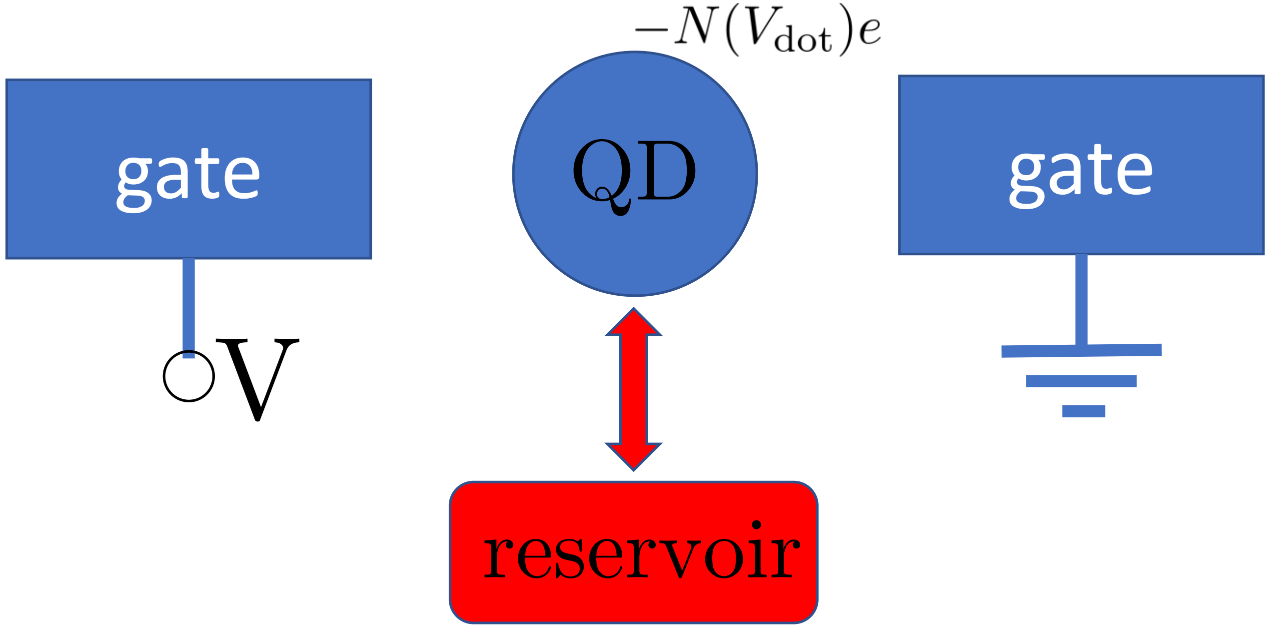
offset

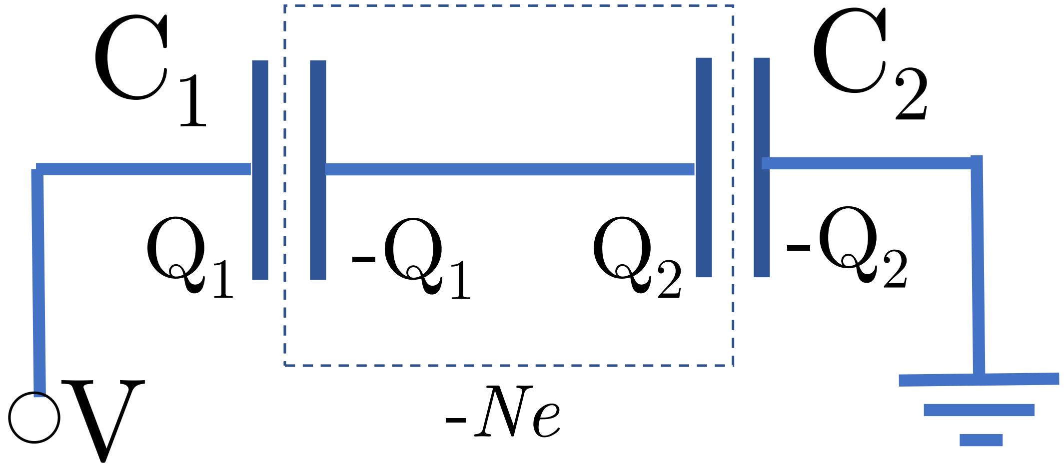
$$I = C_{\mathrm{diff}} \dot{V}$$

$$C_{\mathrm{diff}} = \frac{dQ_1}{dV} = C_{\mathrm{g}}$$

N is fixed

Quantum capacitance

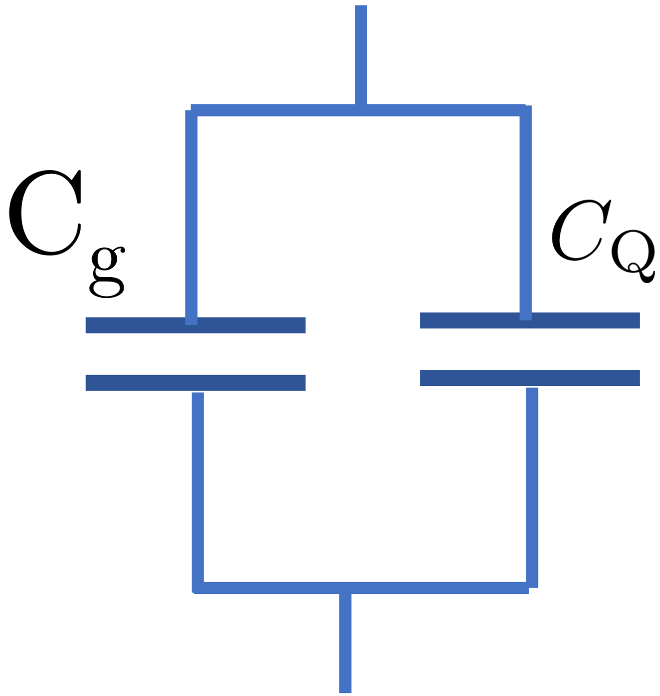




$$Q_1 = C_{\mathrm{g}} V + \frac{C_{\mathrm{g}}}{C_2} N e$$

$$C_{\mathrm{diff}} = \frac{dQ_1}{dV} = C_{\mathrm{g}} + \frac{C_g}{C_2}\frac{dN}{dV} e$$

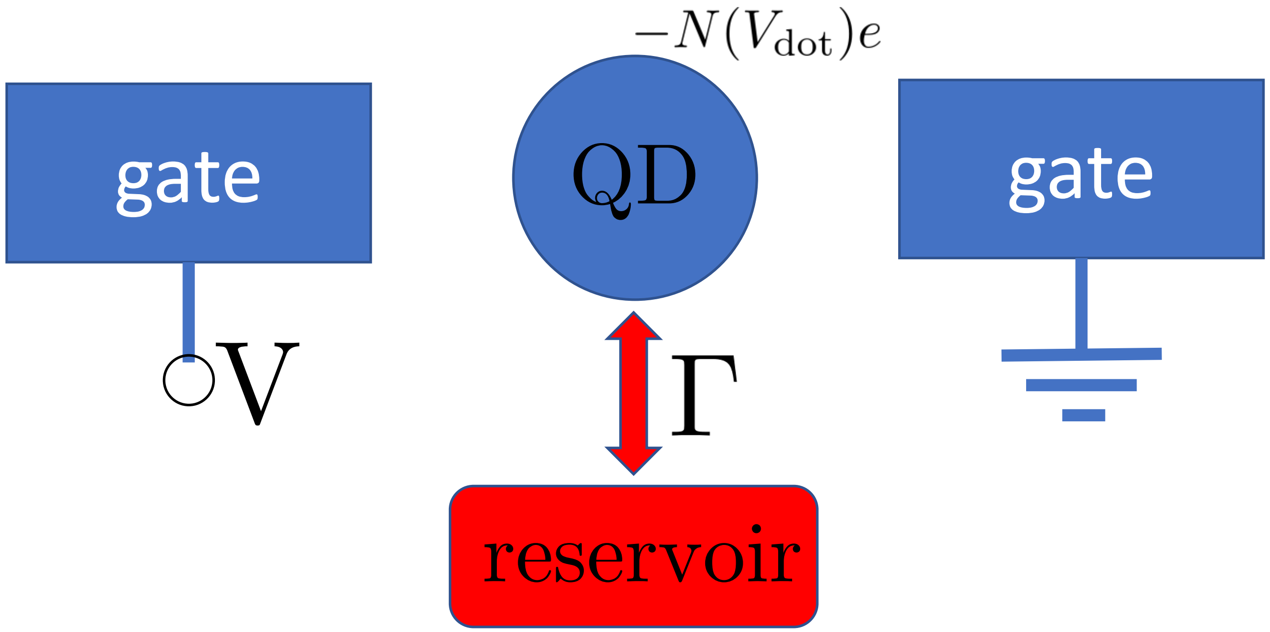
C_Q: quantum capacitance

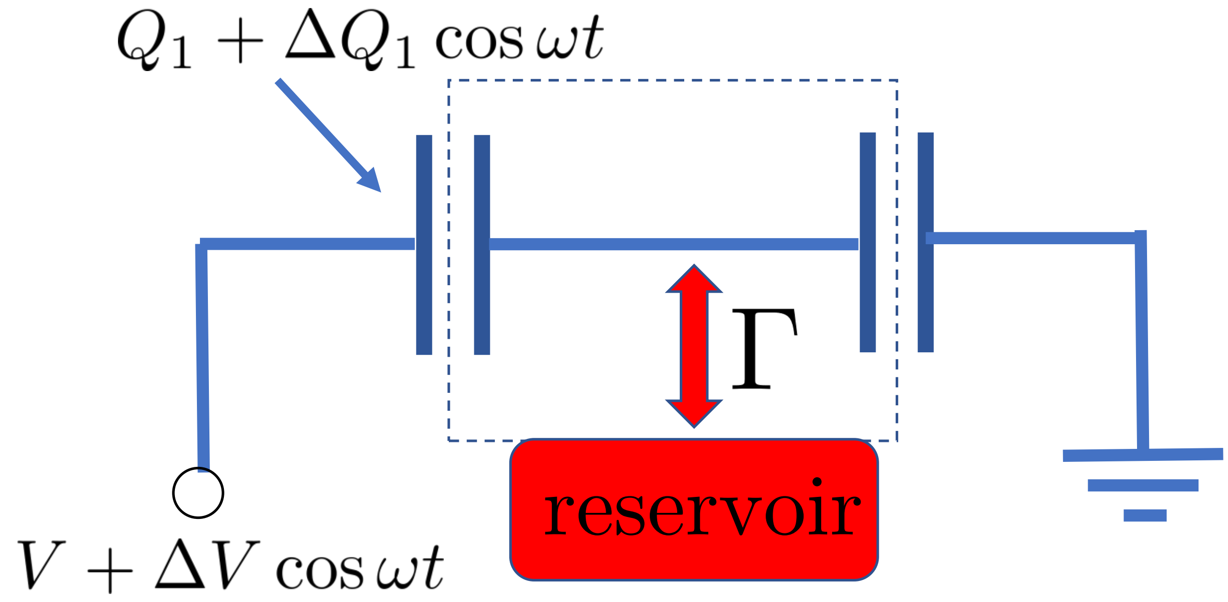


$$C_{\mathrm{Q}} = \frac{C_g^2}{C_2^2}\frac{dN}{dV_{\mathrm{dot}}} e$$

parallel

Quantum capacitance





$$\omega \gg \Gamma \;\rightarrow\; C_Q = 0$$

$$\Delta Q_1 = (C_g + C_Q(V))\,\Delta V$$

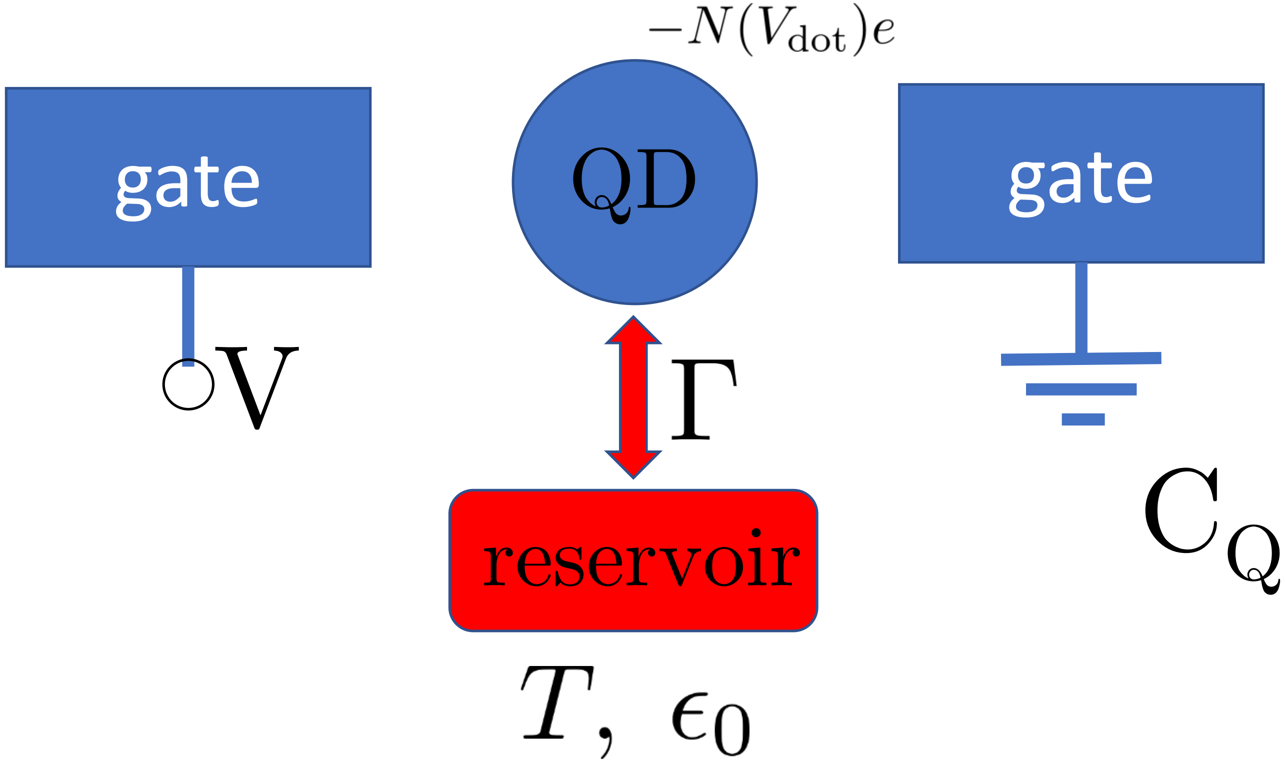
$$\omega \ll \Gamma \;\rightarrow\; \text{adiabatic}$$

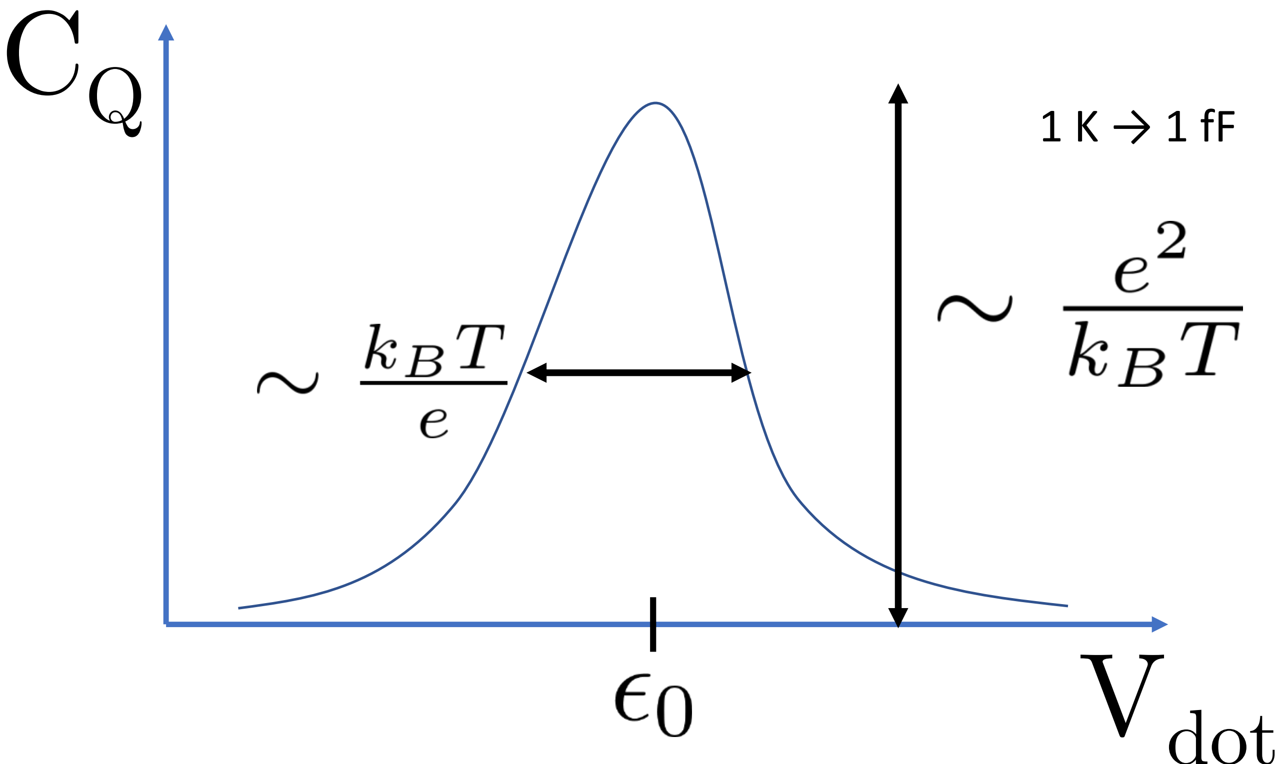
Quantumdot + Fermi-sea

One level in the QD

Orbital energy $= 0$

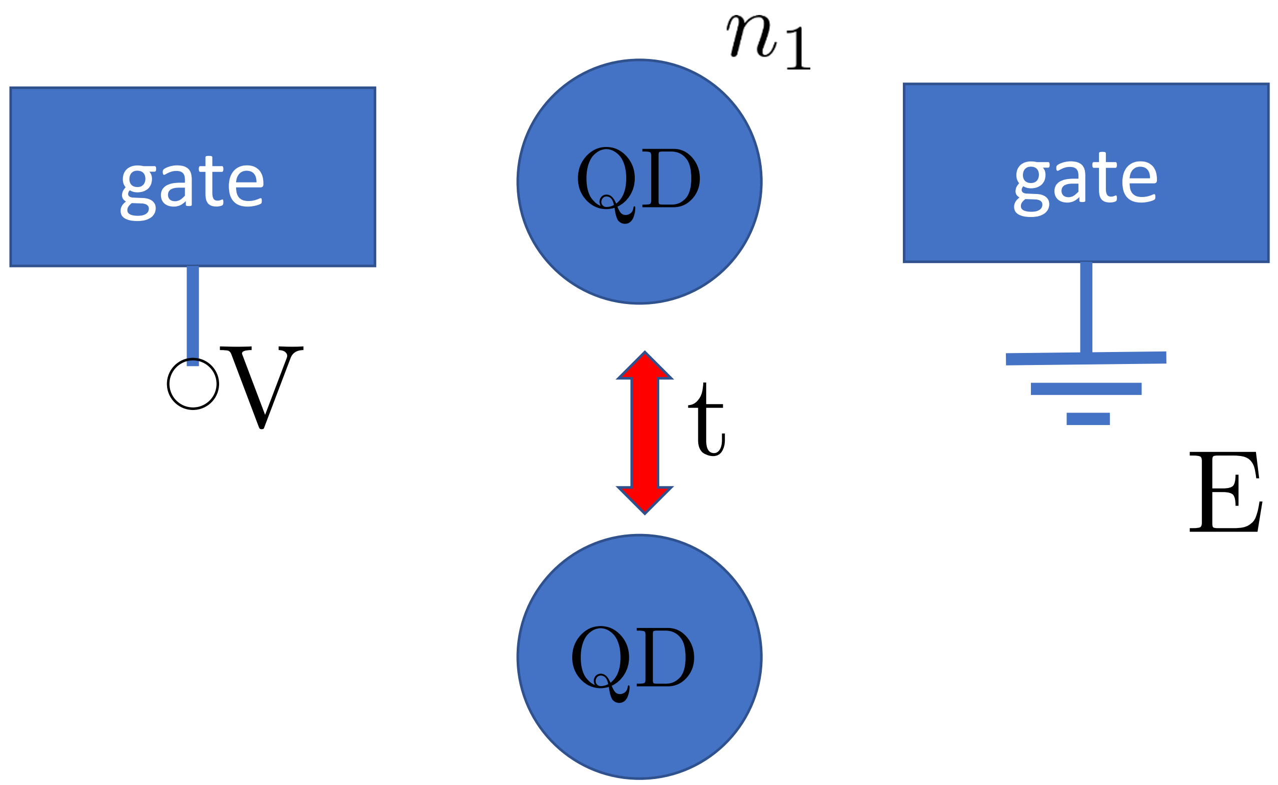
$$N(V_{\mathrm{dot}}) = \frac{1}{e^{\frac{eV_{dot} - \epsilon_0}{k_B T}} + 1}$$



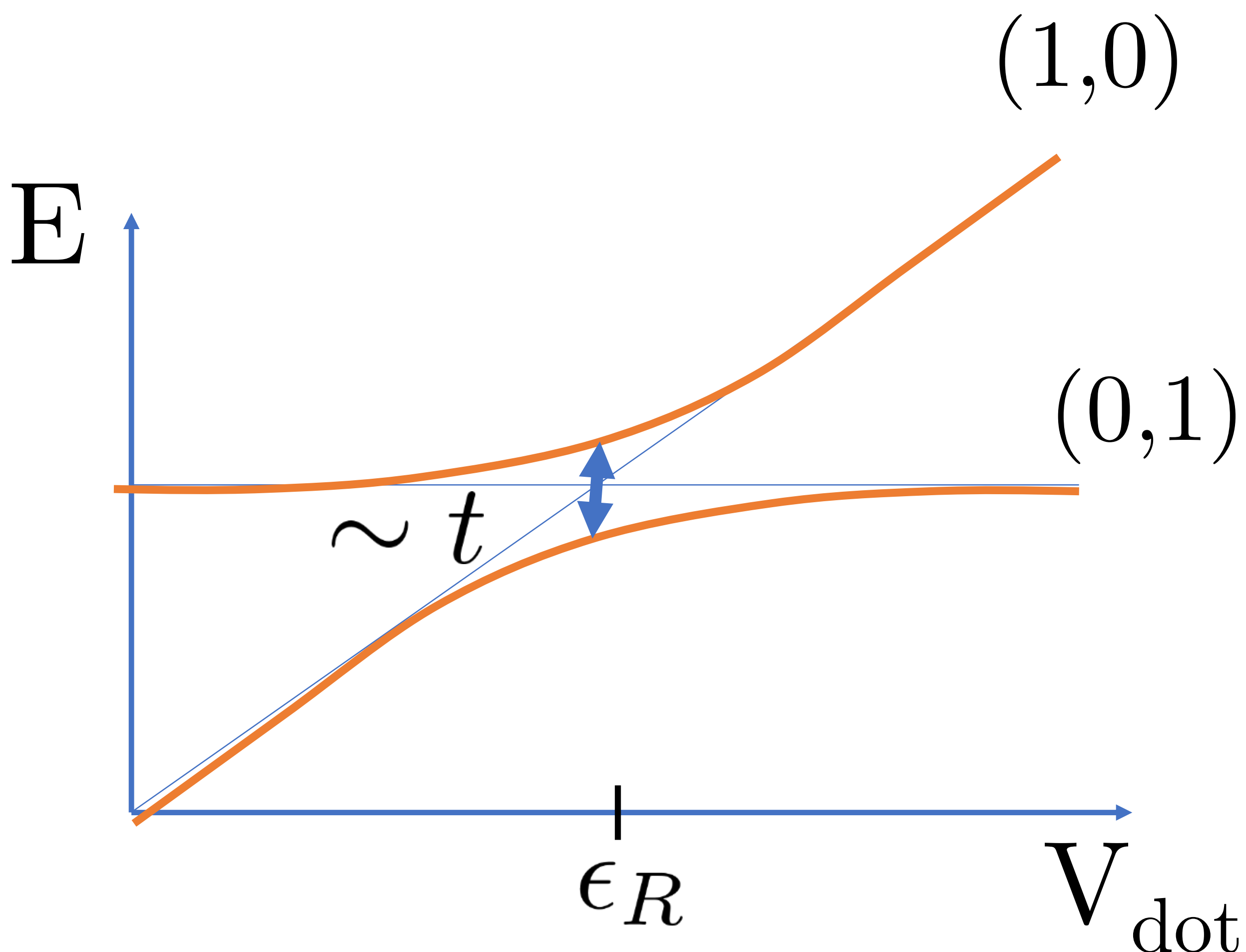


$$C_Q = \frac{C_g^2}{C_2^2} \frac{dN}{dV_{\mathrm{dot}}} e$$

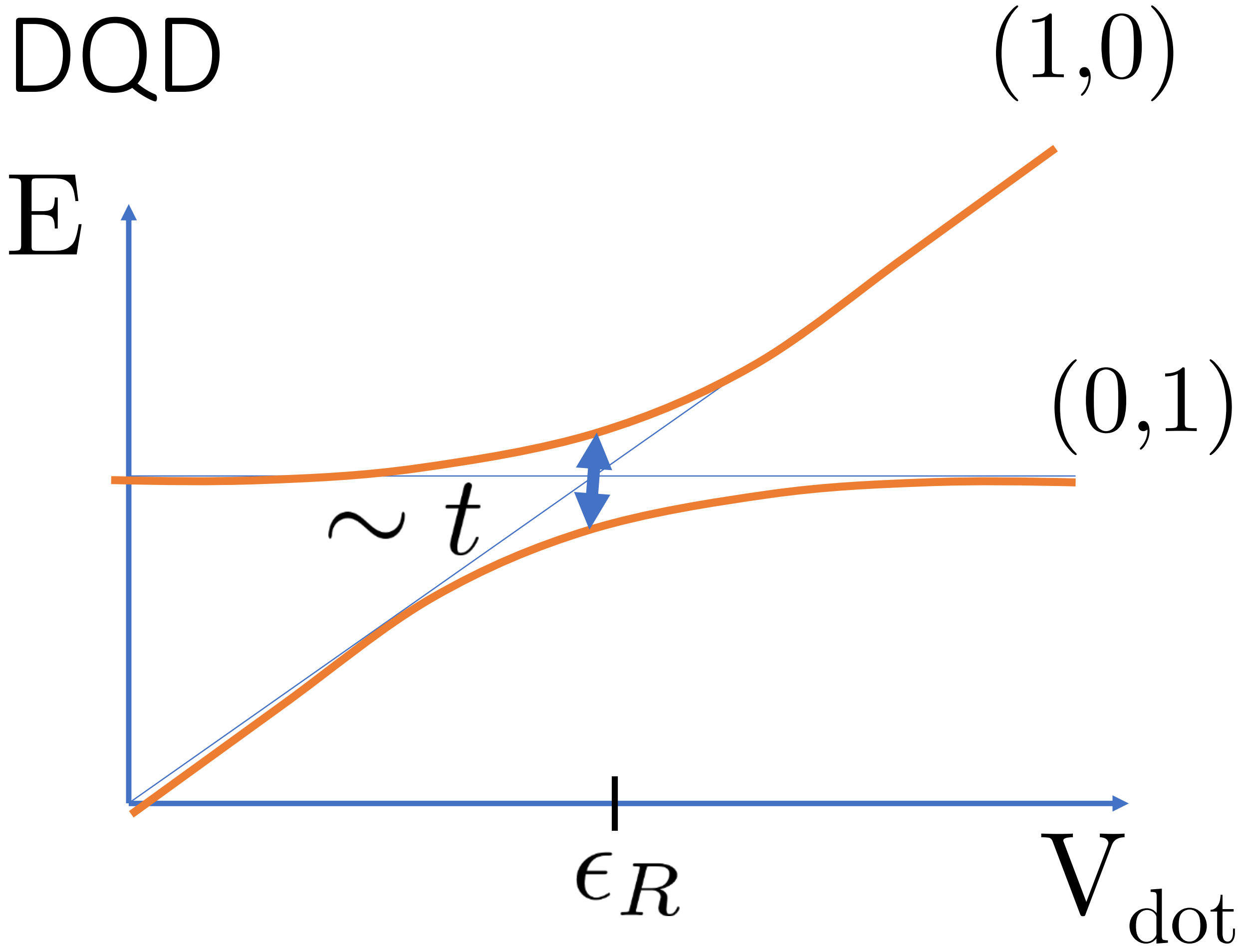
DQD



$$H = \begin{bmatrix} eV_{\mathrm{dot}} & t \\ t & \epsilon_R \end{bmatrix}$$



$$C_{\mathrm{Q}} = \frac{C_g^2}{C_2^2}\frac{dn_1}{dV_{\mathrm{dot}}}e$$

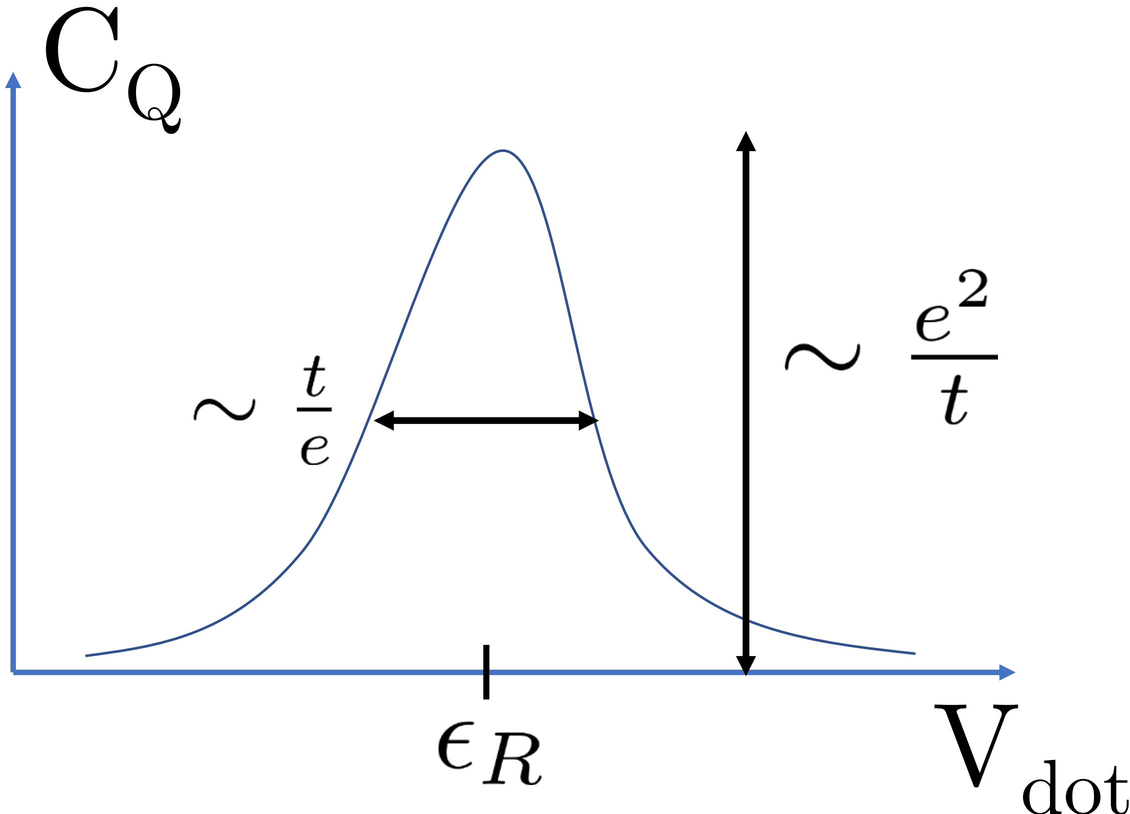


Excited state:

$$C_Q^{\text{ground}}(V) = -C_Q^{\text{excited}}(V)$$

Adiabatically follow the ground state
no excitation, termalization

$$C_{\mathrm{Q}} = \frac{C_g^2}{C_2^2}\frac{2t^2e^2}{(4t^2+(eV_{\mathrm{dot}}-\epsilon_R)^2)^{3/2}}$$



DQD result

Mesoscopic admittance of a double quantum dot

Audrey Cottet, Christophe Mora and Takis Kontos

Physical Review B 83, 121311(R) (2011)

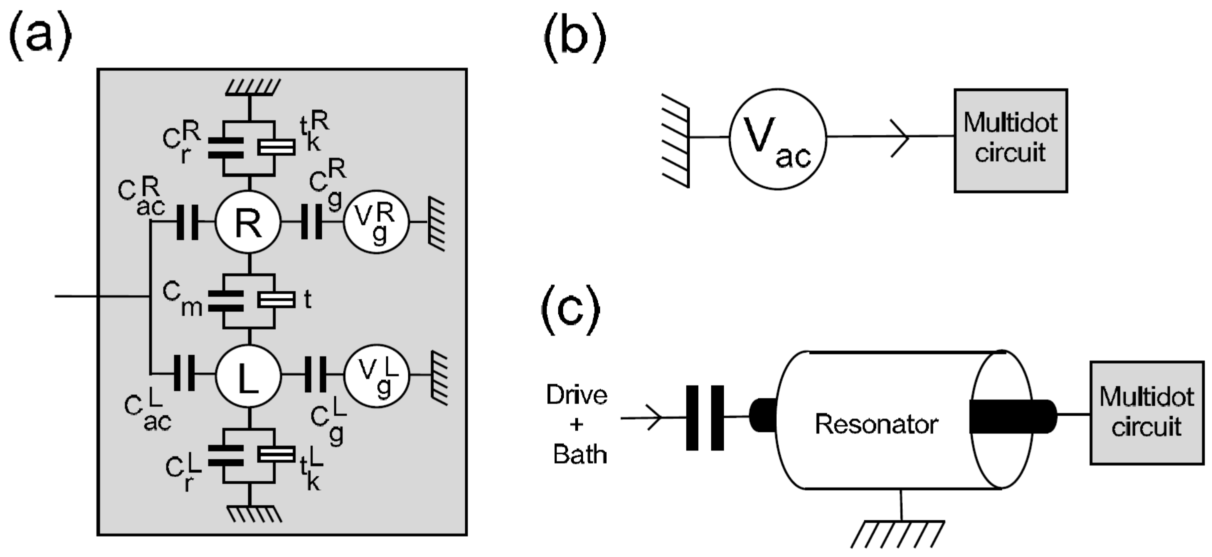
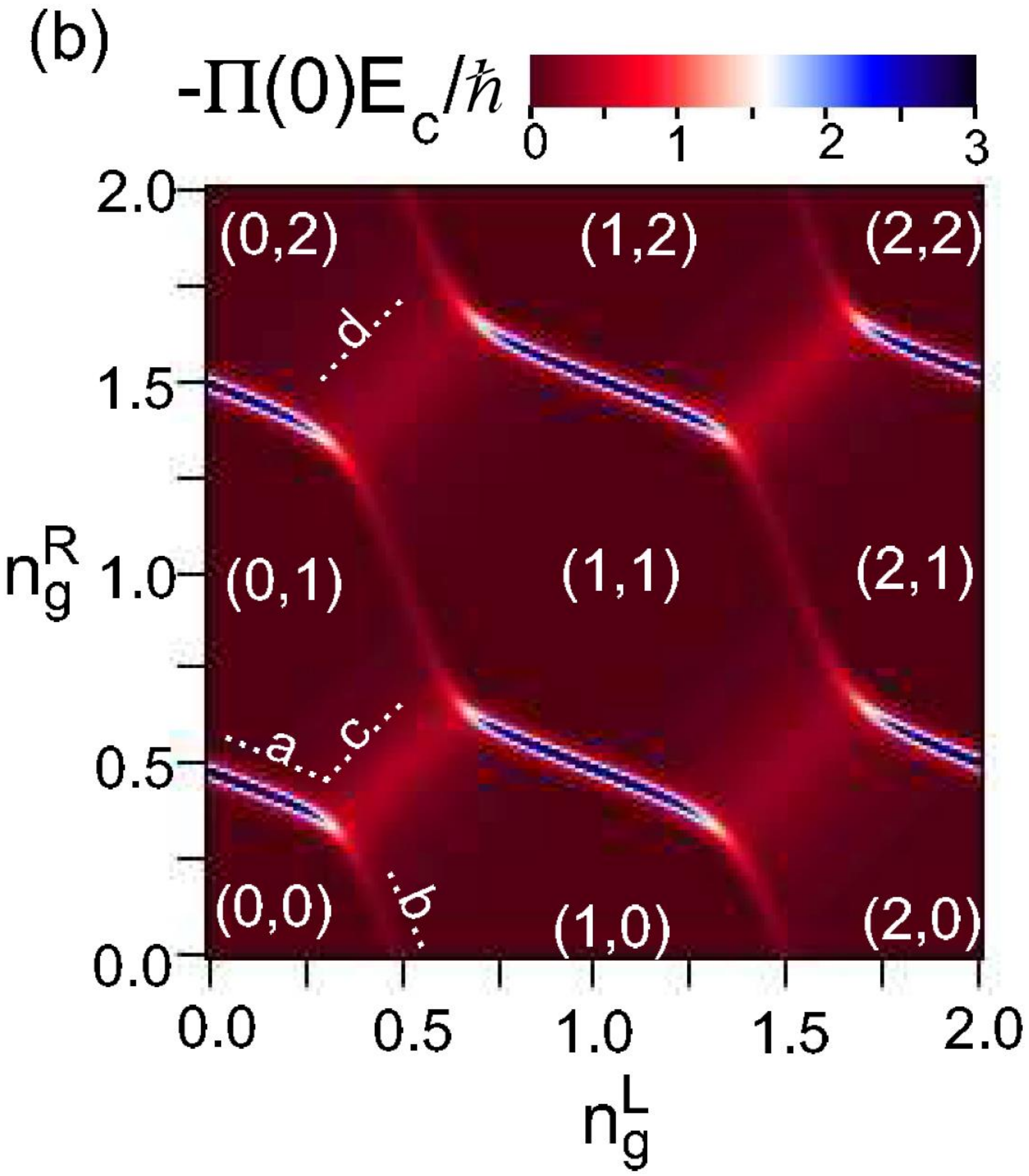


FIG. 1: (a) DQD circuit considered in this article (b) Configuration used for the measurement of the DQD mesoscopic admittance (c) Coupling scheme to a photonic resonator



Tunneling capacitance

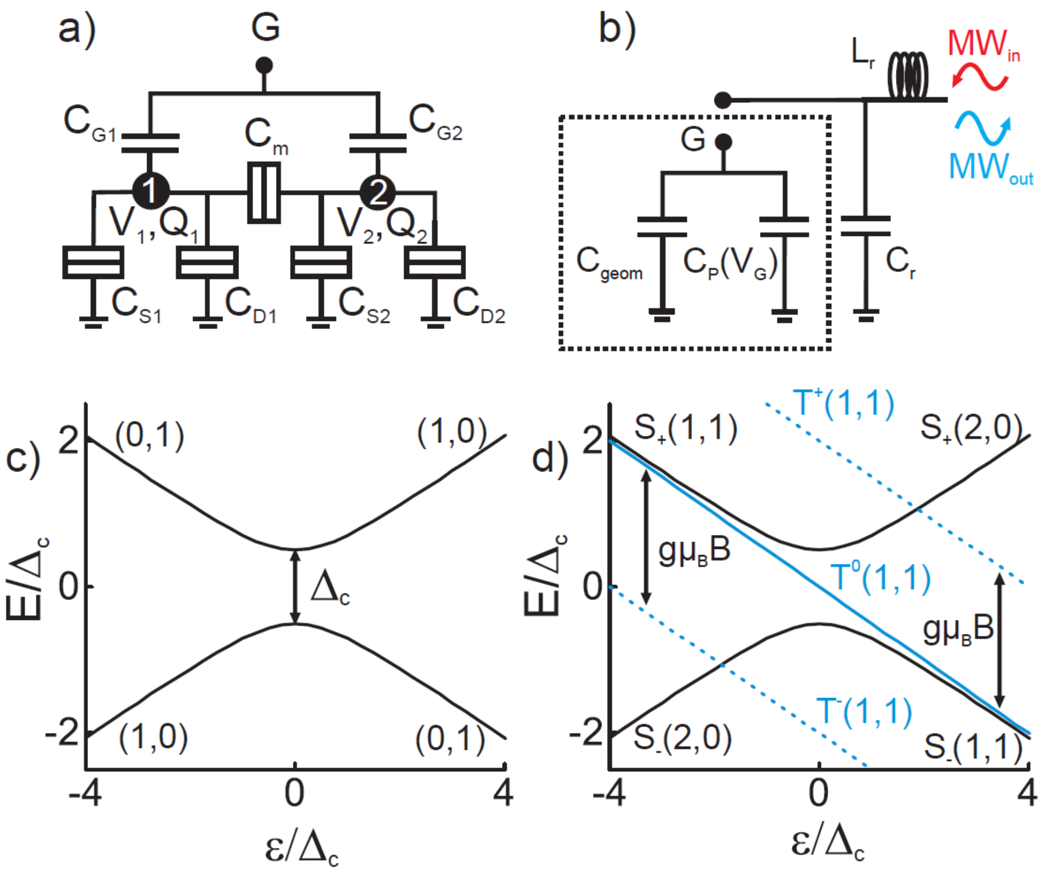
Quantum and Tunnelling Capacitance in Charge and Spin Qubits

R. Mizuta, R. M. Otxoa, A. C. Betz, and M. F. Gonzalez-Zalba

Phys. Rev. B 95, 045414 (2017)

P_+

P_-



Slow-relaxation regime

thermal population constant

$$C_p = C_0(P_- - P_+)\frac{\Delta_c^3}{(\varepsilon^2+\Delta_c^2)^{3/2}}$$

Fast-relaxation regime

instantanious thermal population

$$(P_- - P_+) = \tanh\left(\frac{\sqrt{(\varepsilon^2+\Delta_c^2}}{2k_BT}\right)$$

Tunneling capacitance

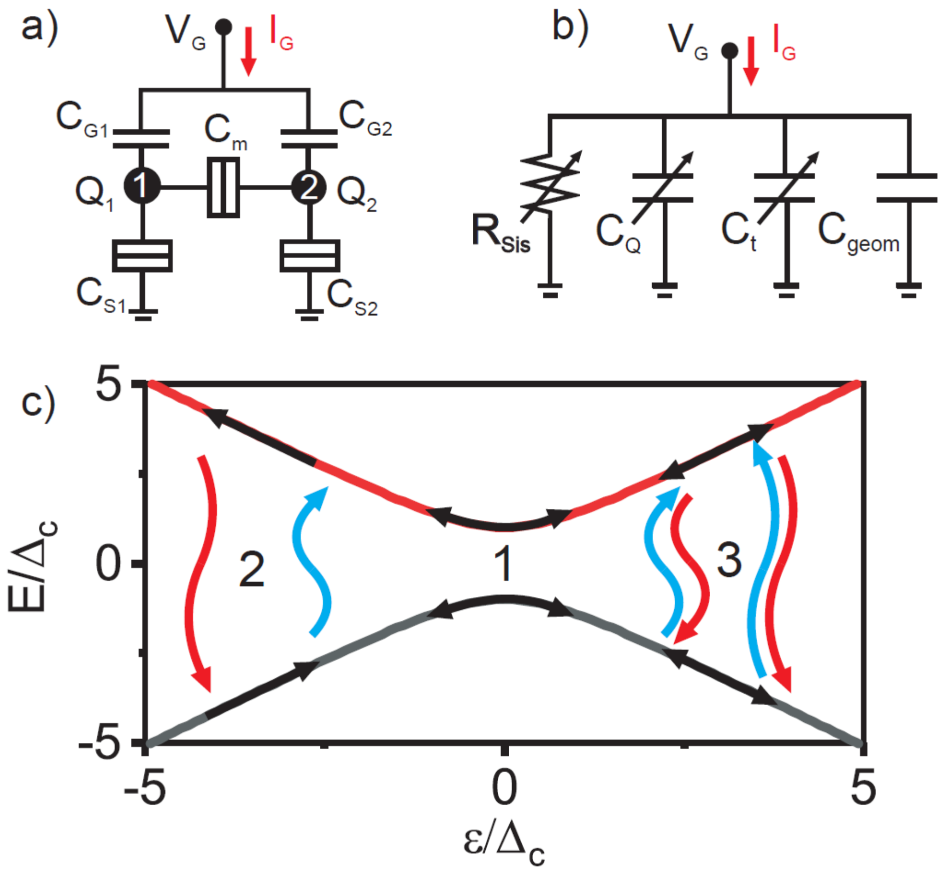
$$C_p = C_{\text{quantum}} + C_0\frac{\varepsilon\Delta_c}{\sqrt{\varepsilon^2+\Delta_c^2}}\frac{\partial(P_- - P_+)}{\partial\varepsilon}$$

Sisyphus resistance

Small-signal equivalent circuit for double quantum dots at low-frequencies

M. Esterli,[1] R. M. Otxoa,[1,2] and M. F. Gonzalez-Zalba[1]

arXiv:1812.06056

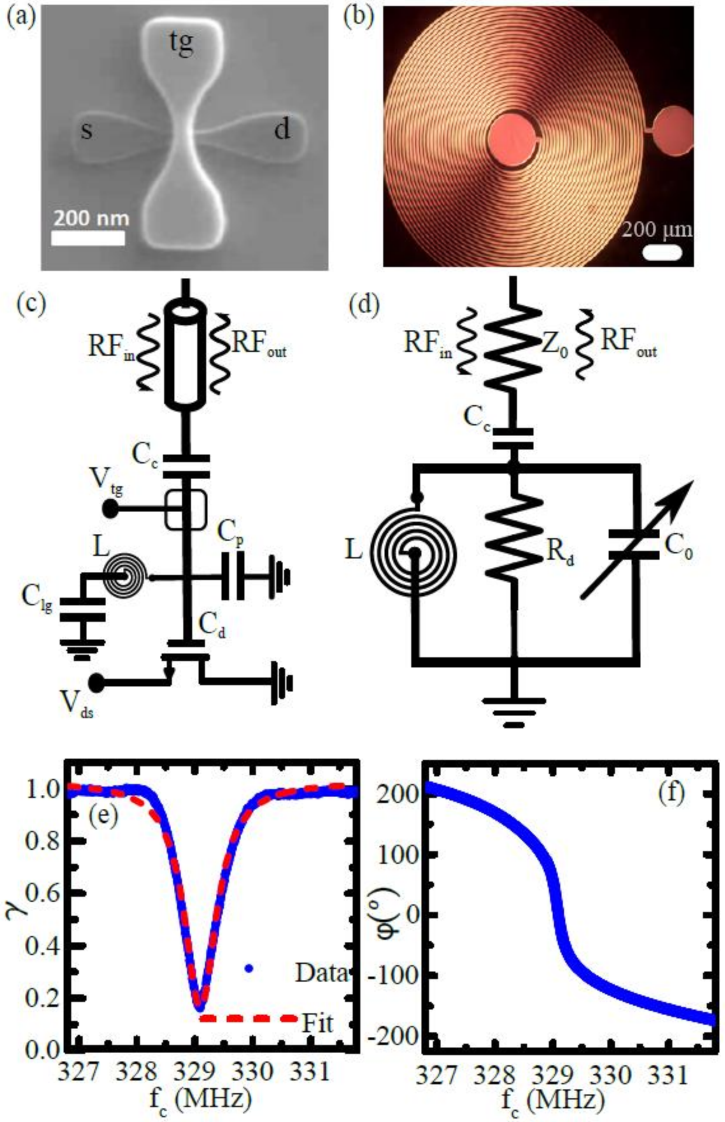


characteristic rate of relaxation: γ

$$C_t = \frac{(e\alpha')^2}{2}\frac{1}{2k_BT}\left(\frac{\varepsilon_0}{\Delta E_0}\right)^2\frac{\gamma^2}{\omega^2+\gamma^2}\cosh^{-2}(\Delta E_0/2k_BT).$$

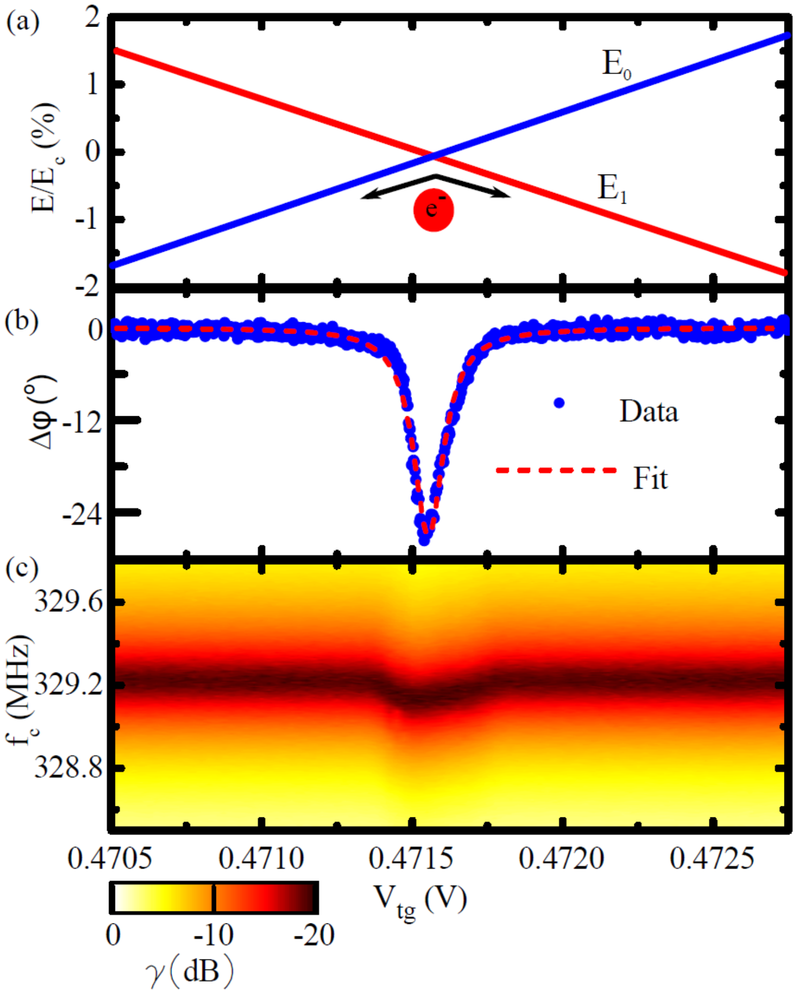
$$R_{Sis} = \frac{4R_Q}{\alpha'^2}\frac{k_BT}{h\gamma}\left(\frac{\Delta E_0}{\varepsilon_0}\right)^2\frac{\omega^2+\gamma^2}{\omega^2}\cosh^2(\Delta E_0/2k_BT).$$

Measuring quantum capacitance



Radio-frequency capacitive gate-based sensing

Imtiaz Ahmed,[1,*] James A. Haigh,[2] Simon Schaal,[3] Sylvain Barraud,[4] Yi Zhu,[5] Chang-min Lee,[5] Mario Amado,[5] Jason W. A. Robinson,[5] Alessandro Rossi,[1] John J. L. Morton,[3,6] and M. Fernando Gonzalez-Zalba[2,†]



$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\Delta f = -f_0 \frac{\Delta C}{2C}$$

$$\Delta \phi = -2Q \frac{\Delta C}{C}$$