Resonator-qubit coupling

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Qubit-resonator coupling

Jaynes-Cummings Hamiltonian

$$H = \frac{1}{2}\omega_q\sigma_z + \omega_r a^\dagger a + g(\sigma_+ a + \sigma_- a^\dagger)$$

- transverse qubit-resonator coupling, $\langle 0|\sigma_+|1\rangle \neq 0$
- high detuning, dispersive limit

$$H_{\mathrm{disp}} = \chi \hat{a}^\dagger \hat{a}\hat{\sigma}_z$$

$$[H_{\mathrm{disp}}, \hat{\sigma}_z] = 0$$

- more generally,

$$\hat{H}_\perp \approx \hbar g_\perp \left(\hat{a}\hat{\sigma}_q^+ + \hat{a}^\dagger \hat{\sigma}_q^-\right)$$

$$\hbar g_\perp = \mu_q \delta F_{\mathrm{rms}}$$

$$\mu_q \equiv \langle g\,|\hat{\mu}_q|\,e\rangle$$

Disadvantages, limitations

- $[H_{\mathrm{int}}, \hat{\sigma}_z] \neq 0$, H_{int} is QND only perturbatively
- Purcell decay $\gamma_k = (g/\Delta)^2\,\kappa$ (non-QNDness)
- must have $n \ll n_{\mathrm{crit}} = (\Delta/2g)^2$
  $\rightarrow n, \kappa$ is constrained
- sensitive to photon noise
- coupling is always-on, $\propto 1/\Delta$
- must have frequency-tunable qubits $\rightarrow$ noise sensitivity
- virtual photons $\rightarrow$ weak N-qubit interaction (limiting to pairwise entanglement)

N. Didier et al., Fast quantum non-demolition readout from longitudinal qubit-oscillator interaction, PRL 115, 203601 (2015)
P.-M. Billangeon et al., Circuit-QED-based scalable architectures for quantum information processing with superconducting qubits PRB 91, 094517 (2015)

Longitudinal coupling

Longitudinal coupling

- general form

$$\mathcal{H} = \omega_r\, a^\dagger a + \frac{\Delta}{2}\,\sigma_z + g\,\sigma_z(a^\dagger + a),$$

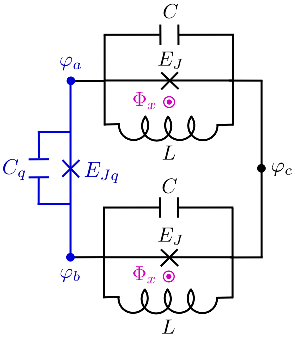
- diagonalized by (*Lang-Firsov transformation*)

$$U = e^{-\theta\sigma_z(a^\dagger - a)}, \qquad \theta = g/\omega_r$$

$$\mathcal{H}' = \omega_r\, a^\dagger a + \frac{\Delta}{2}\sigma_z - \frac{g^2}{\omega_r}1.$$

- no Purcell decay
- exact result
- no dispersive shift
- read-out, single-qubit, multi-qubit gates?
- scalability?

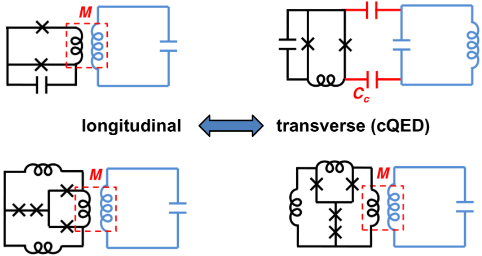
S. Richer et al, Circuit design implementing longitudinal coupling: a scalable scheme for superconducting qubits, PRB 93, 134501 (2016)



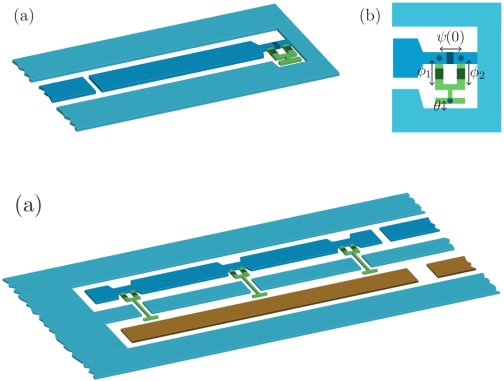
- phase

$$\varphi_i = \left(\frac{2\pi}{\Phi_0}\right)\Phi_i = \left(\frac{2\pi}{\Phi_0}\right)\int_{-\infty}^{t} V_i(t')\,dt'.$$

Examples



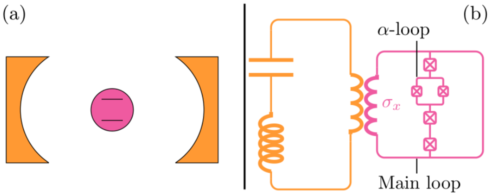
A. J. Kerman, Quantum information processing using quasiclassical electromagnetic interactions between qubits and electrical resonators, New J. Phys. 15 123011 (2013)



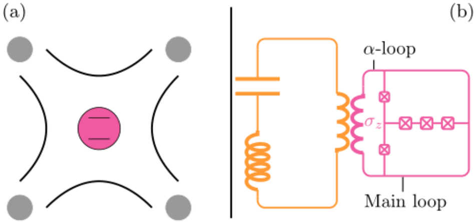
Longitudinal coupling to readout-resonator, transverse to bus resonator

N. Didier et al., Fast quantum non-demolition readout from longitudinal qubit-oscillator interaction, PRL 115, 203601 (2015)

Examples



Transverse coupling



Longitudinal coupling

P.-M. Billangeon et al., Circuit-QED-based scalable architectures for quantum information processing with superconducting qubits, PRB 91, 094517 (2015)

Examples

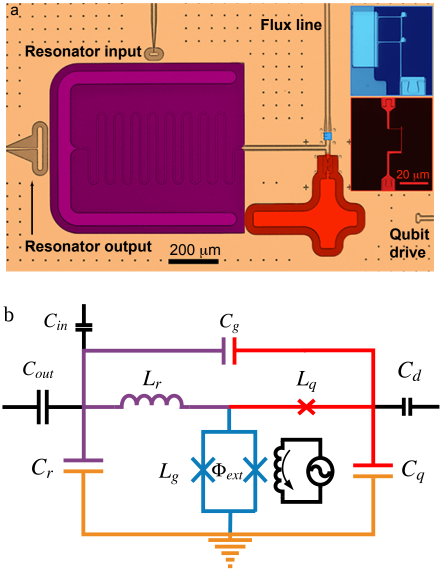
- lumped element resonator
- coupling tunable with Φ_{ext}
- transverse coupling

$$\hat{H} = \omega_r \hat{a}^\dagger \hat{a} + \frac{\omega_q}{2}\hat{\sigma}_z - g_r(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+) - g_b(\hat{a}^\dagger\hat{\sigma}^+ + \hat{a}\hat{\sigma}^-),$$

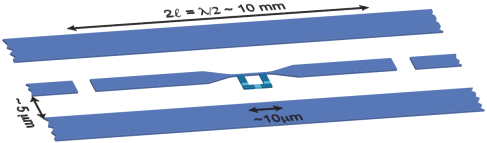
where

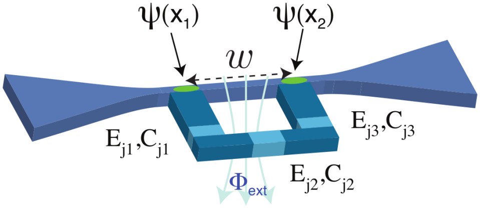
$$g_{r,b} = \frac{L_{g0}}{2\,|\cos(\pi\Phi_{\text{ext}}/\Phi_0)|}\sqrt{\frac{\omega_r\omega_q}{L_r L_q}} \mp \frac{C_g}{2}\sqrt{\frac{\omega_r\omega_q}{C_r C_q}}$$

Yao Lu et al., Universal stabilization of a parametrically coupled qubit, PRL 119, 150502 (2017)



Examples





Direct coupling, proportional to the shared inductance

Flux noise sensitivity scales with the area

Transverse coupling at the sweet spot

J. Bourassa et al., Ultrastrong coupling regime of cavity QED with phase-biased flux qubits, Phys. Rev. A 80, 032109 (2009)

Thank you for your attention!

Qubit-qubit coupling

- Qubit Hamiltonian

$$H = \omega_q a^\dagger a + \frac{\alpha}{2} a^\dagger a^\dagger a a \qquad \left(\to H = \omega_q \frac{\sigma_z}{2}\right)$$

- Transmons, capacitive qubit-qubit coupling

$$H_{\text{int}} = C_g V_1 V_2$$

- For $C_g \ll C_1, C_2$, $V_i = (2e/C_i) n_i \propto i(a_i - a_i^\dagger)$
- Full Hamiltonian

$$H = \sum_{i \in 1,2} \left[\omega_i a_i^\dagger a_i + \frac{\alpha_i}{2} a_i^\dagger a_i^\dagger a_i a_i\right] - g\left(a_1 - a_1^\dagger\right)\left(a_2 - a_2^\dagger\right)$$

- coupling is transverse:
  ${}_i\langle k|a_i - a_i^\dagger|k\rangle_i = 0$
  ${}_i\langle k \pm 1|a_i - a_i^\dagger|k\rangle_i \neq 0$
- two-level approximation

$$H = \sum_{i \in 1,2} \frac{1}{2}\omega_i \sigma_{z,i} + g\sigma_{y,1}\sigma_{y,2}$$

- longitudinal qubit-qubit coupling:

$$H = \sum_{i=1,2} \frac{1}{2}\omega_i \sigma_{zi} + g\sigma_{z1}\sigma_{z2}.$$

  e.g. inductively coupled flux qubits at the degeneracy point
- qubit-resonator coupling

$$H = \frac{1}{2}\omega_q \sigma_z + \omega_r a^\dagger a + g(\sigma_+ a + \sigma_- a^\dagger)$$

P. Krantz et al., A Quantum Engineer's Guide to Superconducting Qubits, Applied Physics Reviews 6, 021318 (2019)